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The vector optimization mechanism for resource planning problems

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Abstract. The mechanism of two-criteria optimization for the resource planning problem is proposed, where the formalization of the problem can be represented by the direct and inverse control problem with a structurally similar description of the objective and cost functions. The set of Pareto-solutions of the two-criteria problem is determined by solution of the direct and inverse problems by the method of dichotomous programming, and the optimum on the set of Pareto-solutions – by the mechanism of complex estimation.

1. Introduction

Many problems of optimal resource planning, solved by industrial companies, are formulated in the form of a scalar direct or inverse control problem [1,2]. Let us consider, for example, the task of planning resources for training personnel operating the ERP system at a mining company [3].

Let $\{\{p_{ji} \mid i=\overline{1,n_j}\} \mid j=\overline{1,m}\}$ be a set of training programs implemented by a consulting company involved in the creation of the ERP-system. Here j – the number of a business process, i – the number of the training program, p_{ji} – the i-th training program for the j-th business process, n_j – the number of training programs for the j-th process, m – the number of business processes.

Let us denote the cost of training of one user within the program by $c_{ji} = c(p_{ji})$, the "expansion" of the user's competence (which we will estimate in points) as a result of training within the program p_{ji} – by $q_{ji} = q(p_{ji})$, the number of users that it is desirable to train within the program p_{ji} – by $k_{ji} = k(p_{ji})$. Let us also assume that the management of each business process imposes a restriction on the minimum number of trained users k_j^*, which allows the initial operation to be started. Let us denote the maximum limit of funds, that the management of a company can spend on the users training, by c^*.

2. The problem formalization and и the solution method

We will introduce a discrete variable x_{ji}, which is equal to 1, if users are to be trained in accordance with the program p_{ji}, and equal to 0 otherwise. Then the direct problem of training costs optimal

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planning is formulated as follows:

$$q = \sum_{j=1}^{m} \sum_{i=1}^{n_j} k_{ji} q_{ji} x_{ji} \to \max \tag{1}$$

$$\sum_{j=1}^{m} \sum_{i=1}^{n_j} k_{ji} c_{ji} x_{ji} \le c^* \tag{2}$$

$$\sum_{i=1}^{n_j} k_{ji} x_{ji} \ge k_j^*, \quad j = \overline{1,m}. \tag{3}$$

The solution of problem (1) – (3) is a user training program $\{\{x_{ji} \mid i = \overline{1,n_j}\} \mid j = \overline{1,m}\}$ that maximizes the total "expansion" of users competencies q with a given restraint c^* for the maximum funds limit on allocated for training and that meets the requirements set by the managers of the business processes for the minimum number of users to be trained.

The opposite to the formulated problem (1) – (3) will be the problem:

$$c = \sum_{j=1}^{m} \sum_{i=1}^{n_j} k_{ji} c_{ji} x_{ji} \to \min \tag{4}$$

$$\sum_{j=1}^{m} \sum_{i=1}^{n_j} k_{ji} q_{ji} x_{ji} \ge q^* \tag{5}$$

$$\sum_{i=1}^{n_j} k_{ji} x_{ji} \ge k_j^*, \quad j = \overline{1,m}., \tag{6}$$

in which it is required to define such training program for users $\{\{x_{ji} \mid i = \overline{1,n_j}\} \mid j = \overline{1,m}\}$, which minimizes the costs c to achieve the given value of the user competence development q *.

Because of the structural similarity of functions c and q, both problems can be effectively solved by the method of dichotomous programming [4,5,6]. We should note the following important feature of this method. When solving each of the sequence of evaluation problems into which the initial problem is broken (because of this structural similarity), a subset of solutions are formed that are permissible by constraints and are Pareto optimal, that is, they are not dominated by other solutions of this set [7,8]. The set of solutions of the last estimated problems of the direct and inverse problem, due to the noted peculiarity, is a subset of Pareto solutions of the vector optimization problem:

$$(q, c) \to \max \tag{7}$$

$$\sum_{i=1}^{n_j} k_{ji} x_{ji} \ge k_j^*, \quad j = \overline{1,m}. \tag{8}$$

Based on their preferences, the decision maker chooses the best option from this subset. The procedure for determining the optimum on a subset of Pareto solutions can be formalized by applying the mechanism of complex estimation widely used in the theory of management of organizational systems [9, 10].

3. An example of a problem solving and results presentation

Let us consider the case with three business processes (m = 3) and, respectively, with three $(n_1 = 3)$, two $(n_2 = 2)$ and two $(n_3 = 2)$ training programs for these processes. The initial data for the training programs are given in table 1.

Table 1. The initial data of training programs.

	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{31}	p_{32}
q_{ji}	4	3	5	3	4	5	3
k_{ji}	3	2	3	4	3	4	2
c_{ji}	60	64	90	54	90	90	54
	$k_1 \geq k_1^* = 5$			$k_2 \geq k_2^* = 3$		$k_3 \geq k_3^* = 2$	

We will take for the direct problem $c^* = 1100$, and for the inverse problem $q^* = 55$.

Solving the direct and inverse problem by the method of dichotomous programming, we obtain, respectively, the Pareto solutions presented in tables 2 and 3.

Table 2. Pareto optimal solutions of the direct problem.

q_{123}	45	51	39	36	<u>59</u>	50	53	56
k_{123}	12	14	11	11	<u>14</u>	13	13	15
c_{123}	774	902	722	632	<u>1026</u>	884	974	992
x_1	101	111	011	110	<u>101</u>	110	011	110
x_2	10	10	10	10	<u>10</u>	10	10	10
x_3	01	01	01	01	<u>10</u>	10	10	11

Table 3. Pareto optimal solutions of the inverse problem.

q_{123}	83	77	71	65	59	<u>56</u>
k_{123}	21	19	18	16	14	<u>15</u>
c_{123}	1532	1404	1262	1134	1026	<u>992</u>
x_1	1 1 1	1 0 1	1 1 1	1 0 1	1 0 1	<u>1 1 0</u>
x_2	1 1	1 1	1 0	1 0	1 0	<u>1 0</u>
x_3	1 1	1 1	1 1	1 1	1 0	<u>1 1</u>

The optimal solutions of the direct and inverse problems are underlined.

By uniting the solutions of both problems and excluding those that are dominated by others, we obtain the Pareto solution of the vector optimization problem, table 4 (a and b), figure 1.

Table 4a. Union of Pareto solutions for the direct and inverse problem.

q_{123}	83	77	71	65	59	56

k_{123}	21	19	18	16	14	15
c_{123}	1532	1404	1262	1134	1026	992
x_1	1 1 1	1 0 1	1 1 1	1 0 1	1 0 1	1 1 0
x_2	1 1	1 1	1 0	1 0	1 0	1 0
x_3	1 1	1 1	1 1	1 1	1 0	1 1

Table 4b. Union of Pareto solutions for the direct and inverse problem.

q_{123}	53	51	50	45	39	36
k_{123}	13	14	13	12	11	11
c_{123}	974	902	884	774	722	632
x_1	0 1 1	1 1 1	1 1 0	1 0 1	0 1 1	1 1 0
x_2	1 0	1 0	1 0	1 0	1 0	1 0
x_3	1 0	0 1	1 0	0 1	0 1	0 1

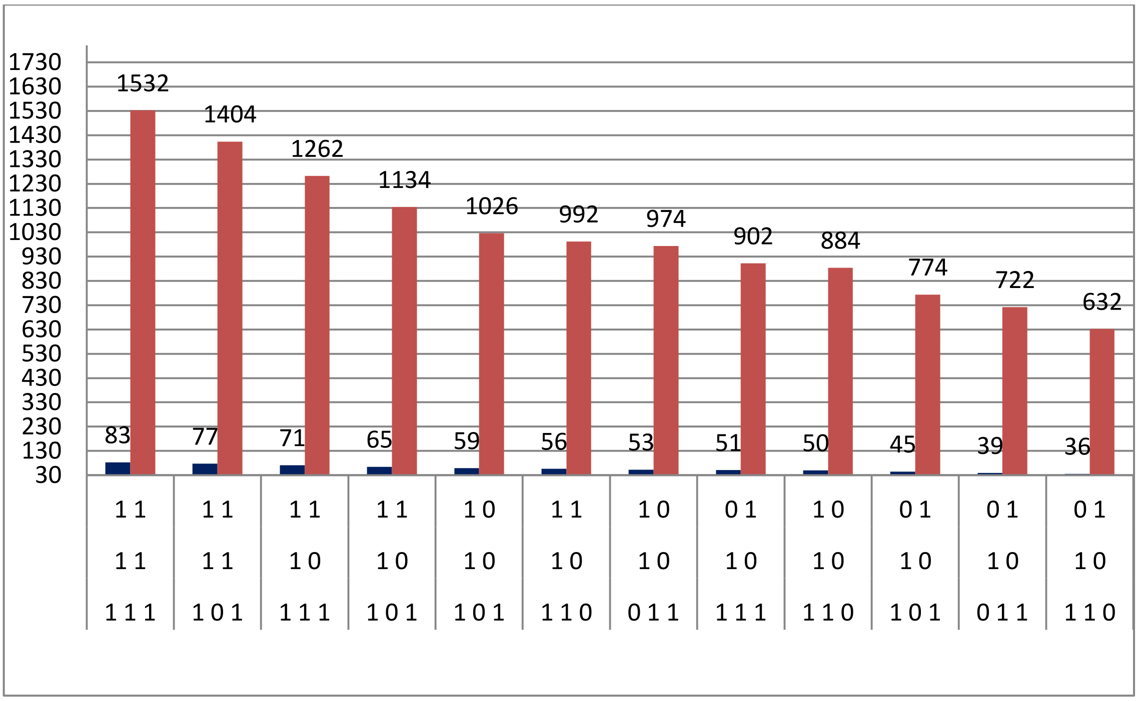

Figure 1. Histogram describing the set of Pareto-solutions to the vector optimization problems.

To formalize the choice of the best Pareto-solution by the decision make, we will apply the mechanism of complex estimation, which, in the annex to the problem under consideration, will include the following procedures:

1. Bringing the scales for measuring local efficiency indicators q and c to a single discrete scale of a given rank R (we assume R = 100):

1.1 Defining the measurement ranges q and c (in our example: q_{min}=36, q_{max}=83, then $q \in [35,85]$; c_{min}=632, c_{max}=1532, we assume $c \in [630,1540]$).

1.2 Defining the lengths of intervals q and c per one point of the measurement scale.

$$\delta_q = \frac{85-35}{100} = 0.5, \qquad \delta_c = \frac{1540-630}{100} = 9.1.$$

2. Calculation of values $q^{á}(q)$ è $c^{á}(ñ)$ in the new measurement scale (the direct scale of measurement for $q^{á}(q)$, the inverse scale for $c^{á}(ñ)$).

3. Calculation of the average values of the integral indicator $(q,c)^{á} = \alpha_q q^{á} + \alpha_ñ ñ^{á}$ describing the effectiveness of the solution, obtained on the basis of additive convolution of local indicators and their relative weights given by the person making the decision ($\alpha_q + \alpha_c$=1).

Table 5 shows the results of converting the values of the indicators in table 4 from the natural scales of measurement to a single 100-point scale (the integrated indicator $(q,c)^{á} = \alpha_q q^{á} + \alpha_ñ ñ^{á}$, ($\alpha_q = 0{,}34$ è $\alpha_c = 0{,}66$) is in the place of indicator k).

Table 5a. Evaluation of Pareto solutions in points.

q^{b}_{123}	96	84	72	60	48	42
$(q,c)^{b}$	98.64	85.32	71	57	45.36	40.68
c^{b}_{123}	100	86	70	56	44	40
x_1	1 1 1	1 0 1	1 1 1	1 0 1	1 0 1	1 1 0
x_2	1 1	1 1	1 0	1 0	1 0	1 0
x_3	1 1	1 1	1 1	1 1	1 0	1 1
No.	1	2	3	4	5	6

Table 5a. Evaluation of Pareto solutions in points.

q^{b}_{123}	36	32	30	20	8	2
$(q,c)^{b}$	37.32	30.68	28.68	17.36	9.98	1.34
c^{b}_{123}	38	30	28	16	11	1
x_1	0 1 1	1 1 1	1 1 0	1 0 1	0 1 1	1 1 0
x_2	1 0	1 0	1 0	1 0	1 0	1 0
x_3	1 0	0 1	1 0	0 1	0 1	0 1
No.	7	8	9	10	11	12

Figure 2 shows a histogram describing the Pareto solutions obtained in a single point scale for q and c.

Solutions 3 and 4 are of interest. The third solution (x_1 = 111, x_2 =10, x_3 =11) is the most balanced and is characterized by the fact that spending 70% of the resource maximum we get 72% of the maximum possible result. The fourth solution (x_1 = 101, x_2 =10, x_3 =11) is the best in terms of resource efficiency: 56% of resources provide 60% of the value q (that is, 4% more than resource consumption).

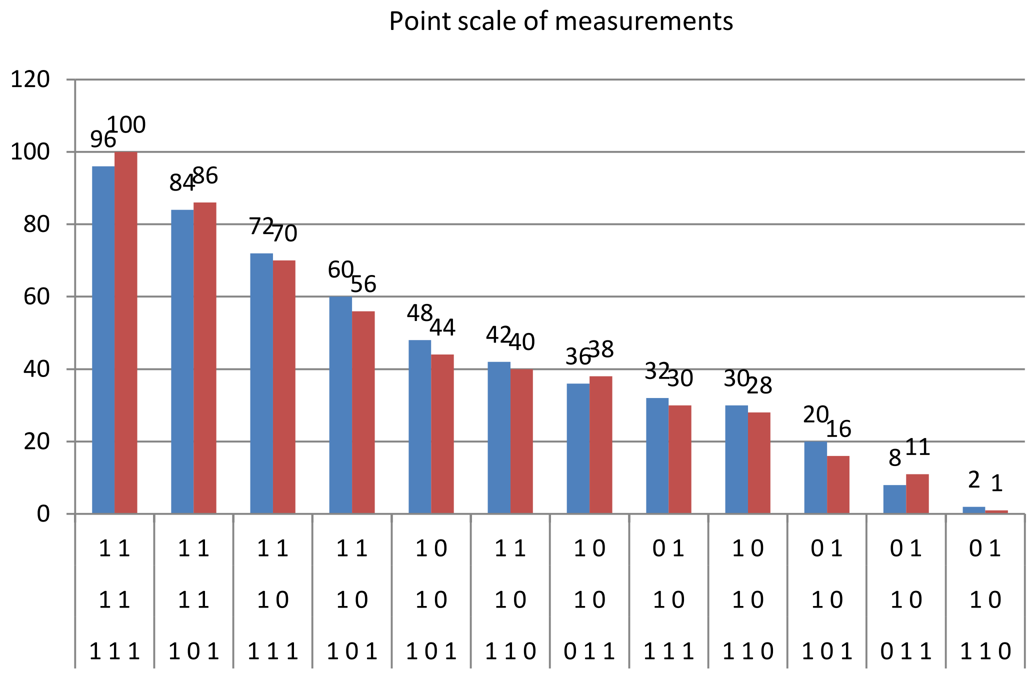

Figure 2. A histogram describing the set of Pareto solutions for the vector optimization problem in a single point scale of measurements.

4. Conclusion

The two-criteria formulation of the resource planning problem is preferable for the decision-maker, as he usually searches for a compromise between costs and the objective indicator. The stated mechanism of two-criteria optimization in many cases allows the decision maker to find such a compromise.

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