

Transient and Optimization Analysis of Vibration Isolation Device Based on Geometric Nonlinear Theory

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Abstract. The vibration isolation device based on geometric nonlinear theory is a hotspot in the field of dynamics in recent years. In order to deeply explore the application of this technology in the field of vibration isolation for power grid equipment, we establish a transformer-casing coupling nonlinear dynamic model considering seismic wave excitation, and use the classical fourth-order Runge-Kutta method to calculate and analyse the transient response behaviour and characteristics of the coupled system under different conditions. The analysis results reveal that the influence of the three parameters, including stiffness, length and damping of springs in shock absorber, on the anti-seismic performance of this coupling structure, which lays a theoretical foundation for the application of geometric nonlinear theory in the field of large-scale transformation equipment vibration isolation.

1. Introduction

With the development of power grid technology in China in recent years, the voltage level has been continuously improved. The isolation problem of high-precision and large-volume equipment represented by transformers has always been an important field of concern for scholars [1,2]. Most of the isolation equipment for traditional transformers adopt linear principle. Due to the different bearing capacity of linear springs, and often have certain characteristics of the main frequency interval, under the complex vibration excitation with time-varying amplitude and frequency characteristics caused by earthquakes, the separation is shown. The effect of the earthquake is often unsatisfactory. In some cases, it will even show a magnifying effect, which will aggravate the damage to the equipment.

Geometric nonlinearity refers to the phenomenon that the external force and the induced displacement do not have a simple linear relationship. Because of its broad application prospects, it has been discovered and deeply explored by researchers as a hot direction. Cao[3,4] proposed a geometric nonlinear SD oscillator theory. By introducing the smoothing parameter α , the transition process and characteristics of the SD-containing oscillator subsystem and the corresponding dynamic behaviour of the corresponding attractors are analysed in detail. On the basis of this theory, Zhang [5] established a quasi-zero stiffness model of transformer-SD oscillator, and Zhu [6] constructed a three-degree-of-freedom vibration isolation structure with more complex motion directions in three-dimensional infinite coupling. The theoretical application and analysis of the large-scale transformer equipment isolation field have been carried out. The results show that the theoretical isolation indicators exhibited by the



isolation devices designed by them are better than the traditional isolation equipment, no matter for the peak or average of the acceleration response, as well as the initial response frequency.

It should be noted that the transformer is not a simple mass or rigid body structure, and the upper casing acts as an additional long flexible structure, which also has a significant impact on the response of the system. However, the analytical models in the existing literature have not considered. In this paper, based on the above-mentioned work, the geometric nonlinear theory is modelled in the transformer-casing coupling system, and the nonlinearity describing the coupled motion of rigid body and long flexible structure is obtained. The dynamic equations are used to analyse the influence of the three parameters of the damper spring stiffness, length and damping on the transient response behaviour of the above-mentioned coupling mechanism under the excitation of the measured seismic wave. The optimization goal is proposed and the corresponding optimization scheme is proposed, which lays a theoretical foundation for the application of geometric nonlinear theory in the field of vibration isolation of large-scale substation equipment.

2. Coupling Mechanism Modelling Based on Geometric Nonlinear Theory

Transformer is a large-scale equipment commonly used in power grid systems. The structure is shown in Figure 1.



Figure 1. Transformer and casing above it (including corona ring)

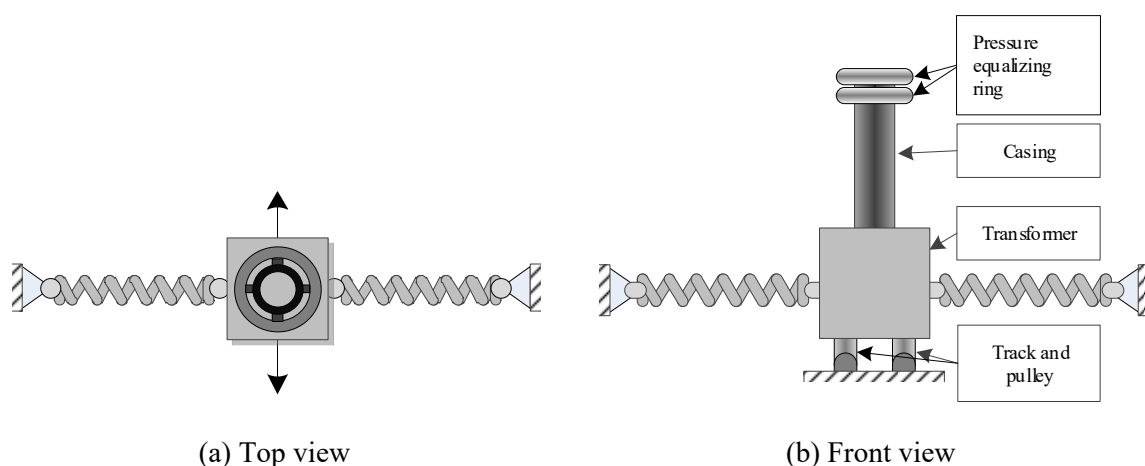


Figure 2. Actual installation of transformer and its vibration isolation device

According to the research method and results of the pre-zero-frequency vibration isolation of the transformer, considering the actual structure of the transformer, the actual installation effect of the vibration isolation device is as Figure 2.

During the reciprocating motion as arrows shown in Figure 2(a), the transformer itself can be regarded as a mass point, and the casing on the transformer can be regarded as a cantilever beam with a circular section, and the corona ring is fixed at Upper end of the casing. When the low frequency first-order mode is considered, the corona ring can be regarded as the additional modal mass.

Relative to the fixed point far away, the absolute displacement of the transformer is set to $x(t)$, the absolute acceleration function of the foundation is set to $y''(t) = \text{ACC}(t)$ in the earthquake state; the relative displacement of the transformer is $z(t)$ relative to the base, and the lower part of the casing is fixed with the transformer, and the displacement of a point relative to the transformer is $w(s, t)$, and the absolute displacement along the axis is the absolute displacement $u(s, t)$, where s is the longitudinal projection distance between fixed end and the section, and t is time.

At this moment, the kinetic energy T , potential energy V and virtual work W_c (variational form) of the coupling system in vibration process can be expressed as:

$$\begin{aligned} T &= mx'(t)^2 / 2 + m_1 \left(u'(s, t)^2 + (w'(s, t) + x'(t))^2 \right) \Big|_{s=l_1} / 2 + \rho A \int_0^{l_1} (w'(s, t) + x'(t) + u'(s, t))^2 ds / 2 \\ V &= k_s z(t)^4 / 4 + EA \int_0^{l_1} (\dot{u} + \dot{w}^2 / 2)^2 ds / 2EI \int_0^{l_1} \ddot{w}^2 ds / 2 \\ \delta W_c &= \int_0^{l_1} c_1 u'(s, t) ds \delta u + \int_0^{l_1} c_2 w'(s, t) ds \delta w + c_3 z'(t) \delta z \end{aligned} \quad (1)$$

Where: m is the overall mass of the transformer (including pulley); $A\rho$ is the line density of casing; m_1 is the mass of corona ring; l_1 is the length of casing; k_s is the cubic stiffness coefficient of vibration isolation device; EA and EI are the tension and compression and respectively bending stiffness of casing, respectively; c_1 , c_2 , and c_3 are the material viscous damping of the casing transverse, longitudinal, and spring. The corner "'" stands for the first derivative of t ; the superscript "." stands for the first derivative of s .

According to the relationship of motion vectors:

$$x(t) = z(t) + y(t) \quad (2)$$

$$\text{and Hamilton principle: } \int_0^t (\delta T - \delta V + \delta W_c) dt = C \quad (3)$$

For each formula of the (1), by taking variation on them with respect to the variation of t , variational formulations are obtained and then could be substituted into the above(3), and:

$$\begin{aligned} \int_0^t & \left((-m(z'' + y'') - \rho A l_1 (z'' + y'') k_s z^3 - c_2 z') \delta z + \left(-\rho A \int_0^{l_1} W'' ds - \right. \right. \\ & \left. \left. \rho A l_1 (z'' + y'') + EI \int_0^{l_1} \ddot{W} ds - m_1 W'' \Big|_{s=l_1} - c_1 \int_0^{l_1} W' ds \right) \delta w \right) dt = 0 \end{aligned} \quad (4)$$

So, separate the correlation terms and get the equation with two degrees of freedom coupling:

$$\begin{aligned} m(z'' + y'') + \rho A l_1 (z'' + y'') + c_2 z' - k_s z^3 &= 0 \\ \rho A \int_0^{l_1} W'' ds + \rho A l_1 (z'' + y'') - EI \int_0^{l_1} \ddot{W} ds + c_1 \int_0^{l_1} W' ds + m_1 W'' \Big|_{s=l_1} &= 0 \end{aligned} \quad (5)$$

According to the boundary conditions of the cantilever beam:

$$w|_{s=0} = 0; \frac{\partial w}{\partial s} \Big|_{s=0} = 0; \frac{\partial^2 w}{\partial s^2} \Big|_{s=l_1} = 0; EI \frac{\partial^3 w}{\partial s^3} \Big|_{s=l_1} = 0; \quad (6)$$

Set the first order vibration mode function as the following form:

$$W_i(s, t) = w_i(t) \phi_i(s) \quad (7)$$

$$\text{Where: } \phi_i(s) = \left(\cos(\beta_i s) - \cosh(\beta_i s) \right) - \frac{\cos(\beta_i l_1) + \cosh(\beta_i l_1)}{\sin(\beta_i l_1) + \sinh(\beta_i l_1)} (\sin(\beta_i s) - \sinh(\beta_i s)) \quad (8)$$

Where: modal order $i = 1, 2, 3, \dots$. According to Euler–Bernoulli beam vibration theory of cantilever beam vibration, there is an i -th order mode:

$$\beta_1 l_i = 1.8751; \beta_2 l_i = 4.6941; \beta_3 l_i = 7.8548 \dots \quad (9)$$

Substituting (7) and (9) into (5), and using the Galerkin method to discretize the first modal vibration of (5), equations with decoupling and simplification forms can be obtained:

$$\begin{aligned} z'' + \frac{c_2}{m + A\rho l_1} z' - \frac{k_s}{m + A\rho l_1} z^3 = -ACC \\ w'' - \frac{9.68EI}{l_1^3 (0.78A\rho l_1 + 2.55m_1)} w - \frac{A\rho l_1 c_2}{(0.78A\rho l_1 + 2.55m_1)(m + A\rho l_1)} z' - \\ \frac{A\rho l_1 k_s}{(0.78A\rho l_1 + 2.55m_1)(m + A\rho l_1)} z^3 + \frac{c_1 l_1}{(A\rho l_1 + 3.69m_1)} w' = 0 \end{aligned} \quad (10)$$

It can be observed from (10) that the coupling between the transformer and the casing elastomer is not particularly obvious after linear partial decoupling. The reason is that the transformer is a rigid body, and the acceleration excitation term $ACC(t)$ from the ground seismic wave can not directly affect the vibration of the casing, but the passive response generated by the transformer, the source of indirect excitation, has obvious influence on reciprocating motion of the casing (relative to the transformer). so the direction of energy flow is shown below.



Figure 3. Energy flow

Using the classical dynamics method, the fourth-order Runge-Kutta method, to solve the equations (10) numerically, the time domain solutions of $z(t)$ and $w(t)$ can be obtained. In order to characterize the efficiency and effectiveness of the isolation scheme, the maximum acceleration transfer rate T_z of the transformer is the first optimization function, and the maximum acceleration transfer rate T_w of the casing is the second optimization objective function (see equation (11) below). Three parameters of the spring, damper coefficient, stiffness and length, can be optimized and analyzed to obtain their best value.

$$T_z = \frac{\max(Abs(z''(t)))}{\max(Abs(ACC(t)))}; T_w = \frac{\max(Abs(w''(t)))}{\max(Abs(ACC(t)))} \quad (11)$$

3. Numerical example

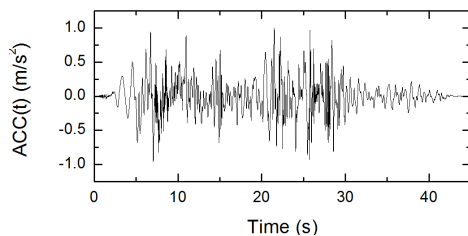


Figure 4. Measured seismic wave acceleration curve with 0.1g acceleration peak

Table 1. Values of the parameters

parameters	Values
Mass of transformer/kg	200
Mass of corona ring/kg	20
Linear density and length of casing/kg/m,m	10,12
Bending stiffness of casing/ $N \cdot m^2$	1.85

As shown in Figure 3, the measured seismic wave with 0.1g acceleration peak is used as the excitation data. The parameters used in the example are shown in Table 1, and then based on the four order Runge-kutta method, the Matlab program is compiled to obtain the corresponding acceleration transfer rate under the different spring length, stiffness and damping coefficient.

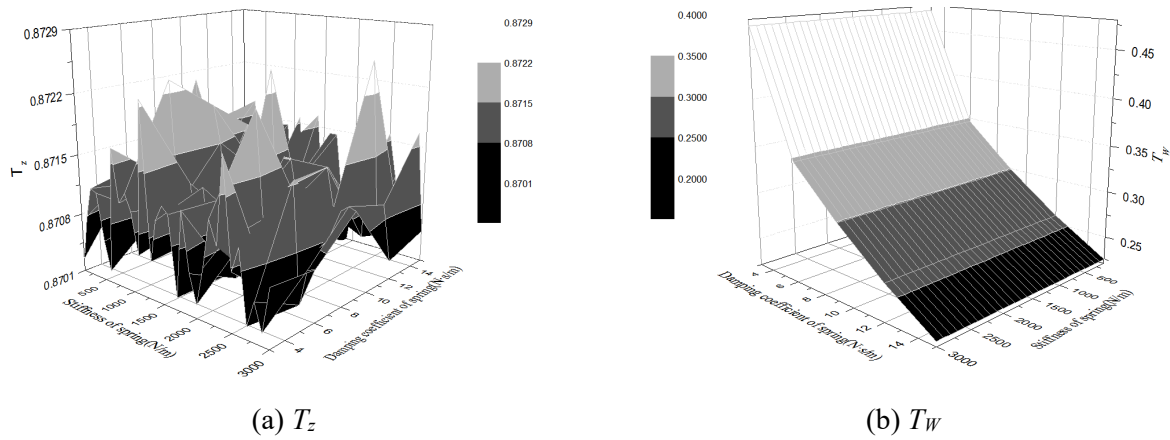


Figure 5. surfaces that show the acceleration transfer rate varies with spring stiffness and damping coefficient (spring length is 0.5m)

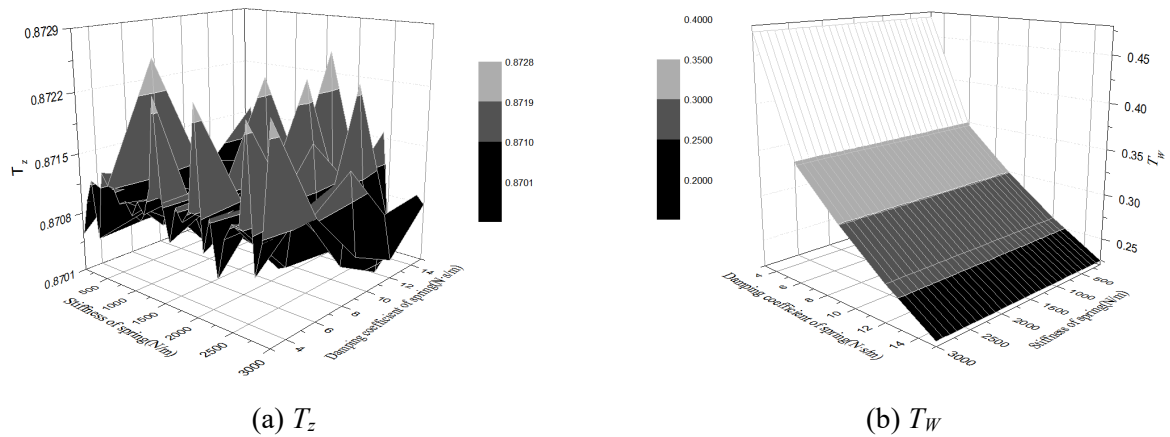


Figure 6. surfaces that show the acceleration transfer rate varies with spring stiffness and damping coefficient (spring length is 0.75m)

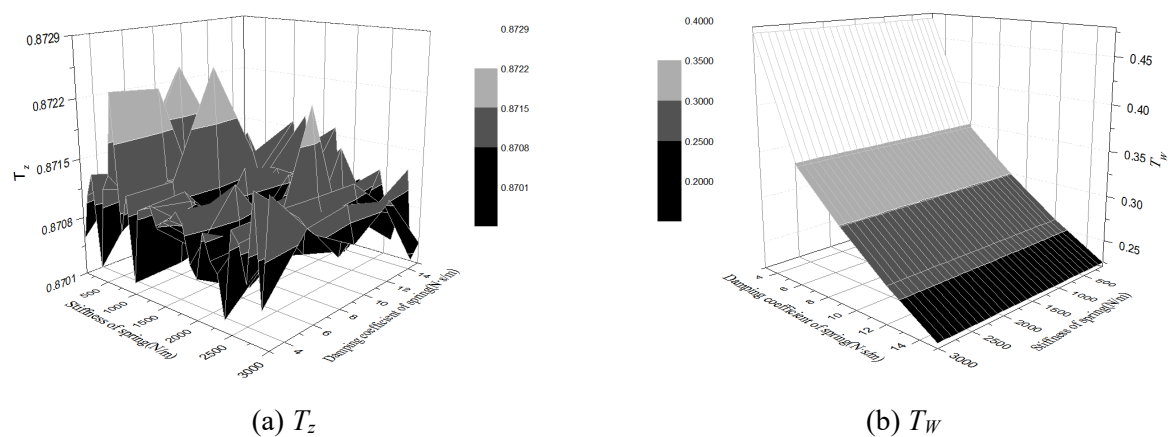


Figure 7. surfaces that show the acceleration transfer rate varies with spring stiffness and damping coefficient (spring length is 1m)

Through comparison, it's not difficult to find that the variations of the acceleration transfer rate of the transformer and the casing have the following rules and characteristics as the spring stiffness, damping coefficient and length change:

(1) The change of spring length between 0.5 and 1 m has no obvious effect on the two kind of acceleration transfer rates. The acceleration transfer rate of the transformer is about 0.87, and the casing acceleration transfer rate is about 0.2~0.5.

(2) When the length of the spring is constant, the spring stiffness and damping coefficient have a great influence on the two acceleration transmission rates, and each has its own characteristics. The acceleration transfer rate of the transformer shows great volatility, and it needs to be deeply analysed under individual design conditions to obtain the best isolation effect. However, the influence of the spring acceleration rate on the casing is not very significant, and it decreases rapidly with the increase of the damping, which indicates that increasing the spring damping is advantageous for transformer vibration isolation;

In general, appropriate increase of spring stiffness and damping is beneficial to the vibration isolation of the transformer as well as casing. The length of the spring can be set according to the actual conditions of the site, etc., which is also suitable for the vibration isolation maintenance and modification of the existing transformer. The analyse method mentioned above can effectively improve the seismic capacity of the equipment and maintain long-term safe and stable operation.

4. Conclusions

In this paper, based on the geometric nonlinear theory, a transformer-casing coupling nonlinear dynamic model is established on the basis of the geometric nonlinear theory, and a typical example is used as a quantitative analysis condition. The optimization method and analysis conclusion of the vibration isolation model are given, and the following are summarized as follows:

(1) Reasonable analysis of key parameters, including appropriate stiffness and damping value of the single side spring and the proper selection of the spring length according to the site conditions, should be taken to get large increase of the isolation capacity of the transformer, which is greatly significant.

(2) the increase of spring damping has a very important effect on vibration isolation, because the larger damping coefficient can reduce the vibration of the casing until the minimum, which can be achieved by adding a subsidiary viscous damping device.

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