Validation value of cable force using the accelerometer

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Abstract. Cable is a tensile element on cable stayed bridges that works to support the load on the bridge deck due to the traffic load. In the design, it has been calculated the maximum load that can be received by each cable based on dead load, live load or environmental load. When the bridge operates, it is necessary to monitor the behavior of the bridge structure due to the work load based on the time function. It is very important for the bridge management to examine the cable forces that occur in real time, so that it can be immediately known if there is a cable force exceeding the maximum limit. One of the sensor tools that can be used to determine the frequency of the cable is accelerometer. The frequency data from accelerometer are calculated into the approach of formulas developed by Wei-Hua Hu, 2005. In this study, calculation cable forces consider the effect of sag cable or bending stiffness using Wei-Hua Hu formula. As a control data in cable force calculations, it used measurement data conducted by consultants. From the results for cable C7 located on left side of bridge needs to be re-tested. This happens because the value of the resulting cable force does not show similarity with the right-side (cable C7 located on right side). The cable C7 in left side of bridge has a frequency value of 1.24 and generate cable forces tends to decrease, where it should be increase for cables forces values. For other cable forces, the value of the cable force obtains a range difference between 0.28% - 9.23%. Longer cable such as cable M7, the cable force formula is less influenced by bending stiffness effects in calculation. In other words, the stiffness of the cable is small or does not affect the cable force. As for shorter cables in cables C2 and M2, the bending stiffness of the cables affects the calculation of the cable force.

 Keywords: cable force, accelerometer, sag cable, bending stiffness

1.   Introduction

Cable is a tensile element that works as a support in increasing the stiffness of structure. The dimension and stiffness of the cable is determined by the magnitude of the force that occurs in the cable. Based on the theory of flat taut string, the cable force is influenced by the effective length of the cable, the mass of the cable, the frequency and vibration mode. Generally, the element of cables is composed of strand and combined more than one strand, it called tendon which have a higher tensile strength. Structures with cable elements are usually widely used in bridge structures that have a long span. The cable on the long-span bridge has a main function, it is as the support of the loads that work on the bridge, therefore the cable used on the bridge had to resist load in the deck bridge. The main factor to be calculated in the

cable forces for bridge structure is the tensile force acting on the cable where the force should not exceed the maximum limit of specifications on the cable. Therefore, the tensile strength of the cable should be checked, and it is not to exceed the maximum allowable limit.

The empirical approach in determining the force of a cable on the bridge has been done and has been used by engineers in the field. The factors that affect the amount of force the cable on the bridge which has been developed by previous researchers are mass, length, frequency, vibrate mode, cable sag, cable bending stiffness, the characteristics of two non-dimensional parameters of the cable (λ_2). Based on the results of field measurements through a dynamic test of bridges with tests performed by VSL consultants and indirect testing (accelerometer) to obtain the value of existing cable force that has a value difference of more than 10%. The question raises, what the method is most valid in determining the force of the cable and the formula used whether in accordance with the case of the cable being calculated. It is necessary for testing and other calculations on the parameters that influence the force of the cables as a parameter that already exists in the empirical formula.

2. Study Literature

2.1 Cable Force on The Bridge

The focus in this study, how to get the cable forces in cable stayed bridge. The magnitude of cable force is very sensitive to the cross section of cable. The relationship cable force and cross section of cable shown in the formula below.

The basic formula determines the force is:

$$\sigma = P/A \tag{2.1}$$

The basic formula determines the force based on Newton's Law II:

$$P = m.a \tag{2.2}$$

The basic formula determines the force based on displacement and rigidity:

$$P = k.U \tag{2.3}$$

Where, cable force (P), cross section of cable (A), strength of cable (σ), stiffness of cable (k), mass of cable (m), acceleration of cable (a), displacement of cable (U)

2.2 Relationship Between Force and Natural Frequency

In calculating the magnitude of force, the cable need to consider the dynamic characteristics and the physical behavior of the cable. The cable force in bridge structures depending on the value of the sag cable and bending stiffness parameters. Currently, the vibration method that has been used is classified into 5 categories:

2.2.1 Flat taut string theory

In this theory the effect of sag cable and flexural stiffness parameters of cable is ignored, so that the relationship between the frequency and the force on the cable. Where f_n is denoted as nth of the natural frequency in units of Hz. For the notation T, m and L are the values of the cable force, density, and length of the cable. Measurement of frequency and the amount of vibration mode can be directly used formulas and it is assist for initial approach in the calculation of cable force. However, this formula has limitations for long cables, so it does not consider the sag cable and the bending stiffness of the cable. The approach formula for this theory is as follows:

$$T = 4\text{m}L^2 \left(\frac{f_n}{n}\right)^2 \tag{2.4}$$

2.2.2 Theory Shimada et al., 1989 (Calculate the flexural stiffness of the cable)

In this theory, the formula considers the bending stiffness of the cable, where EI is a notation of the bending stiffness of the cable. The cable force values and flexural strength are unknown in this approach formula, then the linear regression procedure was applied to measure the frequency value and its relation to the vibration mode. Thus, it can be calculated the value of the flexural stiffness and tensile strength of the cable. Although the results of the calculations reliable for single short cables, but this formula not

appropriate for cable has large sag cable. For that purpose, it required to put in vibration mode level n^{th} can provide better accuracy in determining of cable force. Despite this formula has a limitation, but the engineer in field often used it for calculation because of its simple and fast in determine cable force.

$$T = 4\text{m}L^2 \left(\frac{f_n}{n}\right)^2 - \frac{EI\pi^2 n^2}{L^2} \tag{2.5}$$

2.2.3 The theory proposed by Fang et al, 2012 (High vibration mode of the cable)

The practical formula of the cable force is proposed in a simple form, where the flexural stiffness of the cable is calculated, and the effect of the sag cable is negligible for simplification by using relative frequency to anti-symentric or cable with high vibration mode. The effect of the sag cable on the tensile strength measurements discussed by Zui el al (1996) with parameters ξ and Γ [1].

$$T = 4\pi^2 \text{m}L^2 \left(\frac{f_n}{\gamma_n^2}\right)^2 - \frac{EI}{L^2}\gamma_n^2 \tag{2.6}$$

Where γ_n is the circular frequency of the Zui el al cable (1996) which concludes only the 1st degree vibration mode can be used when Γ> = 3. For cables with Γ <3, the 2nd level vibration mode and so on should be used in the tensile strength measurement [1]. The proposed formula covers the case of beam theory and string theory and has sufficient accuracy for computed tensile strength.

2.2.4   Theory that calculated sag cable (Γ) and the effect of bending stiffness of the cable (ξ)

This theory incorporates the value of the sag cable and the flexural stiffness of the cable in the calculation of the cable force. Based on Zui el al (1996), the criteria in the use of calculations refer to:
When Γ> = 3, vibration with level 1 vibration mode can be used because of the influence of sag cable and the tendency of small and negligible cable tilt angle for vibration mode of level 1.

$$T = 4m(f_1 L)^2 \left[1 - 2.2\frac{c}{f_1} - 0.55\left(\frac{c}{f_1}\right)^2\right] (17 < \xi)$$

$$T = 4m(f_1 L)^2 \left[0.865 - 11.6\left(\frac{c}{f_1}\right)^2\right] (6 \le \xi \le 17)$$

$$T = 4m(f_1 L)^2 \left[0.828 - 10.5\left(\frac{c}{f_1}\right)^2\right] (0 \le \xi \le 6) \tag{2.7}$$

When Γ <= 3, the effect of sag cable is large, and the value of Γ is small, then the level 2 vibration mode is required in the measurement of the cable force in calculation. Here are cases with cables that have small sag cable values.

$$T = m(f_2 L)^2 \left[1 - 4.40\frac{c}{f_2} - 1.10\left(\frac{c}{f_2}\right)^2\right] (60 \le \xi)$$

$$T = m(f_2 L)^2 \left[1.03 - 6.33\frac{c}{f_2} - 1.58\left(\frac{c}{f_2}\right)^2\right] (17 \le \xi \le 60)$$

$$T = m(f_2 L)^2 \left[0.882 - 85.0\left(\frac{c}{f_2}\right)^2\right] (0 \le \xi \le 17) \tag{2.8}$$

Determination of the cable force is determined by the vibration mode, sag and the ratio of the span of the cable, the flexural stiffness of the cable, and the axial rigidity. Based on previous information the axial and flexural rigidity of the cable system is important to determine the appropriate practical method. However, flexible stiffness of the cable is often available or invalid because of the bending and shear mechanisms of the cross-sectional area of the cable that may differ from the whole beam. Practically the ratio between sag cable and spans is often not available because static form measurements of cables require high costs.

2.2.5 An empirical formula for determining the tensile force of a cable using the base frequency (Wei-Xin et al, 2004)

The empirical formula for determining the tensile strength of the cable uses the fundamental frequency proposed by Wei-Xin et al (2004), where sag cable and bending stiffness are taken for calculation [2].

The application of the proposed formula depends on the size of the two parameter non-dimensional characteristics of the cable with λ2 is the characteristic effect of sag cable and ξ is a notation of the effect of bending stiffness of the cable. The formula becomes simple and makes it easier for the engineer to evaluate the tensile strength of the cable.

$$T = 4m^2 f^2 l^2; \ (\lambda^2 \le 0.17)$$

$$T = \sqrt[2]{ml^2(4f^2 T^2 - 7.569mEA)}; \ (0.17 \le \lambda^2 \le 4\pi^2)$$

$$T = m^2 f^2 l^2; \ (4\pi^2 \le \lambda^2)$$

$$T = 3.432 m^2 f^2 l^2 - 45.191 \frac{EI}{l^2}; \ (0 \le \xi \le 18)$$

$$T = m\left(2lf - \frac{2.363}{l}\sqrt{\frac{EI}{m}}\right)^2; \ (18 \le \xi \le 210)$$

$$T = 4m^2 f^2 l^2; \ (210 < \xi) \qquad (2.9)$$

3. Methodology

3.1 The Object of Research

This research will be conducted on the Merah Putih Bridge located in Ambon City, Maluku, Indonesia. This bridge on the Teluk Dalam Ambon Island, which connects Desa Rumah Tiga (Poka) in Sirimau Subdistrict on the north side, and Hative Kecil/Galala village in Teluk Ambon Subdistrict on the south side.
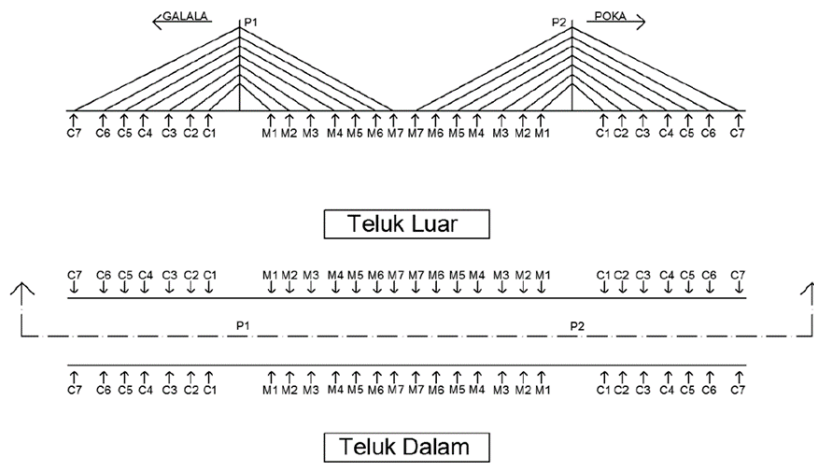

Figure 1. The View of Merah Putih Bridge and Number of Cable

3.2   Data collection technique

In determining the natural frequency of the cable, a device that can capture the vibration acceleration of the cable element is required. The sensor of the device can be connected to the computer, and it can convert its vibration acceleration using Fast Fourier Transform (FFT) method to get the natural frequency of the cable. Accelerometer is a tool that can capture acceleration. This technique of vibration used a hammer tool that was hit to the cable, so that causing the vibration to be captured by the accelerometer sensor. The technique of collecting natural frequency data by vibrating technique using rubber hammer which is hit to cable, and it caused vibration to be captured by accelerometer sensor, then the sensor can read acceleration of vibration on cable which converted to frequency. The testing on cable has been done only on certain cables which has tested by VSL Consultants, so it can be compared with the calculation results.
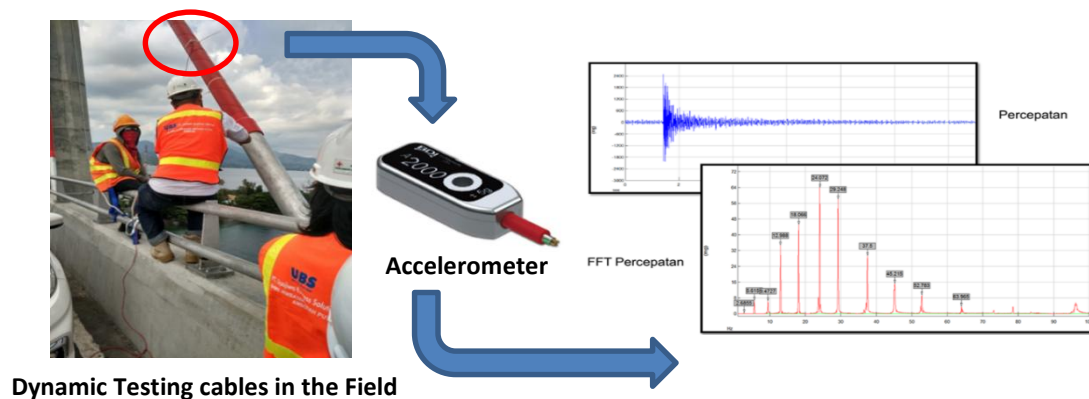
Figure 2. The activity of cable testing

3.3   Research Stages

Initial stages in this research is to identify the problem that exist in research, that is looking for other method in determining of cable force. The next step is to do field data collection, the Merah Putih Bridge. After the data obtained, then is to choose which cable to be tested. The cable to be tested is a cable that has the force data provided by the consultant. The results captured by the accelerometer are then converted using the FFT technique resulting in a natural frequency. After that, the calculation of the cable force using natural frequency parameters using the formula equation. After the results are obtained, the next step is to make comparisons with the field data to analyze.

4. Results and Discussion

4.1 Cable Data

The cables which is used as the object of research are cable type C7, C5, C2, M2, M5, M7. The general data for the Merah Putih bridge as follows.

Table 1. General data cable of Merah Putih bridge

Cable Type	Mass Cable (t/m)	Length (m)	Modulus of Elasticity (kN/m^2)	Inertia (m^4)
C2	0,028175	39,52	197.000.000	8,25E-07
C5	0,033075	67,95	197.000.000	1,14E-06
C7	0,042875	87,42	197.000.000	1,91E-06
M2	0,028175	38,28	197.000.000	8,25E-07
M5	0,033075	65,98	197.000.000	1,14E-06
M7	0,042875	85,11	197.000.000	1,91E-06

4.2 Accelerometer Measurement Results Frequency Value in the Field

Based on the measurement result, it obtained the natural frequency value of some vibration mode based on vibration acceleration using Fast Fourier Transform (FFT) method. In this study, the calculation only considered vibration mode 1. As an example, the output of the FFT method shown in figure 3 and 4, for cable C5 on the outer bay, the value of natural frequency mode 1 is 1.87 Hz. As for the cable C5 on the inner bay is 1.91 Hz.
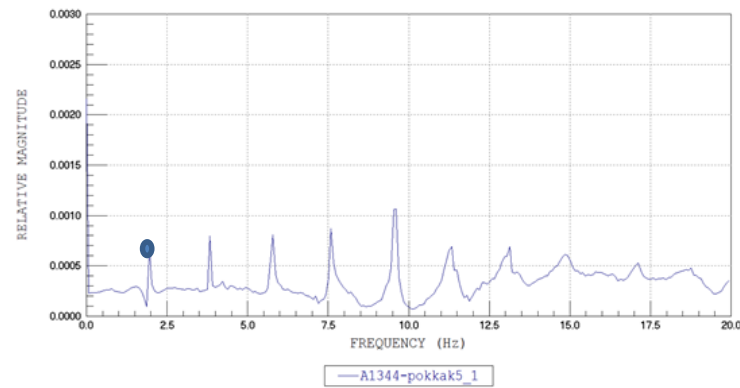
Figure 3. Natural frequency cable C5 Poka, outer bay (vibration mode 1)
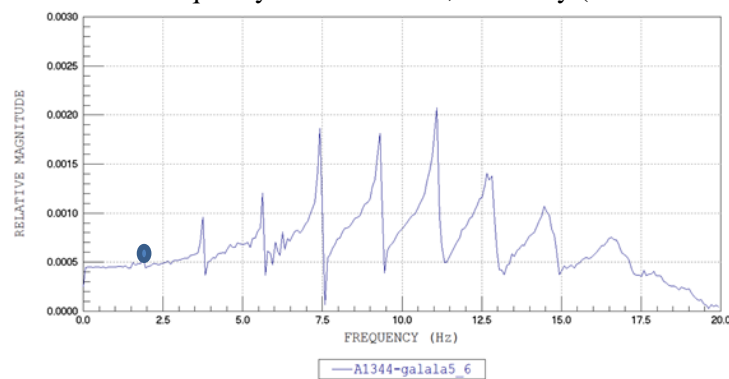

Figure 4. Natural frequency cable C5 Poka, inner bay (vibration mode 1)

The natural frequency for other cables can be seen in table 2 below.

Table 2. The frequency of cables in vibration mode 1

No	Side	Cable	f (Hz)	Side	f (Hz)
1	Teluk Dalam	C7	1.24	Teluk Luar	1.30
2	Teluk Dalam	C5	1.91	Teluk Luar	1.87
3	Teluk Dalam	C2	3.13	Teluk Luar	3.22
4	Teluk Dalam	M2	3.22	Teluk Luar	3.57
5	Teluk Dalam	M5	1.96	Teluk Luar	1.79
6	Teluk Dalam	M7	1.50	Teluk Luar	1.49
7	Teluk Dalam	M7	1.36	Teluk Luar	1.45
8	Teluk Dalam	M5	1.79	Teluk Luar	1.41
9	Teluk Dalam	M2	3.24	Teluk Luar	3.22
10	Teluk Dalam	C2	2.96	Teluk Luar	2.70
11	Teluk Dalam	C5	1.86	Teluk Luar	1.94
12	Teluk Dalam	C7	1.36	Teluk Luar	1.37

4.3 Influence of Non- Dimensional Parameters λ2 and ξ

To get an initial assume of the effect of cable sag and bending stiffness on the cable force, it can be done using the formula approach written by Wei-Hua Hu, 2005 through the calculation of Non-Dimensional Parameters λ^2 and ξ. Non-Dimensional Parameters λ^2 represents the effect of sag on the cable which consists of 3 categories i.e. no effect ($\lambda^2 \leq 0.17$), quite influential ($0.17 \leq \lambda^2 \leq 4\pi^2$), and influential ($4\pi^2$

$< \lambda^2$). While Non-Dimensional Parameter ξ represents the effect of bending stiffness on cable. And it also consists of 3 categories have no effect ($0 \leq \xi \leq 18$), quite influential ($18 < \xi \leq 210$), and influential ($210 < \xi$).

Formula for Non-Dimensional Parameters λ^2:

$$\lambda^2 = \left(\frac{m\,g\,l}{H} \right)^2 \frac{E\,A\,L}{H\,L_e} \; ; \; L_e = l\left\{ 1 + \frac{1}{8}\left(\frac{m\,g\,l}{H} \right)^2 \right\} \qquad (2.10)$$

Formula for Non-Dimensional Parameters ξ:

$$\xi = \sqrt{\frac{T}{E\,I}}\,l \qquad (2.11)$$

The mass of cable (m), gravity (g), elastic modulus (E), cross-sectional area (A), length of cable (l), cable force (H), length of cable effective (Le), moment of Inertia (I), tension of the cable (T). From the calculation results by using the cable force (H) from the measurement control data performed by consultant obtained values of non-dimensional parameters λ^2 and ξ. The results can be seen in the table below.

Table 3. Calculation of non-dimensional values of parameters λ^2 and ξ

No	Cable	Non-dimensional parameter (λ^2)		Non-dimensional parameter (ξ)	
		λ^2 inner bay	λ^2 outer bay	ξ inner bay	ξ outer bay
1	C7	0.153	0.097	211.71	218.67
2	C5	0.059	0.031	212.85	215.47
3	C2	0.020	0.044	119.14	106.89
4	M2	0.030	0.025	109.10	112.33
5	M5	0.031	0.084	206.59	175.06
6	M7	0.049	0.084	236.36	216.27
7	M7	0.098	0.052	210.77	233.98
8	M5	0.061	0.051	184.74	190.00
9	M2	0.025	0.015	112.60	122.89
10	C2	0.035	0.018	111.12	121.71
11	C5	0.046	0.067	201.49	208.92
12	C7	0.094	0.123	219.95	219.70

From figure 5, based on the value of the cable force from VSL measurement, it can be calculated the value of non-dimensional parameter λ^2 of each cable based on Wei-Hua Hu formula, 2005. By using this formula can be known how big the effect of sag cable on the calculation of cable force. Based on the calculation, the whole of Non-Dimensional Parameters λ^2 cable overall cables below 0.17 ($\lambda^2 \leq 0.17$). These results illustrate, the Merah Putih Bridge cable can be said not to be affected by the sag cable in the calculation of the cable force.
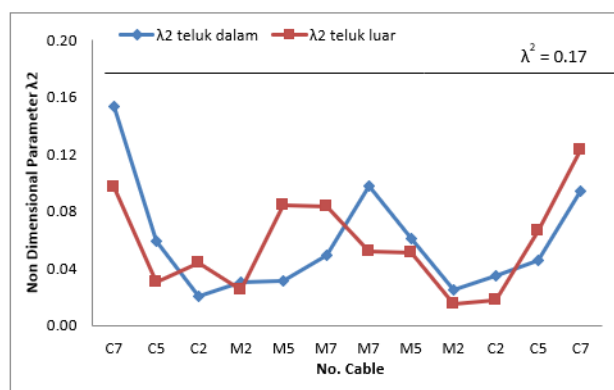
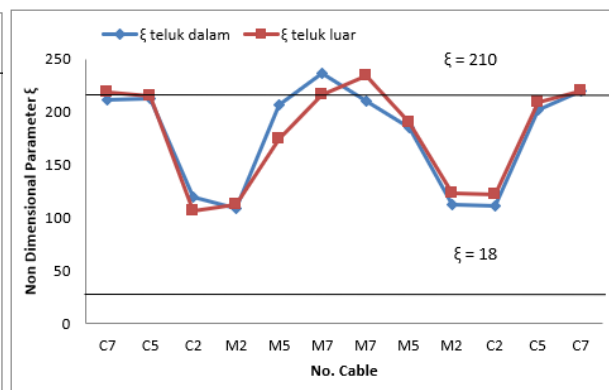**Figure 5.** Non-dimensional value parameters λ^2 cables

Figure 6. Non-dimensional value of parameter ξ cable

From Figure 6, based on the value of cable measurement force from consultant can be calculated the value of non-dimensional parameter ξ each cable based on Wei-Hua Hu formula, 2005. By using this formula can know how big influence of bending stiffness on calculation of cable force. Where, this formula divided into three categories in illustrating the effect of bending stiffness. The third category is the flexural stiffness of large, medium and no influence. Based on the calculation, the value of Non-Dimensional Parameters ξ Merah Putih Bridge cables are categorized into 2 categories, intermediate bending stiffness ($18 \leq \xi \leq 210$) and no bending stiffness ($210 < \xi$). Cables that are not affected due to bending stiffness effects are cables C7, C5 and M7. While, other cables are influenced by medium category stiffness bending.

4.4 Calculation and Analysis of Cable Force

The cable force calculation based on the value of natural frequency in vibrating mode 1 into the formula force cable Wei-Hua Hu, 2005 as shown in the figures below.
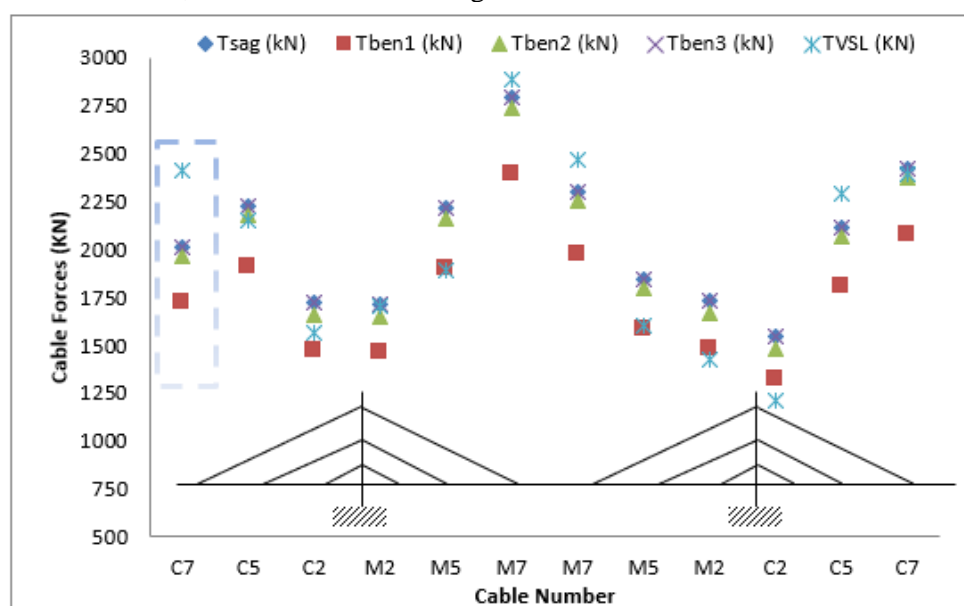

Figure 7. Cable forces in inner bay

Figure 7 represents the cable force on the Merah Putih bridge calculated by consultant as the control data. This research performs indirect test by using accelerometer sensor instrument, which is this tool will get the frequency value of each cable. The value of the frequency obtained from the calculation is entered into the formula of the cable force calculated by Wei-Hua Hu. From the figure 7, the pattern of cable force on the cable C7 on left side bridge seen different than cable C7 on right side bridge. The cable force of C7 which located on the left side of the bridge is decreased, while the cable forces of C7 which located on the right side is increased. Based on measurements from consultant, the values of cable force on cable C7 shown is increased. It can be concluded based on this pattern, cable C7 located on left side bridge cable has irregularities for the measurement of the value of frequency. Where the frequency based on the measurement using the accelerometer on the cable C7 in left side cable is 1.24 Hz. The frequency of cable C7 in left side is different from the value of the measurement frequency of the cable C7 in right side cable is 1.36 Hz. Since the cable C7 in left side cable and the cable C7 in right side have a geometrically symmetrical, it should produce a frequency that has a similarity. So, the results of the frequency of the cable C7 in left side can be said for this value is less valid. So, it is necessary to re-test on cable C7 in left side bridge.

For other cables, the results of the cable force using the Wei-Hua Hu formula have a difference of range between 0.28% - 9.23% against the consultant as a control data. For longer cables such as cable M7, the calculation of the cable force formula is less affected by the bending stiffness effect. In other words, the stiffness of the cable is considered small or does not affect the cable force. As for shorter cables like cable C2 and M2, the stiffness of the cables affects the calculation of the resulting cable force.
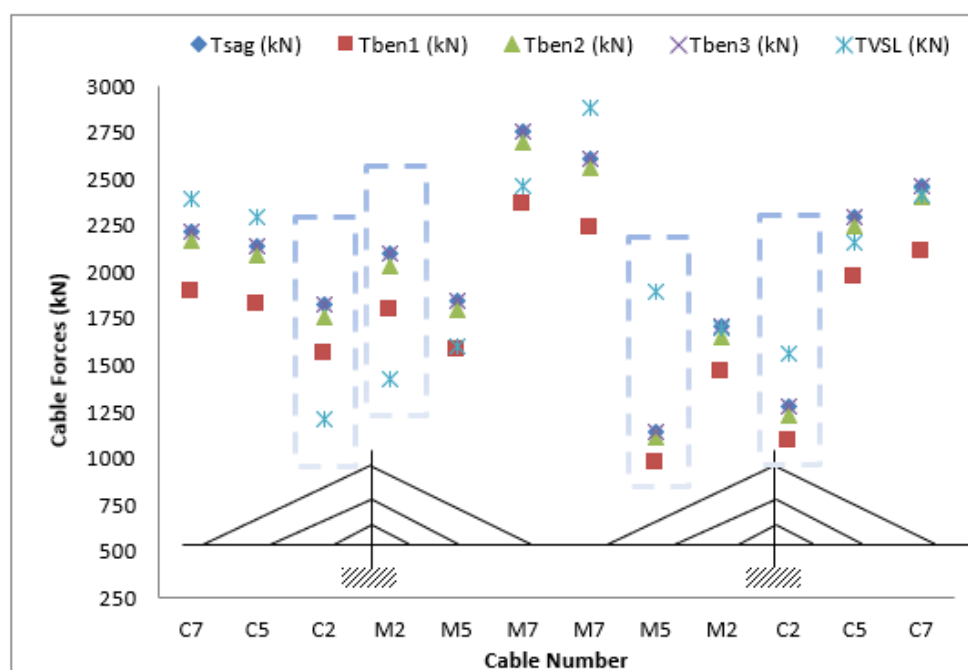

Figure 8. Cable forces in outer bay

From figure 8, the pattern of cable force that occurs in the consultant measurement data as a control data and measurement used accelerometer are different at tower (P1) and tower (P2), especially on cable C2, M2 and M5. This may occur because at the time of measurement that has been done by consultant, the traffic load on the right side of tower is greater than the other side. As for the frequency measurement by using accelerometer means greater traffic load occurs on the left side of tower (P1).

Based on the results of the analysis, the difference in cable force between the measurements by consultant and the testing with the accelerometer, more than one cable that has the difference in cable force with a percentage of more than 10%. Therefore, it is advisable to make measurements and calculations of the bridge inspection cable bridge force to observe the loads that have the same behavior so that the comparative validation between the two measurement methods can be compared to obtain a more appropriate relationship on each cable force.

5 Conclusion

Based on the results of research conducted several conclusions that can be drawn:
- Non-dimensional value parameters λ^2 for the overall cable is below 0.17 ($\lambda^2 \leq 0.17$). These results illustrate that, the cable forces in the Merah Putih bridge can be said not to be affected by the sag cable in the calculation.
- Non-dimensional value parameters ξ for cables at Merah Putih Bridge are categorized into 2 categories, intermediate bending rigidity ($18 \leq \xi \leq 210$) and not affected by bending stiffness ($210 < \xi$). Cables that are not affected due to bending stiffness effects are cables C7, C5 and M7. While other cables are influenced by medium category bending stiffness.
- It is necessary to re-test the natural frequency on the cable C7 in left side bridge using the accelerometer because it should increase for cables forces values. The pattern of cable force in cable C7 on right side tend to increase, this results in accordance with consultant calculation data.
- The values of the cable force have a difference of range between 0.28% - 9.23%. For longer cables such as the cable M7, the calculation of the cable force formula is less or less affected by the bending stiffness effect. In other words, the stiffness of the cable is considered small or does not affect the value of the cable force. As for shorter cables like C2 and M2, the rigidity of the cables affects the calculation of the resulting cable force.

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