

Application of the ideal soil model and tube hydraulics to study filtration processes

A A Khairullin

Industrial University of Tyumen, 38 Volodarskogo St., Tyumen, 625000, Russia

E-mail: khayrullin.azat@mail.ru

Abstract: Some regularities of filtration processes are considered in the article. On the basis of experimental studies of fluid motion in capillary tubes of various types, tortuosity is modelled. These experiments demonstrate that the model of ideal soil in the form of cylindrical tubes should take into account the rheological dependence of the Newtonian fluid on the pressure gradient, that is, deviation from linearity. Determination of the permeability of a porous medium is ambiguous and depends on the conditions, scales, and methods of investigation. The Darcy's law requires refinement because of the nonlinearity of viscosity at different filtration velocities. Based on the analysis of modelling results, a non-linear dependence is suggested that resembles the Leverett function suitable for both linear filtration and beyond linearity and has concave and convex parts.

1. Introduction

Permeability is the most important property of a formation, which is included in most of the parameters used for calculations when developing the field. But permeability is one of the most difficult to determine parameters when studying the reservoir properties of a formation. This raises a dilemma. On the one hand, geological research requires more information to accurately describe the elements of the formation flow and uses a variety of technical resources to model this task. On the other hand, development engineers often sacrifice without hesitation the results of such a description at the modelling stage in favour of adapting the formation behaviour model [1].

There is ambiguity in the definition of permeability, since during the filtration the action of capillary, interphase, viscous, gravitational, hydrostatic and other forces is manifested simultaneously. The methods used to determine the numerical values of the coefficients can give different values due to the difference in the conditions of the static and dynamic effects, in addition, the scale of measurements is important. To explain the known facts, models are used, and a good model, according to A. Kh. Mirzadzhanzadeh, is the one that allows you to look around the corner, i.e. describes the unknown in advance. Since only such things can be measured, according to A. Einstein, which the theory "admits" and "understands" ([2], p 276), it is necessary to act by definition, the most powerful method in mathematics.

In this paper, on the basis of an ideal soil model and accepting the nonlinearity of "Newtonian" fluids, it is proposed to investigate other characteristics of porous media in addition to the permeability coefficient, using a nonlinear dependence of the type of the Leverett function.



2. Research

Using the ideal soil model in the form of cylindrical tubes and the law of linear filtration of A. Darcy, we investigate the main parameters that affect filtration in real seams.

It is assumed that: 1 - the pressure difference remains constant during filtration; 2 - the tortuosity remains constant during modelling; 3 - the used liquid is the same for different experiments; 4 - experimental data are processed by linear and quadratic dependencies.

3. Results and discussion

As is known, the law of linear filtration of A. Darcy is used as the basis for determining the permeability coefficient k , expressing a linear relationship between the filtration velocity v and the pressure gradient $grad p$:

$$\vec{v} \equiv \frac{Q}{S} = -\frac{k}{\mu} grad p, \text{ or as } v = \frac{k \Delta p}{\mu l} \quad (1)$$

where μ - coefficient of dynamic viscosity; Q - liquid flow through a cross-section of the area S ; Δp - pressure difference between layers at the distance l .

This law is limited to the fulfilment of the linearity of the law of filtration. As a rule, Darcy's law is valid under the following conditions: 1) the porous medium is fine-grained and pore channels are rather narrow; 2) the filtration velocity and the pressure gradient are small; 3) the change in the filtration velocity and the pressure gradient are small.

These processes are discussed in detail in [3], using experimental studies by J. Fenchler, J. Lewis and C. Burns, Lindquist, G.F. Trebin and others. The main nonlinear filtration laws are also presented. As the velocity of the liquid increases, Darcy's law is violated due to the increase in pressure losses due to inertial forces: formation of vortices, zones of flow disruption from the surface of particles, hydraulic impact on particles, etc. This is the so-called upper limit. Darcy's law can also be violated at very low filtration velocities, i.e. the lower limit, in the process of the beginning of fluid motion due to the manifestation of non-Newtonian rheological properties of the fluid and its interaction with the solid skeleton of the porous medium.

The filtration velocity v_{cr} , at which Darcy's law is violated, is called the critical filtration velocity. Violation of the filtration velocity does not mean a transition from laminar to turbulent motion but is caused by the fact that the inertia forces arising in the fluid due to the tortuosity of the channels and the change in the cross-sectional area, for $v > v_{cr}$, become commensurable with the frictional forces.

At very low velocities, as the pressure gradient increases, the change in the filtration velocity does not obey Darcy's law. This phenomenon is explained by the fact that at low velocities, the force interaction between the solid skeleton and the liquid becomes significant due to the formation of anomalous, non-Newtonian systems. There are many rheological models of non-Newtonian fluids, the simplest of them is the model with the limiting shear gradient.

Reliability of the well survey data and determination of the parameters of the formation depend on the accuracy of the filtration law used. In this regard, in the field of violation of Darcy's law, it is necessary to introduce more general, nonlinear filtration laws.

For a long time, the efforts of many researchers were aimed at deriving the universal dependence (by analogy with tube hydraulics) of the coefficient of hydraulic resistance λ on the Reynolds number ($Re = vdp/\mu$), on the basis of modelling the phenomenon of filtration, including physical. Various models of porous media have been proposed, including mock soil models (in the form of spheres) and ideal soil models (in the form of stacks of tubes).

3.1. Experimental research

In [4,5], some features of fluid filtration in a porous medium were investigated using tube hydraulics formulas, when the case of linear filtration is an analogue of the laminar flow of liquid in tube hydraulics. These processes are described by linear dependence; therefore, they can have certain analogies.

For comparison, experiments were conducted to study the flow of a liquid through tubes with an inner hole diameter (d_{in}) of 3 mm. In tubes of the order of such sizes, capillary phenomena are observed and at the same time, the application of the laws of tube hydraulics is possible. Schematically, the installation is shown in Figure 1.

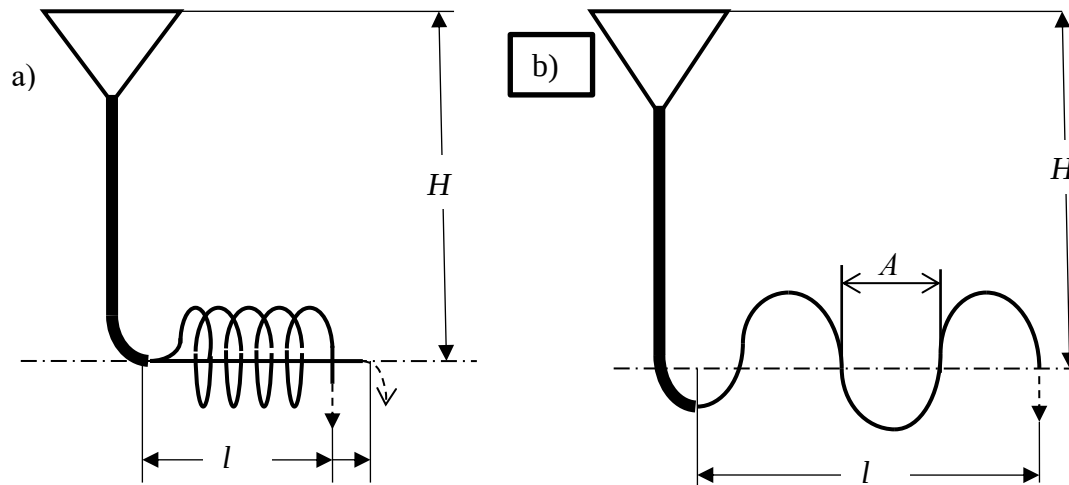


Figure 1. A scheme of installation for studying the flow in pipes of different shapes.

Let us give some geometric estimates. With a tube length $L = 103$ cm, the ratio $L/d_{in} = 1030/3 \sim 343$, if we take a standard cylindrical core sample of height $h = 3$ cm, then at such ratios the corresponding diameter of the capillary will be $d_c \sim 30/343 \approx 0.09$ mm. That is, it approximately corresponds to the size of capillary pores - from 0.5 to 0.0002 mm. We suppose that such a geometric similarity is acceptable.

To simulate the tortuosity of the ideal soil, tubes were taken (as if one from the stacks of tubes), but of different types - rectilinear, helical cylindrical (coil) and helical flat ("snake").

Some results of the experiments are shown in the form of graphs in Figures 2, 3 and 4. All experimental data are approximated by linear and quadratic trends, passing and not passing through the origin. Setting in this form made it possible to obtain small differential pressures $\Delta p/l$.

Of these approximations, the most accurate parabolas are quadratic ones that do not pass through the origin such as:

$$v = v_0 + a \cdot P - b \cdot P^2, \quad (2)$$

where v_0 – velocity at $P = 0$, a and b — parameters for the expression of the average flow velocity v ; $P \equiv \Delta p/l$; $\Delta p = \rho g H$.

On the joint calculated graphs of the dependence of the average flow velocity on the pressure drop from the approximations shown in Figure 5 in different scales, their features are visible. The inverse frequently used dependences $P = P(v)$ are not given here.

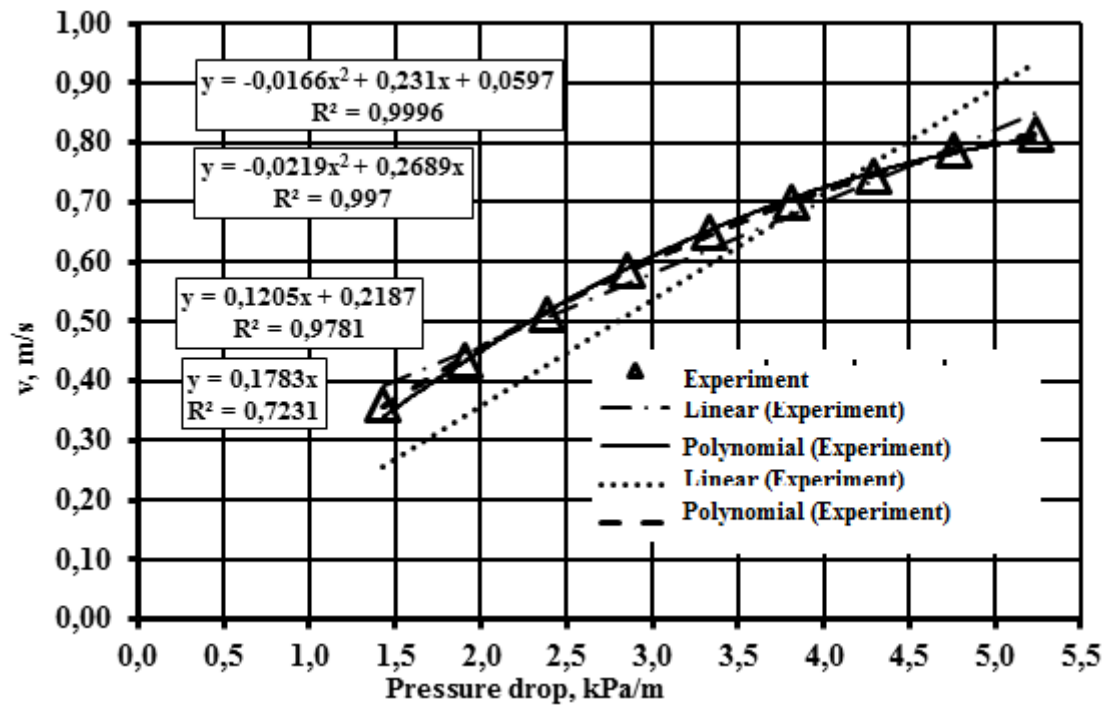


Figure 2. Dependence of the flow velocity on the pressure drop for a rectilinear tube.

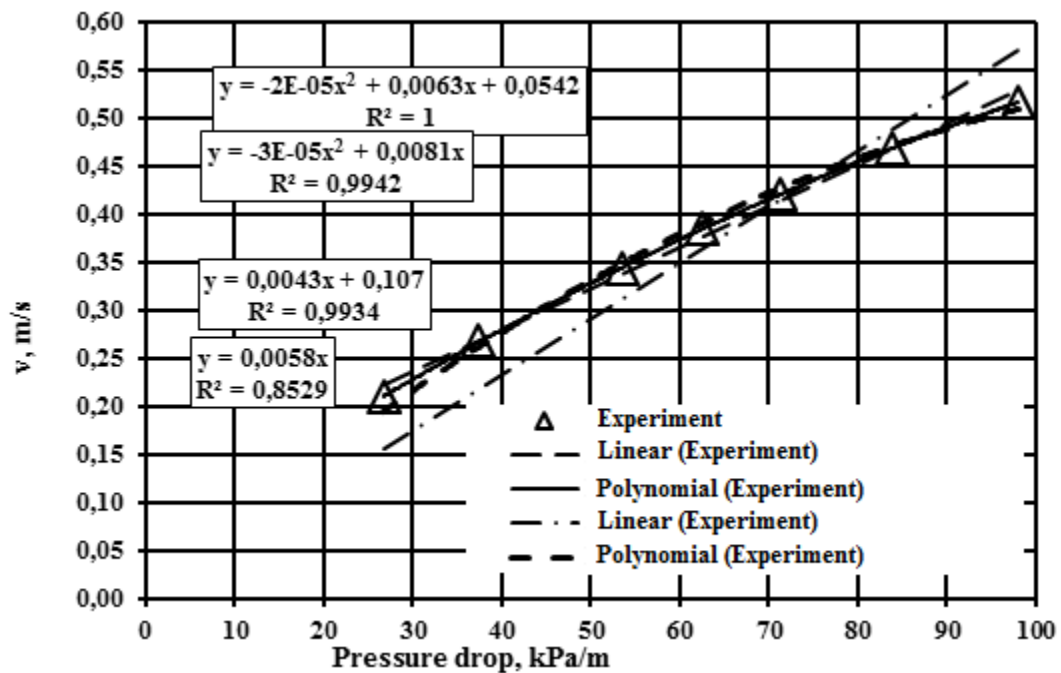


Figure 3. Dependence of the flow velocity on the pressure drop for a helical cylindrical tube (coil, $D=30$ mm).

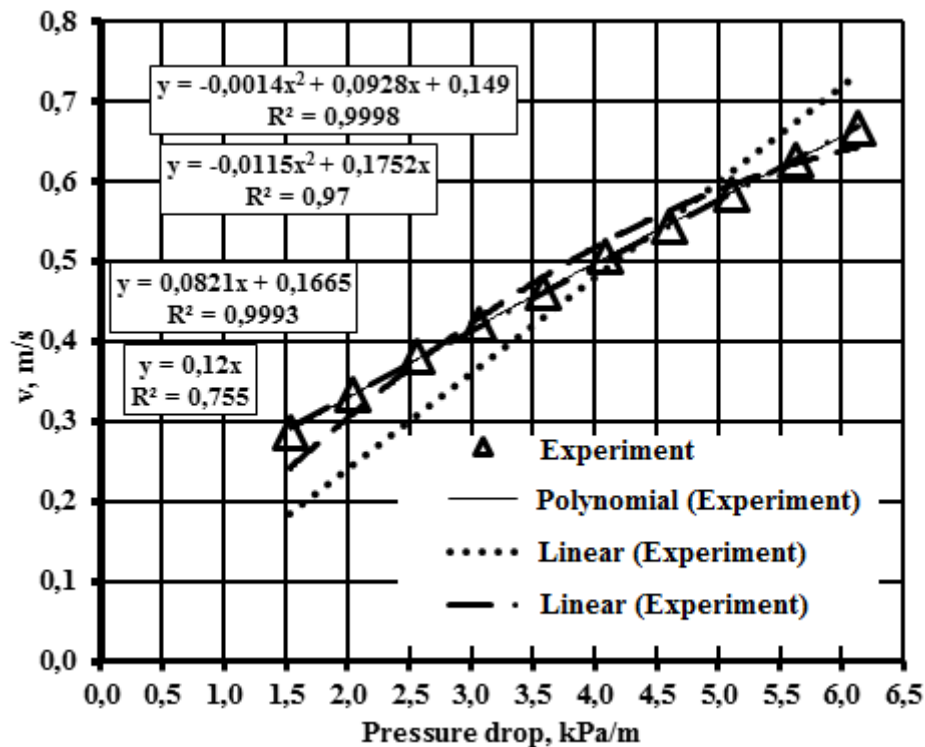


Figure 4. Dependence of the flow velocity on the pressure drop for a helical tube in the vertical plane ("snake", $A=45$ mm).

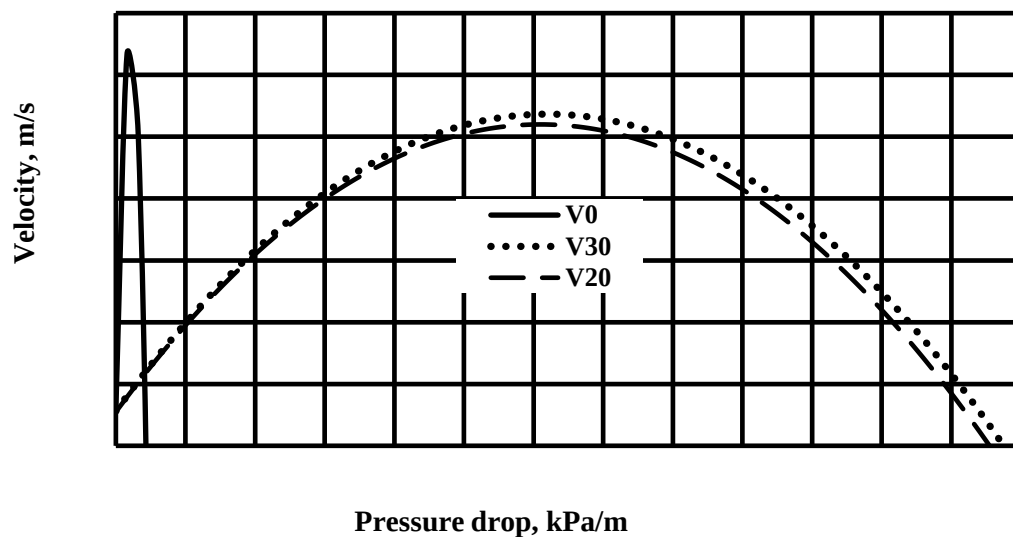


Figure 5. Graphs of the dependence of the fluid velocity on the pressure drop:
V30 and V20 – velocities in coils with diameters of D30 and 20 mm,
V0 – velocity in a rectilinear tube.

3.2. Experimental data processing

It can be seen from the graphical dependences (Figure 5) and equation (2) that all curves are convex and the branches of the parabolas are directed downward. Only the ascending branches have physical meaning, and the descending branches are additional solutions of the chosen approximation and have no physical meaning. Therefore, such approximations are bounded. It should be noted that all the

dependences of the flow velocity on the pressure drop are nonlinear, including for a rectilinear tube, although the flow patterns in all the experiments were laminar.

It is seen from these dependences that, according to (2) for $P = 0$, the flow velocity v is different from zero and is zero only at negative pressure values. Negative pressure values can be caused only by capillary forces. Then the results of the experiments processed according to (2) can be represented schematically in the form of Figure 6.

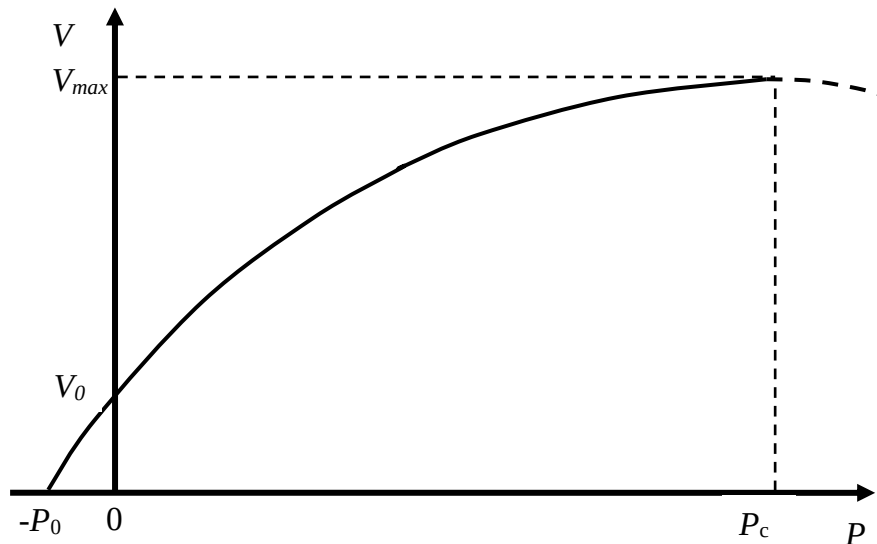


Figure 6. Schematic representation of experimental dependences of flow velocities on the pressure drop

The solution of the quadratic equation (2) with respect to P is represented as:

$$P = \frac{a}{2b} \left(1 \pm \sqrt{1 - \frac{4b}{a^2} (v - v_0)} \right). \quad (3)$$

The maximum velocity is achieved with a pressure drop equal to:

$$P = \frac{a}{2 \cdot b}, \quad (4)$$

that is, when the radicand in (3) equals zero, and the maximum velocity is determined from the equation:

$$1 - \frac{4b}{a^2} (v_{\max} - v_0) = 0, \quad v_{\max} = v_0 + \frac{a^2}{4b}. \quad (5)$$

From the solution of (3) for $v = 0$ we find the roots:

$$P_{1,2} = \frac{a}{2b} \left(1 \pm \sqrt{1 + \frac{4bv_0}{a^2}} \right). \quad (6)$$

The solution of the quadratic equation (3) with a positive sign in front of the square root refers to the right descending branch of the parabola and has no physical meaning. The solution $P = -P_0$ with a negative sign in front of the square root refers to the velocities $v \leq v_0$ with a pressure drop $P \leq 0$. Then for $v = 0$ the pressure drop P is represented as:

$$P_0 = -\frac{a}{2b} \left(\sqrt{1 + \frac{4bv_0}{a^2}} - 1 \right) \approx -\frac{v_0}{a}. \quad (7)$$

And the pressure takes a negative value $P = -P_0$.

3.3. Evaluation of the influence of tortuosity

The length of the coil tube is $L = \pi \cdot D \cdot n$, where D is the diameter of the coils; n is the number of turns. The pressure drop interval is $l = d_{out} \cdot n$, where d_{out} is the outer diameter of the tube. The ratio $L/l = \pi \cdot D/d_{out}$ is expressed tortuosity, for example, in [6]. The average velocity of the fluid flow is:

$$v = \frac{4 \cdot V}{\pi \cdot d_{in}^2 \cdot t}, \quad (8)$$

where V — volume of liquid flowing in time t ; d_{in} — inner pipe diameter.

To estimate the pressure drop and the effect of the curvature of the fluid trajectory on the additional pressure, the following assumptions are made. Centripetal force is determined as:

$$F_{cf} = \frac{2 \cdot m \cdot v^2}{D}. \quad (9)$$

This distributed centripetal force is caused by the turns of the tube forming a cylindrical surface, i.e. there is an additional pressure in the tube equal to:

$$P_{cf} = \frac{F_{cf}}{S_c} = \frac{2mv^2}{D} \cdot \frac{1}{\pi D l} = \frac{\rho \pi d_{in}^2}{2\pi D^2} \cdot \frac{L}{l} \cdot v^2 = \frac{\rho \pi d_{in}^2}{2d_{out} D} \cdot v^2. \quad (10)$$

It can be seen from (10) that the additional pressure is directly proportional to the "tortuosity" in the form of L/l , but the "radius of curvature" in the form of $D/2$ enters the expression inversely proportional to the square. Therefore, a tortuosity representation in the form of L/l may not be sufficient to fully describe the influence of tortuosity on the permeability coefficient; the radius of curvature has a more significant effect. The smaller the radius of curvature, the greater the centripetal force and hydraulic resistance.

The proposed effective diameter of a grain of sand or an open pore in [7] for Darcy's law is supplemented by a constant which depends on the geometric properties of the medium such as porosity, the shape of the medium grains, the fractional composition. Particular formulas for permeability are valid for a limited region of the sand type, so permeability values can vary significantly. It is further noted that the structure of the medium cannot be taken into account by averaging the grain sizes.

Another approach, to agree with laboratory data on filtration, is given by K. Aziz and A. Settari [8] in the form of the tensor δ - the correction coefficient of turbulence. In addition, Darcy's law proposes using an effective viscosity.

The graphs in Figure 5 show that the flow velocity of the liquid in the rectilinear tube (V_0) is much greater than in the coils at identical pressure differences. Estimates show that in a straight tube, the pressure drop is two orders of magnitude smaller than in coils and an order of magnitude smaller than in "snakes" (we take into account the coefficients of only linear terms) at the same average flow velocities. The tortuosity significantly affects the fluid velocity, both in the linear and quadratic terms of approximations. Despite the fact that liquids in tubes move in a laminar pattern, the velocities are nonlinearly dependent on the pressure drop. The definition of tortuosity as L/l is ambiguous; the radius of curvature has a significant effect. So, for $\tau_{30} < \tau_{20}$, the corresponding tortuosity is reversed, that is, it turns out that the greater the tortuosity, the smaller the pressure drop. Using approximation (2), a contradiction arises in the studies performed, which consists in the fact that as the pressure drop increases, the velocity has both ascending and descending branches of the parabola (Figure 5).

Similar phenomena can also be expected during the filtration of liquids through porous substances. We can say that the last branch has no physical meaning and is only an additional solution, or a shortcoming of the chosen approximation. To explain the results of the experiments, the dependence is proposed.

3.4. Nonlinear filtration model.

Let us assume that for $P = 0$ the fluid velocity is different from zero. For $P = -P_0$, the velocity $v = 0$, that is, for this it is necessary to take into account the action of capillary forces. Let us suppose that for $P \rightarrow \infty$ the velocity v tends to the critical velocity v_{cr} . Intermediate values must be smooth (having continuous derivatives), with the left side (in the zero region) concave, and the right side (for large values of P) convex (Figure 7).

For the indicated type, we choose an approximation:

$$v = v_{cr} \cdot \left\{ 1 - \exp[-\alpha \cdot (P + P_0)^2] \right\}. \quad (11)$$

This approximation corresponds to the limiting cases - for small and large pressure drops, and in appearance resembles the Leverett function for impregnation [8]. From the experimental data obtained, the parameters v_{cr} , α and P_0 were adjusted with allowance for the minimum mean square deviation equal to:

$$\sigma = \sum_{i=1}^n \left[v_{\text{exp}}(P_i) - v_{\text{calc}}(P_i | v_{cr}, \alpha, P_0) \right]^2, \quad (12)$$

where n — number of experimental points; v_{cr} , α and P_0 — variable parameters.

The result of the adjustment is shown in Figure 8.

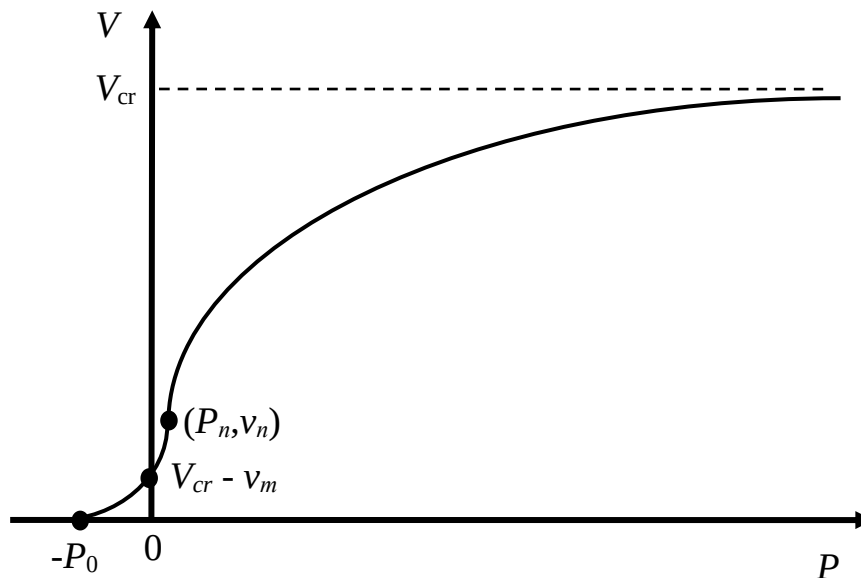


Figure 7. Schematic representation of the dependence of the flow velocity on the pressure drop.

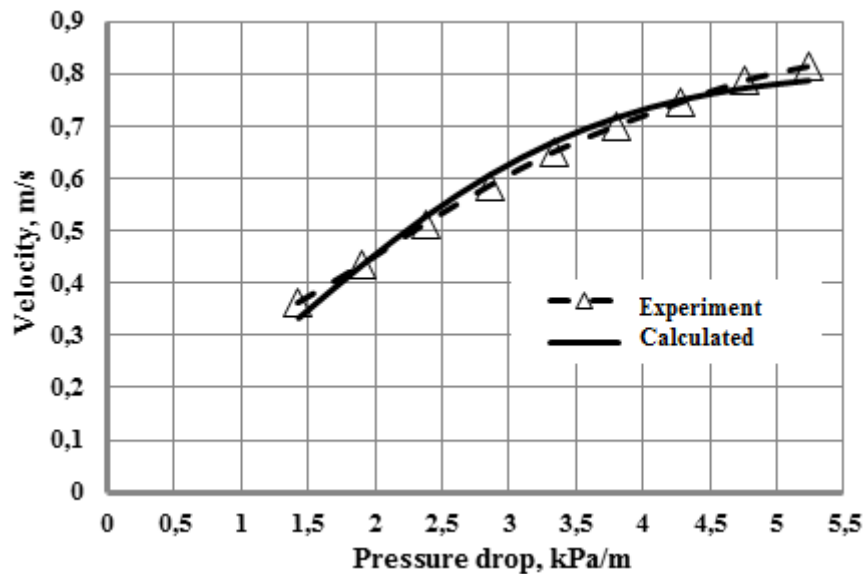


Figure 8. Rectilinear tube (refined data): $v_{cr} = 0.806$ m/s,
 $\alpha = 0.1$ (kPa/m)⁻²; $P_0 = 0.879$ kPa/m; $\sigma = 0.0557$

A comparison of the calculated approximation (11) and the quadratic dependence of the average flow velocity in a rectilinear tube is shown in Figure 9. The presence of negative pressure drops is possibly associated with the appearance of capillary pressure. Similar dependences were obtained for coils and "snakes".

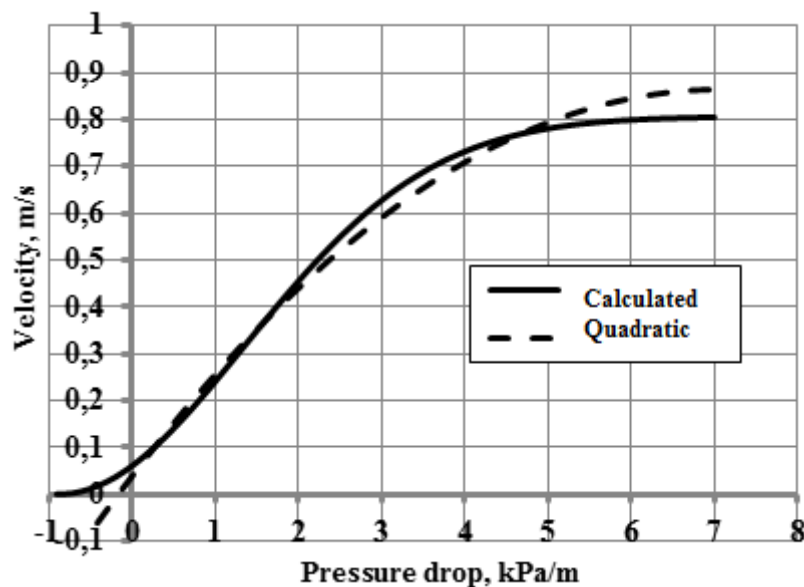


Figure 9. Extrapolation of experimental data on the calculated and quadratic dependences.

4. Conclusion

Thus, to describe the filtration process, it may be more appropriate to use the dependence of the filtration rate in the form (11), since it corresponds approximately to small and large filtration rates. Concavity and convexity can be compared with different laws of filtration, i.e. deviation from Darcy's law, a relatively rectilinear section conforms to Darcy's law. If the permeability is taken as a constant value, then the variable value for the ideal soil model for different constant pressure gradients of variable magnitude can only be viscosity, that is, Darcy's law takes the form:

$$\vec{v} = -\frac{k}{\mu(\text{grad}p)} \text{grad}p. \quad (13)$$

To take into account the influence of the structure of the medium, surface, interphase, inertial and other forces, additional experimental studies are required in combination with the construction of other filtration models.

References

- [1] Kosentino L 2007 *System Approaches to the Study of Reservoirs* (Moscow-Izhevsk: Institute of Computer Sciences, SRC Regular and Chaotic Dynamics)
- [2] Sokolov E T 1988 *Centaur, or How Mathematics Helps Physics* (Minsk: Vyshaya Shkola)
- [3] Basniev K S, Dmitriev N M, Kanevskaya R D and Maksimov I M 2006 *Underground Hydromechanics* (Moscow-Izhevsk: Institute of Computer Sciences)
- [4] Khairullin Az A and Khairullin A A 2013 *Application of the methods of tube hydraulics in the filtration model Non-Newtonian Systems in the Oil and Gas Industry*, Materials of the Intern. Sc. Conf. Devoted to the 85th Anniversary of Acad. Azad Khalil Ogly Mirzadzhanzadeh (Baku) pp 248-50
- [5] Khairullin A A 2016 *Nonlinear Models in Solving Applied Oil Production Problems* (Tyumen: IUT)
- [6] Gimatudinov Sh K and Shirkovsky A I 2005 *Physics of the Oil and Gas Reservoir* (Moscow: Alliance)
- [7] Muskat M 2004 *Flow of Homogeneous Liquids in a Porous Medium* (Moscow-Izhevsk: Institute of Computer Sciences)
- [8] Aziz K and Settari A 2004 *Mathematical Modelling of Reservoir Systems* (Moscow-Izhevsk: Institute of Computer Sciences)