Method of estimating linear dimensions of object by subsurface radar sounding

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Abstract. The methods for recognizing subsurface objects by the length of radar range portraits in a georadar with wideband probing signals, optimized for the maximum likelihood criterion for complex hypotheses, for the case of a known internal noise power, have been developed. It is shown that, using the optimal method, an energy gain is achieved when the threshold value of the probability of correct recognition is reached.

1. Introduction

One of the important tasks solved by the methods of georadiolocation is the search for, determination of the position and shape of subsurface objects and heterogeneities, including the estimation of their depth and linear size. Similar problems of object recognition by using broadband probing signals were successfully solved in radiolocation. A number of sources indicate the results obtained in this area [1-4, 7]. At the same time, these methods seem to be useful for georadiolocation as well. In particular, some results were obtained based on using probabilistic characteristics of subserfaced objects echoes [5].

So in this paper object recognition method using a short probing pulses georadar is proposed.

In typical cases, these can be pulses whose duration is in the range of 0.1 ... 2.0 ns or radio pulses with linear frequency modulation, which are subsequently compressed in the receiver using a matched filter. As a sign of recognizing a subsurface object, it is proposed to use its linear dimensions. The probing impulses of the georadar, reflected from the elements of the extended object, are recorded by the receiving device.

The output response of the receiver is a certain range portrait, the characteristics of which depend on the reflecting properties of the object and its geometry. When short probing pulses are used, the chaotic noise caused by irregularities in the ground surface and lying in the near zone of the antenna's effect disappear after a few nanoseconds and exert only a slight influence on the analyzed radar portrait of the object. An important problem is to provide the necessary electrical isolation between the transmitting and receiving antennas.

However, the signal of the transmitter penetrating the receiving antenna after partial suppression is used as the origin of reference when carrying out subsequent time measurements in the proposed method. To solve the identification problem, the signal from the receiver output is time-sampled, with R samples of the analyzed radar portrait of the subsurface object being formed.

Based on the accepted probabilistic model of the distribution of the received signals of the georadar, two statistical hypotheses are advanced on the linear size of the object. The first hypothesis is that the linear size of the object corresponds to the number of samples of discontinuities over the range in the range of 0 < R < R_1. The second hypothesis, which is alternative, is that the linear size of the object corresponds to the number of samples in the range of values R_1 <R <R_2. Here R_k, k = 1, 2, is the boundary value of the linear dimension corresponding to the number of samples R when the proposed hypothesis is valid. As a result of the comparison with the threshold value, a decision is made to accept or reject the hypothesis of the linear size of the object. If the test hypothesis is rejected, the procedure for estimating the linear size of the object is repeated for each new pair of hypotheses about its linear size.

The implementation of the processing device is reduced to checking the presence of a useful signal in the sample values of the signal at the output of the receiver, sliding estimation of the number of sample values of the envelope square of the output signal of the receiver in the pre-formed time gate for each selected hypothesis and comparing the results with the threshold value determined from the training sample. The analysis of the results of the simulation carried out made it possible to draw conclusions about the adequacy of the proposed method.

2. Results and discussions

We will develop the method under the following assumptions. The received echo is contained in the distance gate, several times larger than the maximum radial size of any subsurface object. The position of the strobe can be determined by the results of preliminary sounding. Then the video signal is sampled with a time interval of $\Delta\mathrm{t} = 1/\Delta F$, where ΔF is the width of the signal spectrum.

Such sampling allows us to consider the signal discrepancies to be statistically independent. Without loss of generality, we assume that the echo signal is taken against the background of a Gaussian noise σ_n^2 with unit variance (in practice this condition is realized with the help of the control on false alarm [6]).

The structure of the envelope of the echo signal (in particular, the amplitude $\{a_i\}$ and the phase $\{\varphi_i\}$ of the sample, $i = \overline{1, R_k}, k = 1{,}2$) is considered as a priori unknown. Here R_k is the maximum size of the distance portrait of a subsurface object of the *k*-th class, *k* = 1,2, measured by the number of sample of the radar range portrait (RRP). The transition to multi-alternative recognition (*k* > 2) is possible when using the proposed method and the procedure for dichotomous division.

We introduce the hypotheses H_1: $0 < R \le R_1$; H_2: $R_1 < R \le R_2$;, where R is the previously unknown length of the subsurface object's RRP, as measured by the number of RRP discrepancies. Obviously, the hypotheses H1 and H2 are complex. The likelihood function of the analyzed sample of a mixture of signal and noise may be shown as:

$$\mathrm{w_i}\,(\vec{\mathrm{y}}_{\mathrm{c}}, \vec{y}_s / \{a_j\}, b_i, \sigma., \{\varphi_j\}) = \frac{1}{\left(\sqrt{2\pi}\sigma\right)^{2N}} \cdot \exp\left\{-\frac{1}{2\sigma^2} \cdot \left\{\sum_{j_1=1}^{b_i-1}\left[y_{cj_1}^2 + + y_{sj_1}^2\right]\right.\right.$$

$$\left.\left. + \sum_{j=b_i}^{b_i+R_i-1}\left(\left[y_{cj} - a_{j-b_i+1} \cdot \cos\varphi_{j-b_i+1}\right]^2 + \left[y_{sj} - a_{j-b_i+1} \cdot \sin\varphi_{j-b_i+1}\right]^2\right) + \sum_{j_2=b_1+R_1}^{N}\left(y_{cj_2}^2 + y_{sj_2}^2\right)\right]\right\}. \quad (1)$$

The optimal method based on the maximum likelihood criterion for complex hypotheses [2.12] consists in the following: the hypothesis H2 is adopted if inequality:

$$\max_{a_i,\varphi_i; i=\overline{1,R_2}} \max_{R; R\in[0,R_1]} \max_{\mathrm{b}_2} \frac{1}{(2\pi)^N} \exp\left\{-\frac{1}{2}\left[\sum_{j_1=1}^{b_2-1}\left(y_{cj_1}^2 + y_{sj_1}^2\right)\right.\right.$$

$$\left. + \sum_{j=1}^{R_2}\left[\left(y_{c\,j+b_2-1} - a_j\cos\varphi_j\right)^2 + \left(y_{s\,j+b_2-1} - a_j\sin\varphi_j\right)^2\right]\right.$$

$$+\sum_{j_2=b_2+R_2}^{N}\left(y_{cj_2}^2+y_{sj_2}^2\right)\Bigg\}\Bigg/\max_{a_i,\varphi_i;i=\overline{1,R_1}}\ \max_{R;R\in[R_1,R_2]}\ \max_{b_1}\frac{1}{(2\pi)^N}$$

$$\times\exp\left\{-\frac{1}{2}\left[\sum_{j_1=1}^{b_1-1}\left(y_{cj_1}^2+y_{sj_1}^2\right)+\sum_{j=1}^{R_1}\left[\left(y_{c\ j+b_1-1}-a_j\cos\varphi_j\right)^2+\left(y_{s\ j+b_1-1}-a_j\sin\varphi_j\right)^2\right]\right.\right.$$

$$\left.\left.+\sum_{j_2=b_1+R_1}^{N}\left(y_{cj_2}^2+y_{sj_2}^2\right)\right]\right\}\rangle\Pi(\alpha),\qquad(2)$$

and H_1 is accepted otherwise. In the expression (1), y_{cj} and y_{sj}, $y_{c\ j+b_1-1}$ and $y_{s\ j+b_1-1}$ are the quadrature components of the received signal, N is the number of discrepancies in the strobe, Π is a threshold that determines the value errors of the first kind α, b_i is the number of sample (i = 1, 2), which determines the initial position for the main and alternative hypotheses about the maximum radial size of the object R_i , i=1,2, in the range gate.

Obviously, the maximum of the likelihood function with respect to R is attained at $R = Ri$. Using the method of the generalized maximum likelihood, we find the maximum likelihood estimates for the unknown parameters $\{a_i\}$ and $\{\varphi_i\}$ in (2) by differentiating and equating the partial derivatives of the numerator and denominator to zero, after substituting them in the expression for the likelihood and logarithmic function, we obtain the following decision rule: H_2, if:

$$\max_b\left[-\sum_{j_1=1}^{b_2-1}\left(y_{cj_1}^2+y_{sj_1}^2\right)-\sum_{j_2=b_2+R_2}^{N}\left(y_{cj_2}^2+y_{sj_2}^2\right)\right]$$

$$-\max_b\left[-\sum_{j_1=1}^{b_1-1}\left(y_{cj_1}^2+y_{sj_1}^2\right)-\sum_{j_2=b_1+R_1}^{N}\left(y_{cj_2}^2+y_{sj_2}^2\right)\right]\rangle\Pi'(\alpha)\qquad(3)$$

where Π' is some threshold different from Π; and H1 is accepted otherwise. We introduce the notation: $S_j^2=y_{cj}^2+y_{sj}^2$, $j=\overline{1,N}$; $S_N=\sum_{j=1}^{N}S_j^2$; $S_k=\sum_{j=b_k}^{b_k+R_k-1}S_j^2$, k = 1, 2. Then rule (3) is represented in the following form: H_2 is adopted, if:

$$S=\max_b\left[-(S_N-S_2)\right]-\max_b\left[-(S_N-S_1)\right]=\max_b S_2-\max_b S_1\rangle\Pi'(\alpha).\qquad(4)$$

If the hypothesis H_1 is true, then the difference value on the left-hand side of (4) is equal to the sum of the amplitude of samples in (R_2-R_1) with only the noise oscillations. If H_2 is true, then in the composition (R_2-R_1) of the samples on the left-hand side of inequality (4) contains the useful signal components belonging to the RRP. The maximum search for the initial position of the alternatives b_i (i=1,2) is performed by a searching procedure. For multi-alternative recognition, the procedure of dichotomous division is reduced to estimate the linear size of the object for a new pair of hypotheses about the size of R_i.

We will perform an analytical calculation of the quality indicators of the method under the following assumptions: 1) the distance from the beginning of the strobe to the beginning of the echo of the object is exactly known; 2) the envelope of the mixture of the useful signal and with the interference are distributed according to the Rayleigh law with the density:

$$w(S_i)=\frac{S_i}{1+q_0^{\,2}/2}e^{-S_i^2/2+q_0^{\,2}},\quad i=\overline{1,R_k},\quad k=1,2$$

where $q_0^2/2$ is the signal-to-noise ratio in the power sample; S_i is the RRP's envelope amplitude. The square of the envelope of a mixture of signal and noise has an exponential distribution as:

$$w(S_i^2)=\frac{1}{2+q_0^2}\cdot e^{\left[-S_i^2/2+q_0^2\right]}$$

which corresponds to the probability distribution $(1+(q^2{}_0/2))\cdot\chi^2$ with two degrees of freedom. Under these assumptions, the statistic S on the left-hand side of (4) has a χ^2-distribution with $2(R_2—R_1)=2\Delta R$ degrees of freedom if H_1 is true, and the probability distribution with density:

$$w(S)=\frac{S^{\Delta R-1}}{\partial(\Delta R)\left(2+q_0^2\right)^{\Delta R}}e^{-S/2+q_0^2},\quad i=\overline{1,R_k},\quad k=1,2$$

if H_2 is true. Here $\Delta R = R_2\text{-}R_1$ is the difference between alternatives in radial size. The expression for *w(S)* can be easily obtained from the known stability property of the χ^2-distribution [2]: the sum of independent random variables distributed according to the law χ^2 also has a χ^2-distribution with the number of degrees of freedom equal to the number of degrees of freedom of the terms. Then if $S_i^2 \propto \propto\chi^2{}_2\cdot[1+(q^2{}_0/2)]$, then:

$$S=\sum_{i=1}^{\Delta R}S_i^2 \ \propto\ \chi^2{}_{2\Delta R}\cdot[1+(q^2{}_0/2)].$$

The probability P_{22} of correct recognition of the object of the second class is calculated by the formula:

$$P_{22}=\int_{n'}^{\infty}w(S)dS=e^{-n'/\left(2+q_0^2\right)}\cdot\sum_{t=0}^{\Delta R-1}\frac{\left[n'/\left(2+q_0^2\right)\right]^t}{t!}. \qquad (5)$$

The threshold Π in (4) is determined by the quantiles of the χ^2-distribution with $2\Delta R$ degrees of freedom in accordance with a given level of significance.

3. Conclusion

Thus, an optimal method for estimating the linear dimension of subsurface objects along the length of their synthesis was used, the principle of generalized maximum likelihood was used. Analytic expressions were obtained for calculating decision thresholds for the Rayleigh distribution model of the signal-noise envelope. The application of the method will reduce energy costs to achieve a threshold value for the probability of correct recognition of subsurface objects.

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