

FORMATION OF OPTIMAL AUTOMATIC SYSTEMS FOR dissemination OF DIFFERENTIAL CORRECTIONS

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Abstract. Currently, local differential subsystems (LDPS) of GNSS GLONASS/GPS operating in the MF frequency range are deployed on the inland waterways of Russia. The paper deals with the problems of creating a continuous field of differential correction associated with the significant influence of the underlying surface on the distance of radio waves propagation in this range. It presents a method of constructing the optimal automatic system of dissemination of differential correction. The method is based on a technique developed by the authors for calculating the range of LDPS stations. The technique allows determining the range of stations by the criterion of a permissible error probability of the symbol-by-symbol message reception for different conditions (sea, land areas with different electrical properties). An algorithm for constructing an optimal differential system is developed. The results of comparison with the analytical solution and experimental data are presented. The deviation of the calculated data from the experimental data at a distance of 200 km does not exceed 5%, which is perfectly acceptable for this case. An example of the construction of the optimal LDPS structure for the Yenisey basin is given.

1. Introduction

The concept of e-navigation accepted by the International Maritime Organization (IMO) provides for the widespread introduction of various telecommunication systems. One of them is the automatic system of differential information of the global navigation satellite system GLONASS/GPS [1]. Local differential subsystems (LDPS) are widely used as such systems in river conditions. They are designed on the basis of control correction stations (CCS) within the range of sea radio beacons (283.5-325.0 kHz) that cover approximately 200÷500 km [2]. Such a large distribution is defined by considerable influence of the underlying surface on the distribution range of radio waves. In turn, this is defined by a large distribution of electric properties (specific conductivity and dielectric permeability) typical for inland waterways in Russia. This causes a problem of optimizing the structure of this system (CCS quantity and placement ensuring continuous coverage of navigable sites of river basins within the specified region).

The synthesis of river-based LDPS belongs to the class of tasks where the goal is achieved at the minimum resources. In this particular case the resource is first of all understood as the power and the number of CCS transmitters.

2. Methodology of designing the optimum system to disseminate the updating information

To optimize the structure of river-based LDPS there is a need to determine its CCS range. The compliance of error probability of a symbol-by-symbol detection of a digital message with its



allowable value is accepted in the calculation of a signal range. The assessment methodology is based on the known correlation between error probability of a symbol-by-symbol detection p_{err} and the signal energy h^2 under the influence of an accidental noise

$$p_{err} = 0.5 \exp(-0.5h^2). \quad (1)$$

Here, the signal energy is defined by the correlation

$$h^2 = \frac{P_r T}{v^2}, \quad (2)$$

where P_r – signal energy in receiving point; T – signaling period; v^2 – spectral density of a white accidental noise.

However, it shall be taken into account that the main requirement to river-based LDPS is continuous coverage of internal waterways (IWW) with the differential correction field. In this regard there is a need to overlap the coverage zones of the neighboring CCS, which leads to considerable mutual interferences within the crossing of these zones.

Within these zones the assessment methodology will be based on the mutual distinction coefficient (MDC) of a signal and a noise [3]

$$g_{rq}^2 = K_0 \left\{ \left[\int_0^T z_r(t) z_{nq}(t) dt \right]^2 + \left[\int_0^T z_r(t) \tilde{z}_{nq}(t) dt \right]^2 \right\}, \quad (3)$$

where $z_r(t)$, $z_{nq}(t)$ – structures of the r -option of a signal and q -concentrated noise; $\tilde{z}_{nq}(t)$ – function associated with $z_{nq}(t)$ according to Hilbert; K_0 – normalizing constant.

The normalizing constant for a digital signal is defined by the expression

$$K_0 = \left\{ \frac{\mu_r \mu_{nq}}{T P_r} \right\}^2 \quad (4)$$

where μ_r , μ_{nq} – amplitude ratios of signal transmission and the narrowband interference respectively.

Then, in case of incoherent reception, the error probability of a symbol-by-symbol detection of a digital message will be defined by the expression

$$p_{err} = -0.5 \exp \left[-\frac{h_{eq}^2}{2} \right] \cdot I_0(R_0 h_{eq}^2) + Q \left[\sqrt{\frac{h_{eq}^2}{2} (1 - \sqrt{1 - R_0^2})}; \sqrt{\frac{h_{eq}^2}{2} (1 + \sqrt{1 - R_0^2})} \right], \quad (5)$$

where Q – Marcum Q-function; $I_0(R_0 h_{eq}^2)$ – modified Bessel function of the first kind, zero order; h_{eq}^2 – equivalent signal energy in case of several mutual interferences; R_0 – radio channel parameter.

For the opposite non-fading signals

$$h_{eq}^2 = h^2 \left[1 - \sum_{k=1}^N \frac{g_{0k}^2 h_{nk}^2}{1 + h_{nk}^2} \right] = h^2 (1 - \delta), \quad (6)$$

where g_{0k} – standard MDC of a signal and k -mutual interferences; h_{nk} – energy of k -mutual interference; N – number of mutual interferences;

$$\delta = \sum_{k=1}^N \frac{g_{0k}^2 h_{nk}^2}{1 + h_{nk}^2}, \quad (7)$$

$$R_0 = \delta / (1 - \delta). \quad (8)$$

3. Method to define the structure of optimum differential subsystem

Depending on whether a signal receiver is only under the influence of an accidental noise or it is exposed to mutual interferences, either (1) or (5) will be used to define the error probability of a symbol-by-symbol detection.

In both cases there is a need to define the signal energy according to (2), and in the second case the energy of every mutual interference. At the same time, to define the power of a signal or a noise

included into (2) it is necessary to find the field intensity of a signal in the receiving point $E(r)$, which for the radiator in the form of a point dipole is defined by the expression [4]

$$|E(r)| = -\frac{3\sqrt{10P_T}}{r}|w(r)|, \quad (9)$$

where P_T – radiated CCS transmitter energy; r – distance from the radiator to a receiving point; $w(r)$ – function of a signal strength loss due to the influence of the underlying surface.

The input energy of the receiver is bound to the field intensity through the known ratio

$$P_r = \frac{A_{eff}E^2}{240\pi}. \quad (10)$$

Here, A_{eff} – antenna effective area determined by the expression

$$A_{eff} = \frac{D_r\lambda^2}{4\pi}, \quad (11)$$

where D_r – directivity index of the receiving antenna; λ – radiation wavelength.

Substituting (9) and (11) into (10) we will receive

$$P_r = \frac{3D_T D_r \lambda^2 P_T w^2(r)}{32\pi^2 r^2}. \quad (12)$$

Here D_T – transmitting antenna directivity factor used in (9) when replacing a dipole with a real antenna.

Hence,

$$r = \frac{\lambda|w(r)|}{4\pi} \sqrt{\frac{3D_T D_r P_T}{2P_r}}. \quad (13)$$

Then the CCS range will be defined by the expression

$$r_{\max} = B|w(r)|, \quad (14)$$

where B – energy potential of a radio channel [5]

$$B = \frac{\lambda}{4\pi} \sqrt{\frac{3P_T D_r D_r}{2P_{r,\min}}}. \quad (15)$$

The $P_{r,\min}$ value, defining the minimum allowable sensitivity of a receiver under the exposure of fluctuation noise only, is determined from (2) following the minimum allowable signal energy h_{\min}^2 , which is calculated from (1) according to the maximum allowable error probability of a symbol-by-symbol detection $p_{err, val}$.

Substituting (2) into (1) taking into account (12), we can express r_{\max} in its explicit form

$$r_{\max} = \frac{\lambda}{8\pi} \left[-\frac{3D_T D_r P_T T}{\ln(2p_{err, val})N^2} \right]^{1/2} |w(r)|. \quad (16)$$

Generally, when the receiver input is exposed to mutual interferences, this algorithm of CCS range calculation is not applicable. Here the most allowable value of error probability of a symbol-by-symbol detection directly serves the limiting condition. At the same time the CCS range r_{\max} corresponds to the condition

$$p_{err} < p_{err, val}. \quad (17)$$

In this case the error probability is calculated according to (5). The energy of a signal and mutual interferences included into (5) are defined according to (2) taking into account (9) - (11). The equivalent energy is calculated according to (6). The standard MDC for a signal and a mutual interference, having similar structure, is defined by the expression

$$g_{0q}^2 = \left\{ \frac{\sin[0.5\Omega_{nq}T]}{0.5\Omega_{nq}T} \right\}^2, \quad (18)$$

where Ω_{nq} – carrier-frequency shift of a signal and q -interference.

In its explicit form the dependence r_{\max} on $p_{err, val}$ in case of incoherent reception of phase-shift keyed signals may be obtained only under the influence of one concentrated interference for a

particular case: non-fading signal – noise faded under the Reyleigh law. Then the expression for error probability of a symbol-by-symbol detection will be as follows

$$p = 0.5 \exp \left[-\frac{h^2}{(2 + h_n^2 g_0^2)} \right]. \quad (19)$$

Hence, taking into account (9) - (11), we will receive

$$r_{\max} = \frac{\lambda}{4\pi} \left[-\frac{3D_{rr}D_rP_rT}{2\ln(2p_{\text{err val}})v^2} \frac{1}{(2 + h_n^2 g_0^2)} \right]^{1/2} |w(r)|. \quad (20)$$

The main difficulty in solving (16) and (20) is to define the attenuation function $w(r)$. For distances of approximately 200÷300 km the attenuation function may be described by the simplified Hufford equation for a spherical surface [6]

$$w(r) = 1 + i \sqrt{\frac{ir}{\lambda}} \int_0^r w(x) \left[\frac{1}{\sqrt{\varepsilon(x)}} + \sin \left(\frac{r-x}{2a} \right) \right] \frac{1}{\sqrt{x(r-x)}} dx, \quad (21)$$

where a – earth radius; $\varepsilon(x)$ – complex dielectric constant of an underlying surface; x – distance from the source to the current integration point.

The expression (21) represents the Volterra integral equation of the second kind and looks as follows

$$w(r) = f(r) + \rho \int_0^r w(x) K(r, x) dx \quad (22)$$

In this case, the constant term identically equals one, and the kernel contains a complex variable (dielectric conductivity $\varepsilon(x)$). Therefore, the (21) will be solved as

$$w(r) = \text{Re}(w(r)) + i \text{Im}(w(r)). \quad (23)$$

After simple transformations let us finally receive

$$w(r) = 1 - \int_0^r w(x) \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left(\frac{\sqrt{|\varepsilon| - \varepsilon''(x)} + \sin \left(\frac{r-x}{2a} \right) |\varepsilon|}{\sin \left(\frac{r-x}{2a} \right) |\varepsilon|} \right) dx + \\ + i \int_0^r w(x) \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left(\frac{\sqrt{|\varepsilon| + \varepsilon''(x)} + \sin \left(\frac{r-x}{2a} \right) |\varepsilon|}{\sin \left(\frac{r-x}{2a} \right) |\varepsilon|} \right) dx. \quad (24)$$

Here $|\varepsilon|$ – complex dielectric constant module defined by the expression

$$\varepsilon = \frac{\varepsilon' + i2c\sigma}{\lambda} \quad (25)$$

where ε' – specific dielectric constant; c – light speed; σ – specific conductivity.

4. Results and discussion

The analytical solution (24) is only possible for certain cases. Therefore, the numerical integration using a quadrature method was used here.

The kernel of the equation (24) looks as follows

$$K(r, x) = \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left[-\left\{ \sqrt{|\varepsilon| - \varepsilon''(x)} + \sin \left(\frac{r-x}{2a} \right) |\varepsilon| \right\} + \right. \\ \left. i \left\{ \sqrt{|\varepsilon| + \varepsilon''(x)} + \sin \left(\frac{r-x}{2a} \right) |\varepsilon| \right\} \right] \quad (26)$$

Then the numerical solution may be found through the known iterative formula

$$w_k = \frac{1 + \sum_{j=1}^{k-1} A_{kj} K_{kj} w_j}{1 - A_{kk} K_{kk}}, \quad (27)$$

where $k = 0, 1, 2, \dots, N$; N – number of nodes; $A_{kj} = p_{kh}$ – set of integration coefficients; h – integration step; $p_{kh} = B a_k$ – weight ratios for various degrees of quadrature formulas; $w = w(r_k)$; $K = K(r_k, x_j)$.

Since the kernel (26) has certain singularity at its end due to a denominator $x(r-x)$, the direct use of (27) is impossible. To eliminate such singularity each integral in (24) is split into three parts. In the first integral a variable according to condition $x=y^2$ is replaced, and in the third one – to condition $x=r-y^2$. Then we get

$$\int_0^r K(r, x)w(x)dx = \int_0^{\sqrt{n_1 h}} K_1(r, y^2)w(y^2)dy + \int_{r-n_2 h}^{r-n_1 h} K(r, x)dx + \int_0^{\sqrt{n_2 h}} K_3(r, r-y^2)w(r-y^2)dy \quad (28)$$

where n_1 and n_2 – number of points, according to which the first and the last integrals in (28) are calculated respectively.

The software in a program shell MATLAB [7] is developed to implement the obtained algorithm. The program used the following source data: earth radius, light speed, wavelength, path subinterval, number of areas with various dielectric capacity and electric conductivity and the number of nodal points of each area. Besides, the array of ε' and σ values are set in nodal integration points.

To test the obtained algorithm and the developed program the solution of a task for a route consisting of two areas (highly conductive soil and sea) was obtained, for which the analytical solution and experimental data are given in [4]. The test results are given in Fig. 1.

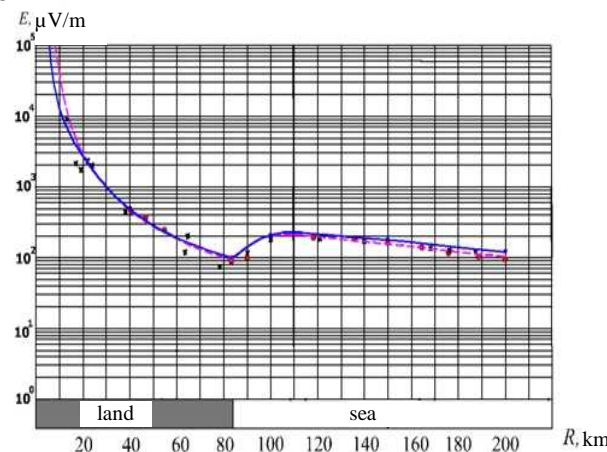


Figure 1. Comparison of theoretical and experimental data with results of numerical integration:

- analytical solution;
- numerical integration;
- × • experiment (1 and 2 series).

Source data for calculation: $P_T=10$ kW; $\lambda=96$ m; land area – 84 km; sea area – 116 km; land: $\varepsilon'=10$, $\sigma=0.009$ Cm/m; sea: $\varepsilon'=80$, $\sigma=4$ Cm/m.

The diagram shows that the developed algorithm of numerical integration of the Hufford equation well coincides with the analytical solution for a spherical surface and the test results.

The developed algorithm and the program were used to optimize the structure of CCS LDPS within the inland waterways of the Russian Federation. The underlying surface was considered as piecewise and uniform. Beams dispersing from the station, along which the area sizes with various electric properties, as well as with ε' and σ values for each of them, were built to define the CCS coverage. The size of the coverage area was determined for all directions from (16) or (20) after the attenuation function was defined via numerical method.

The equation (20) was used if the coverage areas of the neighboring CCS with mutual interferences overlapped in a given direction.

Since (16) and (20) belong to the class of transcendental equation, their solution, and, therefore, the CCS range in each set direction is defined by the method of successive approximations. Whereas, the r_{ass} is set and $w(r_{ass})$ is defined according to (21) at the first stage. Then, according to (16) or (20) the r_{max} is defined. If $r_{max} > r_{ass}$, then on the following stage the r_{ass} increases, if $r_{max} < r_{ass}$, then r_{ass} decreases and the calculation is repeated until the condition $|r_{max} - r_{ass}| < \Delta r$ is satisfied. Here the Δr value determines the specified CCS range accuracy.

The design of the optimum LDPS structure for the Yenisei basin is considered as an example illustrating the described technique. Fig. 2 shows the fragment of designing the CCS coverage areas for the most complex section of the waterway from the point of view of the interfering situation, where three coverage areas of the neighboring CCS overlap at once. The following source data were used in calculations: output power of all transmitters – 400 W; $\lambda=1000$ m; $v^2=10E-12$; $T=0.01$ s; $p_{err, val}=10E-3$; pitch of frequency rifling of CCS transmitters – 500 Hz; $\Omega_{nq}=2\div 20$ kHz; $D_T, D_r=0.9$.

5. Conclusions

The optimization of the LDPS structure made it possible to cover a large Yenisei basin with continuous field of differential correction using seven CCS only (three northern CCS on linear navigable section of the Yenisei are not shown in Fig. 2). Fig. 2 shows that the CCS range in this region extends within the limits of 350–450 km.

In most cases, due to rational distribution of carrier frequencies of the neighboring CCS it was possible to reduce mutual interferences in the fields where the CCS coverage areas overlap, which made it possible to almost avoid the reduction of stable reception range in these directions.

The creation of electric arrays of the underlying surface ε' and σ became the main problem in defining the CCS range significantly influencing the computational accuracy. The existing soil maps of Russia do not contain much details, therefore taking into account the errors of incremental solution of transcendental equations (16) and (20) it makes sense to set the accuracy values Δr not less than 3–5 km.

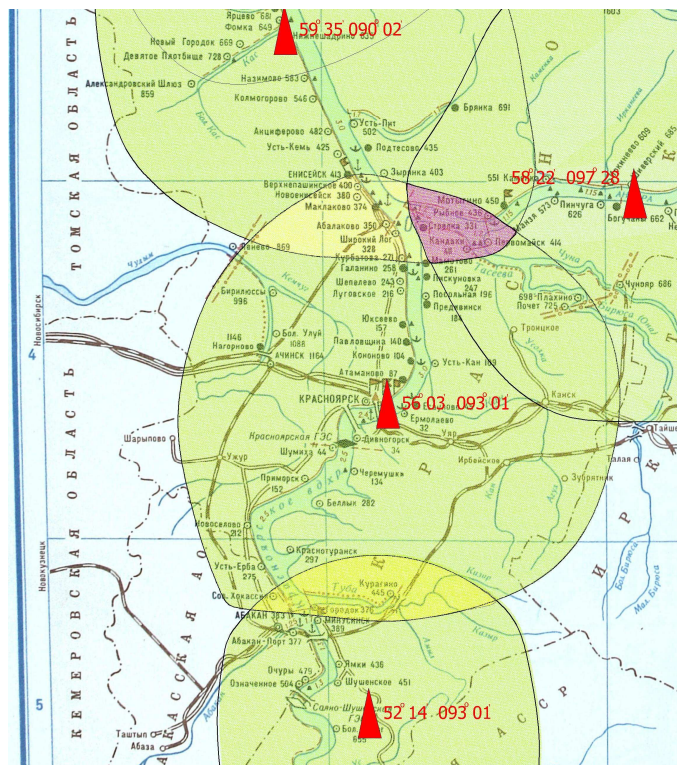


Figure 2. LDPS structure for the Yenisei basin

References

- [1] Jonas M, Oltmann J-H 2013 IMO e-Navigation Implementation Strategy – Challenge for Data Modelling, *TransNav, International Journal on Marine Navigation and Safety of Sea Transportation* **7**, **2** 45-49
- [2] Zhang Y, Bartone C 2004 A General Concept and Algorithm of Projected DGPS for High-Accuracy DGPS-Based Systems *Navigation* **51**, **4** 293-309
- [3] Shahnov S F 2015 *Interference immunity and stability of radiolines of the river differential subsystems of GNSS GLONASS. GPS: monograph.* (SPb.: Iz-vo Politekn. un-ta)
- [4] Fejnberg E L 1999 *Propagation of radio waves along the earth's surface: monograph. 2nd edition.* (M.: Science)
- [5] Karetnikov V V, Sikarev A A 2013 *Topology of differential fields and range of control and correction stations of high-precision positioning on inland waterways.* (SPb.: Admiral Makarov State University of Maritime and Inland Shipping)
- [6] Senchenko A A, Salomatov Y P 2012 A comparison of quadrature methods for a solution of Hufford integral equation, *Proceedings of TUSUR University* **2**, **2** (26) 36-41
- [7] Shahnov S F 2015 Calculation of function field weakening of the control and correction stations taking into account the influence of the underlying surface *Vestnik Gosudarstvennogo universiteta morskogo i rechnogo flota imeni admirala S.O. Makarova* **1**(29) 116-123