

# Automated adaptive drive for sucker rod pump

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**Abstract.** The paper is devoted to the optimization of the cycloidal mechatronic drive of automation and robotic facilities according to the criteria of power and energy consumption minimization. The paper presents the basic mathematical model of a drive and the method of choosing types of motion laws, their parameters, and design features based on the above-mentioned criteria.

## 1. Introduction

A reduction of resources consumption is a highly relevant challenge in any industry. In this case the reduction of resources consumption usually refers primarily to energy saving, as well as a reduction of the drive installed power (a decrease of the standard size of a motor, converters, switching and protective equipment). The paper considers the most common cycloidal (reciprocating) drive [1-5, 6, 7]. It primarily refers to a rack-and-gear drive, which is widely used as a drive of sucker rod pumps, injection molding machines, load lifting and conveying systems [8, 9]. Most of developers design the cycloidal drive based on the average power consumed during a cycle, taking into account maximum force or torque [5, 7, 10, 11]. There are no data in the literature concerning possibility to decrease considerably the peak drive power (frequently taken for the installed power) by means of rational choice of a motion law with specified and unchangeable values of the cycle and the working body travel. Meanwhile, up-to-date mechatronic systems enable one to reproduce almost any motion law [12].

The paper presented is devoted to technical constituent of the issue: determination of motion law rational parameters, ensuring minimum peak power with specified cycle duration, travel value, as well as assessment of the energy amount used.

## 2. The problem and solution

Reciprocal motion of the actuator of the mechatronic drive can be imparted according to different laws, determined by the process requirements and mechanical characteristics of the mover. A force and a torque acting on an output member defines its acceleration, which in practice can be measured by means of rectangular, trapezoidal, triangular, sinusoidal and other laws [10, 11].

At the first stage, the paper considers a standard case, when a working body of a cycloidal drive with mass  $m$  executes a reciprocal motion according to the symmetric trapezoidal speed variation law [13, 14]. Besides, it accelerates and brakes with accelerations identical in module but opposite in the sign. Only inertial loads act on the drive. In this case a half of the working cycle  $T$  – half cycle time (one way movement) can be considered, after introduction of parameters  $\tau$  – acceleration/brakeage time;  $b$  – working body acceleration;  $s$  – working body travel, the speed variation law can be written:

$$a(t, \tau, T, s) = \begin{cases} b & \text{if } \tau \geq t \\ 0 & \text{if } \tau < t \leq T - \tau \\ -b & \text{if } T - \tau < t \end{cases} \quad (1)$$



where  $t$  – current time,  $\tau$  - acceleration/brakeage time.

After double integration (1) and transformation, the equations for speed and motion can be derived, correspondingly:

$$v(t, \tau, T, s) = \begin{cases} bt & \text{if } \tau \geq t \\ b\tau & \text{if } \tau < t \leq T - \tau; \\ b(T - t) & \text{if } T - \tau < t \end{cases} \quad (2)$$

$$x(t, \tau, T, s) = \begin{cases} \frac{bt^2}{2} & \text{if } \tau \geq t \\ \frac{-\tau b(\tau - 2t)}{2} & \text{if } \tau < t \leq T - \tau \\ \frac{-b(T^2 - 2T\tau - 2Tt + 2\tau^2 + t^2)}{2} & \text{if } T - \tau < t \end{cases} \quad (3)$$

The normalization equation of the functions, given above and derived from (3) (coefficient  $b$  determination), takes the following form:

$$x(T, \tau, T, s) = s. \quad (4)$$

Unknown parameter  $b$  can be found from the equation (4) after substituting (3) in it:

$$b(\tau, T, s) = \frac{s}{\tau(T - \tau)}. \quad (5)$$

Final laws of motion (acceleration, speed and movement) are derived by means of substituting (5) in (1), (2) and (3):

$$a(t, \tau, T, s) = \begin{cases} \frac{s}{\tau(T - \tau)} & \text{if } \tau \geq t \\ 0 & \text{if } \tau < t \leq T - \tau; \\ \frac{s}{\tau(T - \tau)} & \text{if } T - \tau < t \end{cases} \quad (6)$$

$$v(t, \tau, T, s) = \begin{cases} \frac{st}{\tau(T - \tau)} & \text{if } \tau \geq t \\ \frac{s}{T - \tau} & \text{if } \tau < t \leq T - \tau; \\ \frac{s(T - t)}{\tau(T - \tau)} & \text{if } T - \tau < t \end{cases} \quad (7)$$

$$x(t, \tau, T, s) = \begin{cases} \frac{s}{2\tau(T - \tau)} & \text{if } \tau \geq t \\ \frac{s(\tau - 2t)}{2\tau(T - \tau)} & \text{if } \tau < t \leq T - \tau \\ \frac{s(T^2 - 2T\tau - 2Tt + 2\tau^2 + t^2)}{2\tau(T - \tau)} & \text{if } T - \tau < t \end{cases} \quad (8)$$

Fig.1 presents motion laws for the cycloidal drive with different parameters.

The value of minimum possible acceleration  $a_0$  with specified parameters of path -  $s$  and period -  $T$  can be found from (6) by means of substituting  $t = 0$  and  $\tau = T/2$  in it:

$$a_0(\tau, T, s) = \frac{4s}{T^2}. \quad (9)$$

In this case trapezoid degenerates into triangle.

Equation for determination of instantaneous power of the cycloidal drive takes the following form:

$$N(t, \tau, T, s, m) = a(t, \tau, T, s)mv(t, \tau, T, s) \quad (10)$$

where  $m$  – mass of moving parts.

Laws of motion with different parameters are shown in Figures 1 and 2. Figure 3 presents instantaneous power variation laws for the cycloidal drive with different parameters of acceleration/brakeage.

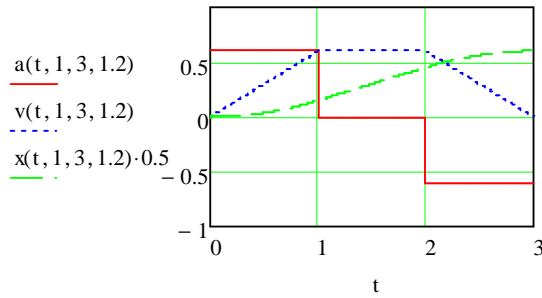
It follows from the graphs that minimum peak power is the case with acceleration/ brakeage time  $\tau = 1$  s.

For analysis of this, the equation for the peak power value can be written as follows:

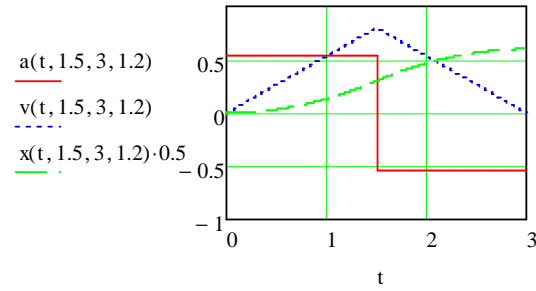
$$N_p(\tau, T, s, m) = v_{\max}(t, \tau, T, s)a_{\max}(t, \tau, T, s)m. \quad (11)$$

Taking into account (6) and (7), equation (11) is transformed as follows:

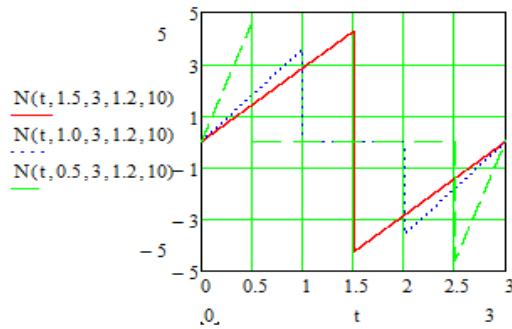
$$N_p(\tau, T, s, m) = \frac{ms^2}{\tau(T - \tau)^2}. \quad (12)$$



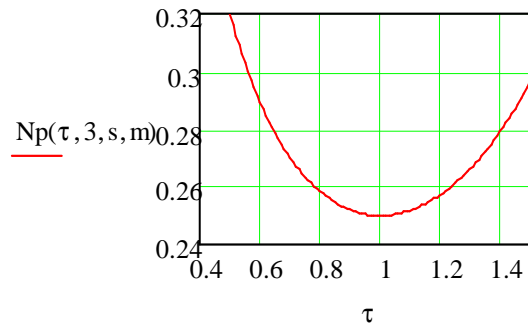
**Figure 1.** Laws of motion for cycloidal drive ( $\tau = 1s$ ,  $T = 3s$ ,  $s = 1.2m$ )



**Figure 2.** Laws of motion for cycloidal drive ( $\tau = 1.5s$ ,  $T = 3s$ ,  $s = 1.2m$ )



**Figure 3.** Dependence of instantaneous power on speed



**Figure 4.** Law of instantaneous power variation

Figure 4 presents law of instantaneous power variation for the cycloidal drive due to acceleration/brakeage time with  $s = 1.2m$  and  $T = 3s$ ,  $m = 10kg$ .

The necessary and sufficient condition for existence of minimum peak power is as follows:

$$\frac{dN_p(\tau, T, s, m)}{d\tau} = 0. \quad (13)$$

Or

$$0 = \frac{-ms^2(T-3\tau)}{\tau^2(T-\tau)^3}. \quad (14)$$

By solving (14) with respect to  $\tau$ , we can get:

$$\tau = \frac{T}{3}. \quad (15)$$

Thus, it is possible to assert that in case of trapezoidal speed variation law, the minimum value of the consumed peak power is the case, if times of acceleration and brakeage equal motion time with constant speed. In case of recovery (for example, while using the asynchronous motor with a frequency converter), usage of cyclogram with rational parameter  $\tau$  enable one to use a drive of minimum standard size.

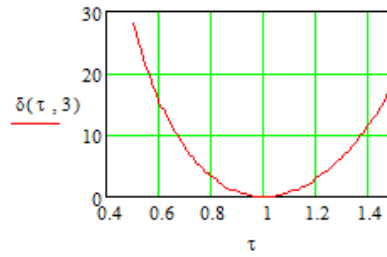
Efficiency of cyclogram optimization can be assessed on the basis of relative instantaneous power:

$$\delta(\tau, T) = \frac{N_p(\tau, T, s, m) - N_p(\frac{T}{3}, T, s, m)}{N_p(\frac{T}{3}, T, s, m)} 100\%. \quad (16)$$

By substituting (12) in (15) and after the required transformations are made, we can get:

$$\delta(\tau, T) = \frac{(T-3\tau)^2(4T-3\tau)}{27\tau(T-\tau)^2} 100\%. \quad (17)$$

Figure 5 presents dependence of relative instantaneous power for cycloidal drive on acceleration/brakeage time with  $s = 1.2m$  and  $T = 3s$ ,  $m = 10kg$ . It follows from the graph and the formula (17) that instantaneous power is determined only by ratio of  $T$  and  $\tau$ . When passing from rational cyclogram (trapezoid) to triangular cyclogram of speed, peak power increases by 18.5%, and in case of passing from parameter  $\tau = T/3$  to  $\tau = T/6$ , peak power increases by 28%. Thus, this phenomenon can be justifiably defined as “rule of one-third”.



**Figure 5.** Dependence of relative instantaneous power on acceleration time

Energy consumption for one cycle without recovery can be obtained by means of integration of peak instantaneous power variation law on the interval  $(0, T/2)$  or by determining the area of left triangle (when braking, kinetic energy dissipation is the case):

$$A(\tau, T, s, m) = 0,5v(t, \tau, T, s)a(t, \tau, T, s)m\tau. \quad (18)$$

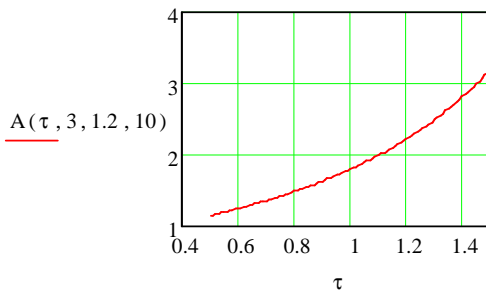
Taking into account (6) and (7), equation (18) can be transformed as follows:

$$A(\tau, T, s, m) = \frac{ms^2}{2(T-\tau)^2}. \quad (19)$$

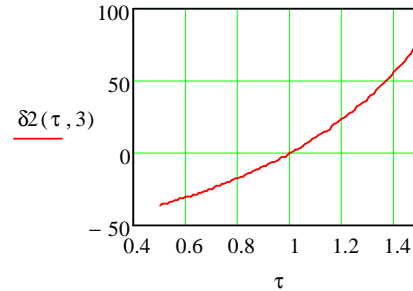
Dependence of relative cycle energy variation compared to energy consumption with optimal value  $\tau$  takes the following form:

$$\delta_2(\tau, T) = \frac{\frac{5T^2}{9} - 2T\tau + \tau^2}{(T-\tau)^2}. \quad (20)$$

Figure 6 presents dependence of energy consumption on acceleration/brakeage time, and Figure 7 presents dependence of relative energy consumption on  $\tau$ .



**Figure 6.** Dependence of energy consumption on acceleration/brakeage time



**Figure 7.** Dependence of relative energy consumption on acceleration/brakeage time

It follows from the graph given in Figure 7 that a decrease of peak power by 18.5%, found above, is attended with deduction of energy consumption per one cycle by 78%, and a decrease of acceleration/brakeage time from  $\tau = T/3$  to  $\tau = T/6$ , it leads to a decrease of the power consumed by 36% that will be attended with an increase of peak power by 28%.

Parameter  $\tau$  can be substituted with parameter  $k$  or  $\xi$ , which are interdependent:

$$k(\tau, T) = \frac{T^2}{2\tau(T-\tau)} \quad (21)$$

$$\xi(\tau, T) = \frac{T-2\tau}{T-\tau} \quad (22)$$

where  $k=a/a_0$  is coefficient of exceedance of minimum possible acceleration,  $a_0$  – acceleration ensuring triangular speed variation law with specified  $T$  and  $s$  (minimum achievable acceleration with those parameters),  $\xi = s_0/s$  – coefficient of motion steadiness of output member (ratio of the length of trajectory section, at which output member moves with constant speed),  $s$  – length of trajectory section, where output member moves with constant speed.

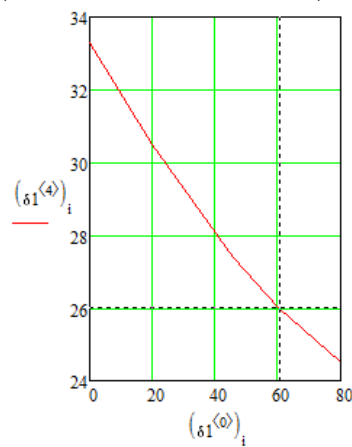
By substituting  $\tau = T/3$  in (21) and (22), we can get the following:

$$k = 9/8 = 1.125 \quad (23)$$

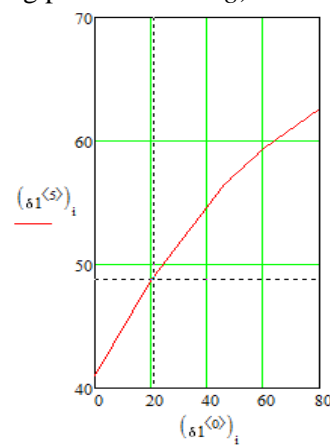
$$\xi = 1/2 = 0.5. \quad (24)$$

It follows from the given relations that in case of an optimal cyclogram on the basis of minimum peak power criteria, the actuator moves halfway with constant speed, or acceleration exceeds minimum possible acceleration by a quarter under such law (as happens with triangular speed variation law). In this case validity of converse propositions should be mentioned, that is, if any of the conditions are met, optimal motion law is the case.

When complicating the task by introduction of constant friction force  $F$ , formulas for analytical calculation of one of the mentioned parameters become too cumbersome, and it is reasonable to solve the task set by means of numerical methods. Results of those calculations are presented at the graphs. Figure 8 presents dependence of relative acceleration/brakeage time  $\tau/T$ , ensuring minimum peak power with specified  $T$  and  $s$  on friction force  $F$  (cycle time  $T = 0.9s$ , travel distance  $s = 1.2m$ , reduced mass of moving parts  $m = 10kg$ ). Figure 9 presents dependence of coefficient of motion steadiness of the output member, ensuring minimum peak power with specified  $T$  and  $s$  on friction force  $F$  (cycle time -  $T = 0.9s$ , travel distance -  $s = 1.2m$ , reduced mass of moving parts -  $m = 10kg$ ).



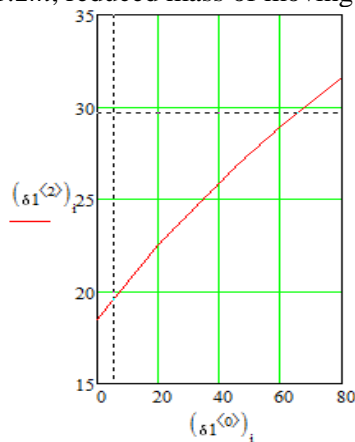
**Figure 8.** Dependence of  $\tau/T$  on friction force  $F$



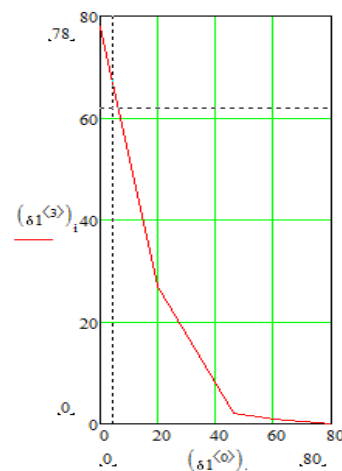
**Figure 9.** Dependence of coefficient of motion steadiness  $\xi$  on friction force  $F$

Figure 10 presents dependence of relative increase of minimum peak power, required for motion  $s = 1.2m$ , cycle time  $T = 0.9s$ , reduced mass of moving parts  $m = 10kg$  on friction force  $F$ , when passing from an optimal trapezoidal speed cyclogram to a triangular cyclogram.

Figure 11 presents dependence of an increase of cycle energy consumption on friction force  $F$ , when passing from the optimal cyclogram to the triangular cyclogram (cycle time  $T = 0.9s$ , travel distance  $s = 1.2m$ , reduced mass of moving parts  $m = 10kg$ ).



**Figure 10.** Dependence of relative power increase on friction force  $F$  when passing from optimal cyclogram to triangular cyclogram

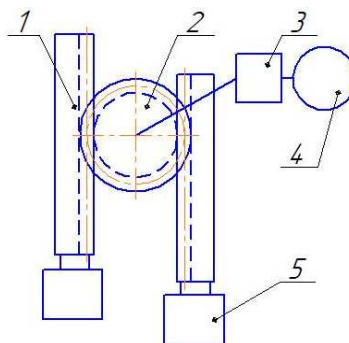


**Figure 11.** Dependence of increase of cycle energy consumption on friction force  $F$  when passing from optimal cyclogram to triangular cyclogram

On the basis of the above-mentioned procedure, dependence of maximum peak power on acceleration/brakeage time was investigated, taking into account constant friction force. Such task happens when developing a rational cyclogram of the rack-and-gear drive of sucker rod pumps with power up to 32kW. In the example, reduced mass of moving parts is 26700kg, constant load - 76000N, travel distance - 1.2m, cycle time - 3s (lifting of pump rod and plunger from lower end position to upper end position). For trapezoidal motion law implementation with acceleration/brakeage time 0.5s, the drive with peak power of 47kW will be required. For triangular motion law implementation (acceleration/brakeage time 1.5s), the drive with peak power of 69kW will be required. Thus, motion law optimization for this example enables one to use the drive with lower power (by 47%).

Traditional static balancing of masses in such machines is a particular problem. Inertial and gravity loads are often prevailing, with gravity load being proportional to unbalanced mass size, and inertial load being proportional to total reduced mass of the drive [15]. It is obvious that the increase of the reduced mass by means of static drive balancing leads to the decrease of gravity component of energy consumption, and increase of inertial one [13, 14]. As a consequence, there is a task, which is reduced to determination of the mechanism balancing level based on criteria of minimum energy consumption or peak drive power. It can be assumed that there are different conditions for implementation of those criteria.

In order to demonstrate the selection procedure of static balancing rational degree, the drive model under consideration is presented in Figure 12. An output member of mechanism 1, being a toothed rack with payload  $m$ , is driven by gear 2, installed at output shaft of the gearbox 3, which is connected to motor 4 (the reduced mass from moment of inertia of the armature  $m_0$ ). A counterbalance rack 5 with mass  $\mu m$  is provided for balancing of output member. The drive is loaded with unbalanced mass and inertial load (friction forces are negligible).



**Figure 12.** Drive model

The paper considers the simplest case, when member 1 reciprocates according to trapezoidal speed variation law. Acceleration and deceleration are executed with the same intensity.

Instantaneous drive power in the forward travel is determined with the following equation:

$$N_0(\tau, \mu, m, m_0) = \frac{s^2 mg}{T - \tau} \left[ \frac{1 - \mu}{s} + \frac{\mu + 1 + \frac{m_0}{m}}{g\tau(T - \tau)} \right] \quad (25)$$

where  $g$  – acceleration of free fall.

Figure 13 presents dependence of peak power on relative acceleration (brakeage) time with completely balanced and unbalanced mass ( $m = 10\text{kg}$ ,  $m_0 = 0\text{kg}$  or  $m_0 = 20\text{kg}$ ,  $s = 1\text{m}$ ,  $T = 0.2\text{s}$ ).

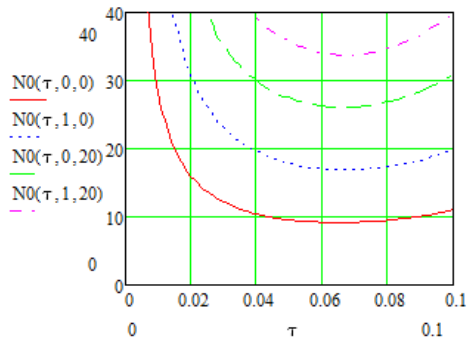
Analysis of graphs shows that for the cycloidal drive, operating with relative small dynamic loads, an extremum position of peak power is determined by acceleration (brakeage) time. Thus, acceleration (brakeage) time is one of conditions of reducing the drive to minimum energy consumption and peak power.

Complete balancing of moving masses for the example under consideration (reduced mass from inertia moment of motor rotor is low and amounts to  $m_0 = 0\text{kg}$ ) leads to an increase of peak power by 30% compared to the case without balancing. Flywheel installation (identical to significant inertia

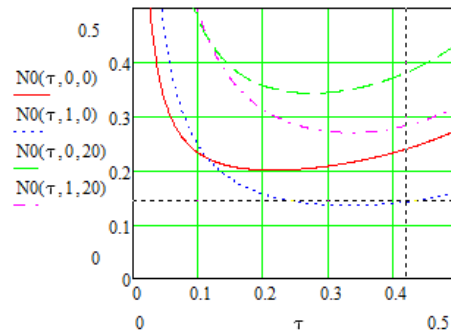
moment of the motor  $m_0 = 20\text{kg}$ ) on the mechanism with unbalanced mass leads to an increase of peak power by 84%. Power increases by 268% compared to the mechanism without counterbalance.

In case there is flywheel and larger inertia of the motor rotor, complete balancing of moving masses, for the example under consideration, leads to peak drive power only of 30%.

In order to assess influence of dynamic component on peak power, cycle time is increased up to 1s. The corresponding graphs of dependence of peak power on relative acceleration (brakeage) time with completely balanced mass and completely unbalanced mass ( $m = 10\text{kg}$ ,  $m_0 = 0\text{kg}$ ,  $20\text{kg}$ ,  $s = 1\text{m}$ ,  $T = 1\text{s}$ ) are presented in Figure 14.



**Figure 13.** Dependence of peak power on relative acceleration (brakeage) time,  $T=0.2\text{s}$



**Figure 14.** Dependence of peak power on relative acceleration (brakeage) time,  $T=1.0\text{s}$

The corresponding graphs in comparison with Figure 13 not only tend to the vertical axis, but also their extremums have changed position on the x-axis.

In case of relatively low dynamic loads without a flywheel and low small inertia of motor armature, complete balancing leads to a decrease of peak power by 33%. In this case, optimal acceleration (brakeage) time increases by 65% and becomes almost equal to one third of half-period, which coincide with extremum for the first case with  $T = 0.2\text{s}$ .

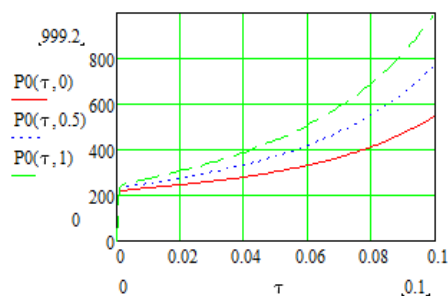
Complete balancing in combination with presence of the flywheel (or higher inertia of motor armature  $m_0 = 20\text{kg}$ ) decreases peak power by 21%. In this case optimal acceleration (brakeage) time increases by 27%.

The energy consumed during forward travel time can be determined by the following equation:

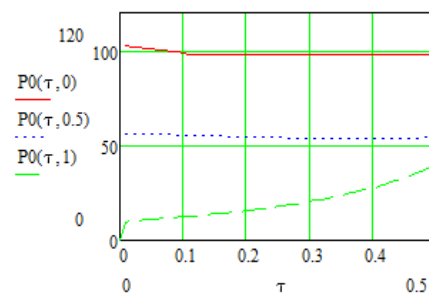
$$P(\tau, \mu) = \int_0^{\tau_0} N0(t, \tau, \mu, m, m_0) dt \quad (26)$$

where interval  $[0, \tau_0]$  corresponds to positive values of the function  $N0(t, \tau, \mu, m, m_0)$  (brakeage is carried out due to energy dissipation at the rest path).

Figure 15 presents dependence of the energy consumed per energy cycle on relative acceleration (brakeage) time with  $m = 10\text{kg}$ ,  $m_0 = 0\text{kg}$ ,  $m_0 = 10\text{kg}$ ,  $s = 1\text{m}$ ,  $T = 0.2\text{s}$  and different values  $\mu$ . Figure 16 presents similar dependences, but with less dynamic load and  $T = 1\text{s}$ .



**Figure 15.** Dependence of peak power on relative acceleration (brakeage) time,  $T=0.2\text{s}$



**Figure 16.** Dependence of peak power on relative acceleration (brakeage) time,  $T=1.0\text{s}$

The graphs of Figures 14, 15 indicate that, in case of relatively high dynamic load (the first case) with any balancing degree of vertically moving masses, increase of acceleration (brakeage) time leads

to significant increase of energy consumption, and in case of less dynamic load (the second case) depending on balancing degree of vertically moving masses, energy consumption can either increase, or decrease.

### 3. Conclusion

1. Analysis of scientific and technical literature has shown that there are currently no design practices of cycloidal drives on the basis of resource-saving criteria.
2. Rational choice of the ratio of acceleration/brakeage time and half cycle time by means of motion process modelling enables the designer to select feasible combination of motor standard size and energy consumption.
3. In case of trapezoidal speed variation law, there is minimum of drive peak power, the value of which depends on the ratio of reduced mass, load weight and balance mass, as well as relative time of acceleration (brakeage).
4. Static balancing of cycloidal mechanisms without considering dynamics can lead to a significant increase of energy consumption and peak power.

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