

Mathematical simulation of rotor tool technological process

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Abstract. Here we present theoretical research of rotor tool technological process for fertilizer spreading. To enhance the quality of fertilizer spreading by rotor spreaders we suggest using multi-bladed rotor tools. In theoretical research, the functioning of suggested multi-bladed rotor tools is described step-by-step: fertilizer capture by rotor blades from the row; fertilizer movement along the blades and free throw of the fertilizer particles and their distribution over the fields' surfaces. Equations are received to determine fertilizer relative amount coming on every rotor blade row. Basing on the scheme of forces acting on a particle during its movement along a rotor blade, we receive differential equations of motion. Taking into account the possibility to install the blades at different angles relative to radial position, equations to calculate absolute velocity and angle of fertilizer exit from the blade were deduced. Free flight of fertilizer particles with air forces acting on them has been studied; and differential equations describing this process have been deduced. The method to solve these equations has been suggested. Algorithm of mathematical program functioning is described in the conclusion of the article. This mathematical scheme is designed with the help of common program Maple 7.

1. Introduction

Nowadays rotor tools are widely used in many national economy sectors (trenching machines, root-extracting, blower-type snow ploughs etc.). In agriculture rotor tools are used in tilthers [1], grain blowers, and in fertilizer spreaders [2, 3]. In fertilizer spreading rotor tools provide high performance, which is several times higher than that one of carbony fertilizer spreaders, which allows to treat larger surfaces at the given agrotechnical terms.

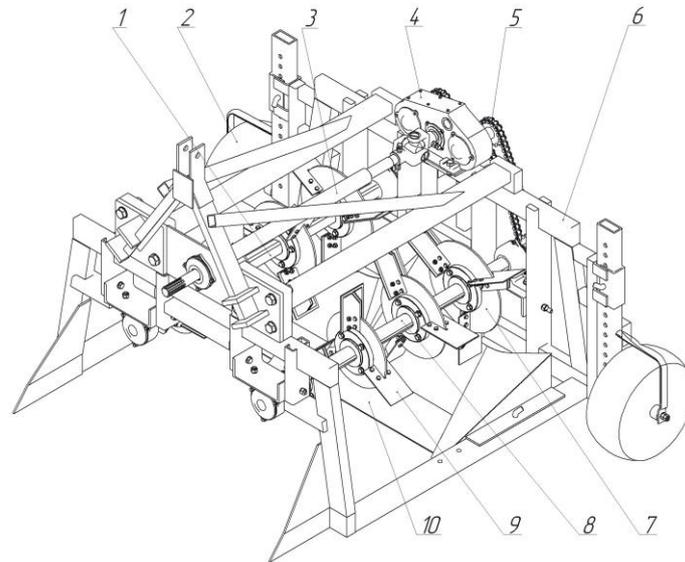
The drawback of serial rotor fertilizer spreaders is high unevenness of material spreading, which does not meet the agricultural requirements. The cause of the rotor swath spreader low productivity is in the imperfection of swathmakers and spreading tools.

Here we present a new multi-blade rotor fertilizer spreader (patent RF № 2222883) and mathematical simulation of its functioning.

2. Materials and methods

To provide high productivity of fertilizer spreading a construction of multi-bladed rotor fertilizer spreader from rows is suggested (picture 1).





Picture 1. Modernized fertilizer spreader with rolls:

1, 8 – multi-bladed rotors; 2 – bearing-adjusting wheels; 3 – driving shaft; 4 – reductor; 5 – chain gearing; 6 – frame; 7, 9 – rotor blades rows; 10 – lifting knife with flow divider.

During spreader movement the fertilizer row goes between the tractor wheels or catapillars to multibladed rotors which provide gradual fetching of fertilizers from the row and its spreading at different distance from longitudinal axis of the dispenser due to different length of blades.

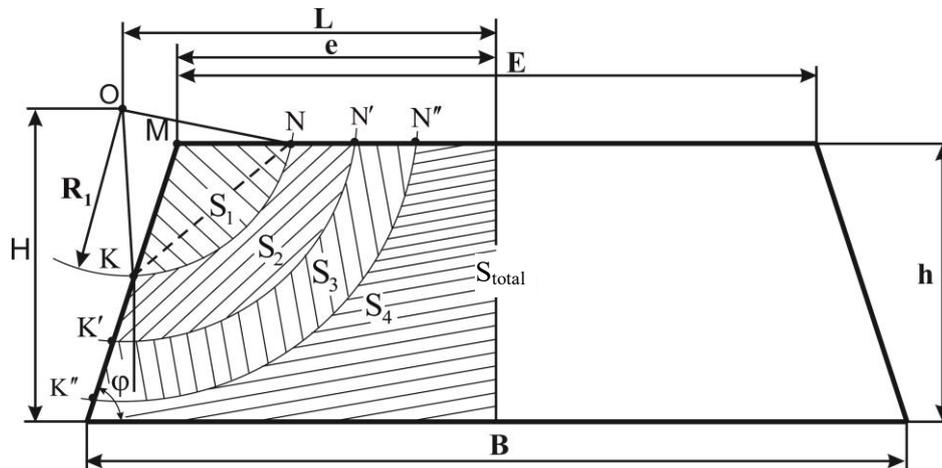
Theoretical research has been conducted to ground the rational constructional parameters and working regimes of tools during fertilizers spreading process. This process consists of several interconnected phases: fertilizer capture by rotor blades from the row; fertilizer movement along the blades and free flight of the fertilizer particles, Air forces acting on the particles during their flight and their distribution over the fields' surfaces.

The first stage of rotor tools operation includes blade movement in fertilizer row. Here the blade fullfills rotary and translatory movement along the row together with a spreader. Whereby, theoretically, the blades cut out the "chips" of fertilizer of a certain volume. At first sight, spatial fertilizer cutting takes place.

Serial rotor spreaders have tools rotational velocity n from 8.3 to 10 s^{-1} , translatory velocity V_t from 1 to 1.4 m/s, blades width b varies from 0.13 till 0.15 m. During movement of a spreader on distance b , the blade fullfills from 1.5 to 2 turns, that is why spatial fertilizer cutting can be neglected in calculations.

In the theoretical investigations conducted by other authors the amount of fertilizer coming to rotor blades was not taken into account. Only the area of rotor blades loading was taken into account. It is due to the fact that fertilizer movement, as number of solids with their restricted sizes, is considered as particles movement, and their movement as that one of material points. That is why in the known mathematical models of fertilizer spreading by rotor blades their amount was randomly chosen. With multi-bladed rotors this will lead to questionable values, as changing of ratio of blades length of different rotor rows will result in changing the ratio of fertilizer amount coming on them. It will influence the material distribution over the field's surface. We consider it is enough to determine the fertilizer amount in percent on the blades of every row in relation to the total mass of fertilizer row, that is in relative value.

The initial data to determine the material amount coming to every rotor blades row are the row parameters (B, E, h), rotors' rotational center coordinates (L, H), amount of blades rows, blades length in every row ($R_1, R_2, R_3 \dots R_z, z$ – amount of rotor blades rows) (picture 2).



Picture 2. Cross-section of a fertilizer row:

B, E are bottom and top width of a fertiliser row respectively, m ; e are half of fertiliser row bottom, m ; h are row height, m ; L, H are rotors' rotational center coordinates, m ; R_1 is the first row of blades radius, m ; φ is the angle of fertilizer natural slope, degrees, S_{total} is fertilizer row cross-section area, m^2 ; S_1, S_2, S_3, S_4 are areas of row section under the action of the first, second, third, fourth rotor row respectively, m^2 .

Let's accept, rotor blades of all rows are radially fixed, first row blades (the shortest) have radius R_1 , then during rotation they will catch the layer of fertilizers equal to S_1 area (picture 2). Basing on the equations of triaxial geometry the row cross-section area (S_1), captured by the first row blades is determined by the equation:

$$S_1 = S_{\Delta KMN} + S_{seg} = \frac{1}{2} \cdot KM \cdot MN \cdot \sin(180 - \varphi) + \frac{\pi \cdot R_1^2}{360} \cdot \arccos\left(1 - \frac{KN^2}{2 \cdot R_1^2}\right) - \frac{1}{2} \cdot R_1^2 \cdot \sin\left(\arccos\left(1 - \frac{KN^2}{2 \cdot R_1^2}\right)\right) \quad (1)$$

where R_1 is the length of the first row blades, m .

Using the formula (1) we can calculate the areas $S_2, S_3 \dots S_z$, determining relative amount of fertilizers, captured by the first, second, third and following blades rows. Then

$$S_2 = S_{\Delta MN'K'} - S_1; \quad (2)$$

$$S_3 = S_{\Delta MN''K''} - (S_1 + S_2); \quad (3)$$

$$S_z = \frac{S_{total}}{2} - (S_1 + S_2 + S_3 + \dots + S_{z-1}), \quad (4)$$

where z is the amount of blades rows.

Material load coordinate L_{01} of the first row blades will be determined by OM section (picture 2)

$$OM = \sqrt{(e - L)^2 + (h - H)^2}. \quad (5)$$

At mutual position of rotor rotation centers and fertilizer row in such a way that during fertilizer capture by first row blades their radius R_1 will not intersect point M , load coordinate L_{01} will be equal

$$L_{01} = H - h. \quad (6)$$

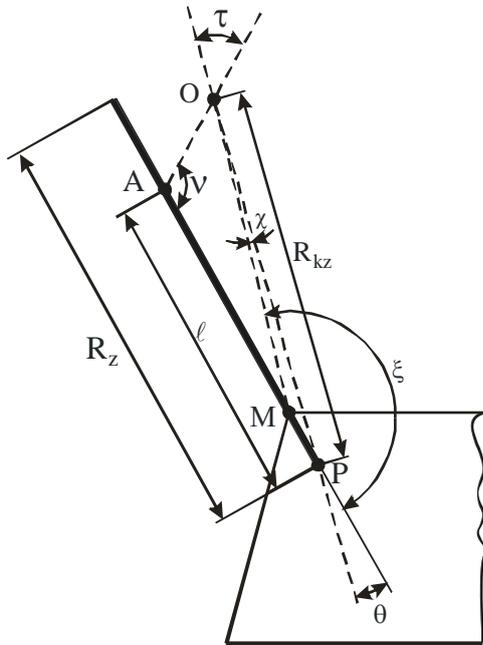
Basing on the assumptions accepted above, loading coordinates of the subsequent rotor rows blades for both studied cases will be respectively equal to

$$L_{02} = R_1; L_{03} = R_2; L_{0z} = R_{z-1}, \quad (7)$$

where L_{0z} is blades loading coordinate of z^{th} rotor row, m ;

R_z is blades radius (length) of z^{th} rotor row, m.

If rotor blades are at a certain angle θ relative radial position (picture 3), there appears a necessity to correct the zone (radius) of fertilizers capture from the row by blades of different rotor rows, as here the radii will decrease and the amount of fertilizer coming onto the blades of different rotor rows will differ. As well as the coordinates of blades loading L_{0z} will change.



Picture 3. The scheme to determine the fertilizer capture zone by rotor blades at their declination from radial position:

A is a pin-joint point, relative to which the blade is fixed at the angle θ to radial position; P is an extreme point of the blade; l is the distance between the pin-joint point A and the blade end, m; R_{kz} is the corrected radius of blades capture of z^{th} rotor row, m; θ is the angle of blades inclination relative to radial position, degrees; τ is the angle between the sections connecting rotor rotational center with the blade pin-joint point and with the extreme edge of the blade, degrees; v is the angle between blade plane and the section, connecting the centre of rotation with the pin-joint point, degree; ξ is the angle between the blade plane and the section, connecting rotor rotational center and the top surface of the row, degrees.

The corrected value R_{kz} for blades of any z^{th} row can be determined by the following way

$$R_{kz} = \frac{\sin v \cdot OA}{\sin \theta} \tag{8}$$

Here, $v = 180 - \tau - \theta$, $\tau = \arcsin\left(\frac{l}{OA} \cdot \sin \theta\right)$. (9)

When the blades are declined from the radial position in formula (1) it is necessary to use the corrected value of blade capture radius R_{kz} calculated by the equations (8) and (9) instead of R_z .

Here, if during fertilizer capture R_{k1} intersects point M (picture 3) then load coordinate L_{01} is equal

$$L_{01} = R_1 - MP; \tag{10}$$

$$MP = \frac{OM \cdot \sin \chi}{\sin \theta} = \frac{OM \cdot \sin (\xi + \theta)}{\sin \theta}; \tag{11}$$

$$\xi = \arcsin\left(\frac{R_{k1}}{OM} \cdot \sin \theta\right); \tag{12}$$

$$\chi = 180 - \xi - \theta. \tag{13}$$

If R_{k1} does not intersect point M then load coordinate L_{01} can be calculated according to the formula (6) and consequent rows blades by formula (7).

The second stage of the rotor tools functioning is characterized by fertilizer movement on rotor blades. Here the material fulfills a complex movement: rotational together with the blade and translator along its plane [4, 5].

At first, let's take several assumptions: fertilizers mass movement, as amount of solids with restricted sizes, will be considered as particles movement at first approximation, their movement as that one of non-interacting material points (hence, single particle movement is isolated from fertilizer

masses); blades surface and particles are considered non-deformable (inelastic); the particle moves along blades plane without rolling; there is no air resistance during particle movement.

Absolute velocity of fertilizer particle movement V_a at the moment of its coming off the blade is expressed by the geometric sum of velocities of relative V_r and translator V_e movement, that is:

$$\bar{V}_a = \bar{V}_r + \bar{V}_e, \quad (14)$$

Particle translatory movement velocity is perpendicular to blades plane and equal to

$$V_e = r \cdot \omega, \quad (15)$$

where r is the current distance from the particle to the axis of rotor rotation, m :

ω is angular velocity of rotor rotation, rad/s .

Relative movement velocity V_r is along the rotor blades. To determine it let's consider forces acting on the materials particle during its movement on the rotor blade, rotating around horizontal axis O with the constant angular velocity. Here, the blade is fixed at the angle $(+\theta)$ to the rotor radius (positive angle – forward to the rotor rotation) (picture 4).

The particle with mass m , which has come onto the blade at distance r_0 from rotor rotational center O , is influenced by [6-8]:

Particle mass force – $m \cdot g$;

Centrifugal force – $m \cdot r \cdot \omega^2$;

The force, caused by Coriolis force – $2 \cdot m \cdot \omega \cdot V_r$;

Friction force – $f \cdot N$,

where g – acceleration gravity force, m/s^2 ;

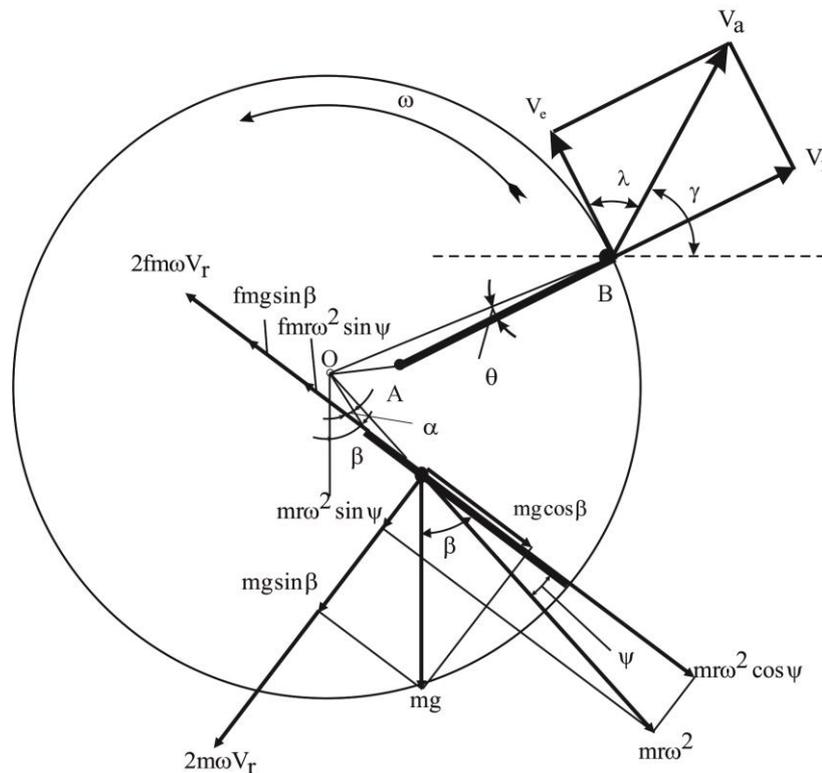
f is friction force between a particle and the blade;

N is normal blade reaction, H .

The centrifugal force and the particle gravity force can be decomposed into the forces acting in the blade plane – $m \cdot r \cdot \omega^2 \cdot \cos \psi$, $m \cdot g \cdot \cos \beta$, and the component forces acting perpendicular to the blades surface – $m \cdot r \cdot \omega^2 \cdot \sin \psi$, $m \cdot g \cdot \sin \beta$.

Total projection of all the forces acting on the particle perpendicular to the blades surface will be equal:

$$N = 2 \cdot m \cdot \omega \cdot V_r + m \cdot g \cdot \sin \beta + m \cdot r \cdot \omega^2 \cdot \sin \psi, \quad (16)$$



Picture 4. Scheme of forces acting on a particle during its movement on the rotor blade

where β is the angle between the mass force and the blade plane, rad;

ψ is the angle between the centrifugal force action and blade plane, rad;

r is the current distance from the particle to the rotor rotation axis, m.

Composing the total of forces projections in blade plane, equaling it to the composition of mass on acceleration with some mathematical transformations, we receive a differential equation of particle movement on the blade:

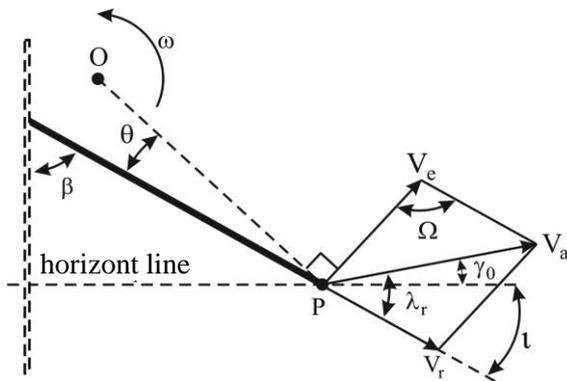
$$\frac{d^2x}{dt^2} = r \cdot \omega^2 \cdot \cos \psi + g \cdot \cos(\omega t) - 2 \cdot f \cdot \omega \cdot \frac{dx}{dt} - f \cdot g \cdot \sin(\omega t) - f \cdot r \cdot \omega^2 \cdot \sin \psi \tag{17}$$

Equation (17) is a linear non-homogenous differential second-order type equation with constant coefficients and the right side. Equation solution (17) was done by Rungge-Kutta method, improved by Feldberg, by the fourth and fifth order with the use of mathematical program Maple 7.

Integrating the equation (17) twice with the initial conditions $t = 0, x = r_0 = L_{oz}, \dot{x} = 0$, we receive the function of this equation solution (equation function of particle relative movement on the inclined blade). Applying the blade edge coordinate to the function we receive a transcendental function relative to movement time t , which is determined by the numerical method.

Having differentiated the function of the relative translation we determine the function of relative velocity of a particle movement. Applying the received value t to it, let's determine the particle velocity at the moment of its coming off the blade.

The third phase of the rotor tool functioning includes: fertilizer particles' coming off the blade with the absolute velocity V_a at the angle to the horizon γ_0 (picture 5); their movement in the resisting medium (the air); particles distribution over the soil surface.



Picture 5. Fertilizer particles exit angle correction: Ω is the angle between translational velocity direction and particle relative velocity projection, degrees; λ_r is the angle between the directions of relative and absolute velocities, degree; ι is the angle between the relative velocity direction and the horizontal line, degrees.

Fixing rotor blades radially the absolute velocity of a particle exit from the functioning tool is equal

$$V_a = \sqrt{V_e^2 + V_r^2} . \tag{18}$$

Let's determine the angle of exit γ_0 between the vector of particle exit absolute velocity and the horizontal line. For this let's determine the angle λ between the directions of translational V_e and absolute V_a velocities (picture 5)

$$\lambda = \arctg \frac{V_r}{V_e} . \tag{19}$$

Then, as picture 3 shows, we receive

$$\gamma_0 = \beta - \lambda = \beta - \arctg \frac{V_r}{V_e} , \tag{20}$$

where β is the rotor turning angle from the moment of a particle coming onto the blade and its exit, rad.

Fixing rotor blades at θ angle relative radial position (picture 4) particle exit absolute velocity from the tool equals:

$$V_a = \sqrt{V_e^2 + V_r^2 - 2 \cdot V_e \cdot V_r \cdot \cos \Omega} = \sqrt{V_e^2 + V_r^2 - 2 \cdot V_e \cdot V_r \cdot \sin \theta} ; \tag{21}$$

$$\Omega = 90 + \theta .$$

Particle exit angle γ_0 in the given case can be determined by the equation:

$$\gamma_0 = \left(\arcsin \left(\frac{V_e}{\sqrt{V_e^2 + V_r^2 - 2 \cdot V_e \cdot V_r \cdot \sin \theta}} \cdot \cos \theta \right) \right) - 90 - \beta . \tag{22}$$

After particle exiting the blade with the velocity V_a at the angle γ it flies freely in the resisting medium. The flight can be described by the system of the equations [8, 9, 10]:

$$\ddot{x} = -k_{\Pi} \cdot V_a^2 \cdot \cos \gamma$$

$$\ddot{y} = -g \mp k_{\Pi} \cdot V_a^2 \cdot \sin \gamma , \tag{23}$$

where k_{Π} is wind resistance coefficient, m^{-1} ;

g is gravity acceleration, m/s^2 .

In such form the equations (23) cannot be integrated. The differential equations of particles movement in the resisting air medium were solved numerically by Rungge-Kutta-Felberg method of the fifth and sixth order, using mathematical program Maple 7.

For the radially fixed blades the initial conditions are as follows:

$$x(0) = R_z \cdot \sin \beta ; \quad y(0) = H - R_z \cdot \cos \beta ; \quad \dot{x}(0) = V_a \cdot \cos \gamma ; \quad \dot{y}(0) = V_a \cdot \sin \gamma \tag{24}$$

For the blades declining from the radial position at θ angle, the initial conditions are as follows:

$$x(0) = R_{kz} \cdot \left(\frac{\sin \beta}{\cos \frac{\beta}{2}} \right) \cdot \cos \left(\frac{\beta}{2} - \theta \right); \quad y(0) = H + R_{kz} \cdot \left(\frac{\sin \beta \cdot \sin \left(\frac{\beta}{2} - \theta \right)}{\cos \frac{\beta}{2}} - \cos \theta \right); \quad (25)$$

$$\dot{x}(0) = V_a \cdot \cos \gamma; \quad \dot{y}(0) = V_a \cdot \sin \gamma. \quad (26)$$

The equations given above form the base of the mathematical model of fertilizer particle distribution process with multi-blades rotor. Basing on this a mathematical program is designed with the help of typical programming tools Maple 7, whose algorithm is shown in picture 5.

The initial data for the program are: rotor constructional parameters (number of blades rows, blades length of every row, angle of blades fixing relative radial position, angle of deflector shield setting under the last row blades, height of rotor setting above the field surface); rotors kinematic parameters (rotation frequency); formed fertilizer parameters (height and width of the row); physico-mechanical properties of fertilizers (natural slope angle, friction coefficient, wind resistance range).

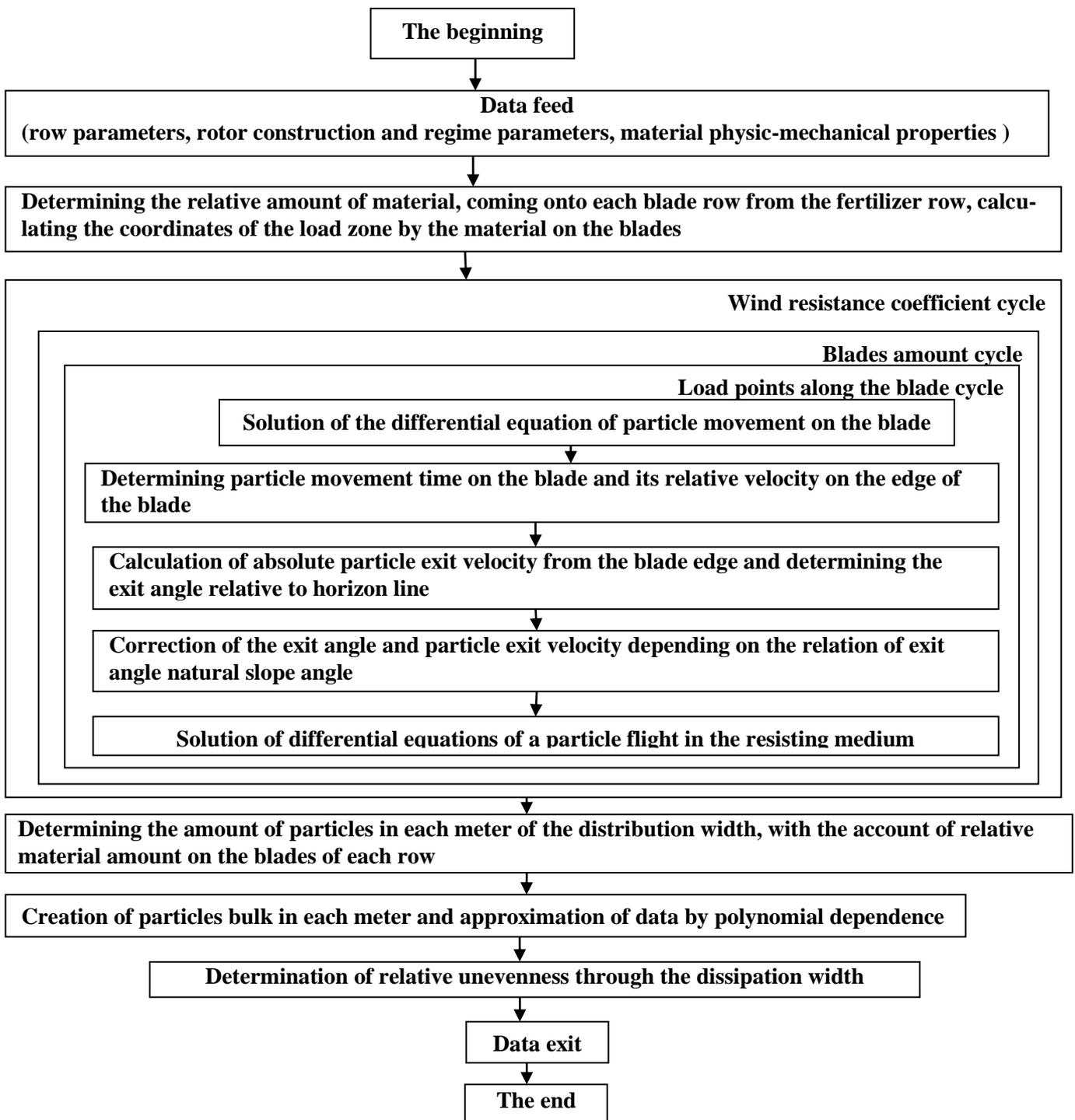
Basing on these parameters, the program calculates how much fertilizers come onto the blades of every row in relative amounts that is how many per cents of total relative amount of fertilizer row come onto the blades of every row. Besides, the zone of blades loading is determined.

Further, with the help of the incorporated programming cycle (inner cycle according to the loading points along the blade) an amount of particles is chosen on the blades of every row in the corresponding loading zone, evenly placed in the loading zone. The program solves differential equations of movement for every particle, calculates the time of movement along the blade, absolute velocity values and particle exit angles relative to the horizon line, also it determines the particle flight distance. Besides, for the last row of blades, which are the longest and have deflector shield, the correction of exit angle and exit absolute velocity is done depending on relation of the exit angle and the angle of deflector shield setting.

In the incorporated outer programming cycle (wind resistance coefficient cycle) variation limit of particle wind resistance coefficient is set, as well as several values of this coefficient, that must be used in the calculations. That is why the program calculates the distribution of all the particles, on all blades rows and for all values of the wind resistance coefficient. So, the bulk of particles is created, which are distributed at different distance from the origin of the coordinates, depending on the given initial parameters. During the calculation we can determine the distance of every particle flight, within the accuracy of 0,1 m. Besides, the, relative amount of the material on row blades as well as amount of particles at any point within the distribution width are determined.

At the given stage, the bulk of particles is created, which are at different points of distribution width k . The received data are approximated by polynomial dependence with the degree of polynom s . The degree value is taken as equal to the number of points received at the dissipation width, limited by coordinates of particle landing point from the origin of coordinate axes at a maximum distance.

Using tools for graphic data display, basing on the approximated data, we can draw a graph of theoretical distribution of fertilizers on the area width.



Picture 6. The algorithm scheme of fertilizer particle distribution process with multi-blades rotor tool

3. Conclusion

The presented mathematical program gives a possibility to vary: fertilizer row parameters; material physic-mechanical properties (friction coefficient, natural slope angle, particle wind-resistance capacity); rotor construction and kinematic parameters.

This program algorithm allows proving rational parameters of distributive rotor type tools at the stage of theoretical research.

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