

# Use of mechanical models for the analysis of antivibration mounting

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**Abstract.** The paper considers a two-stage vibration isolation of rotary machines including rotor-frame elastic linkage. It proposes internal elastically inertial vibration protection inbuilt between a rotor and a frame. The method of dynamic compliance is used to define forces influencing the frame. The proposed vibration motor can be used for rotary actuators of compact devices. The main objective of the study is to increase the motor torque and durability.

## 1. Introduction

The problems of vibration protection arise practically in all areas of modern technology. Their solution leans heavily on the specifics of vibration isolation system. The choice of the laws of motion regarding operating members of machines, mechanisms ensuring such motions, geometry of components and structures, their interface type and mechanical properties, materials and processing modes along with functional specifications shall meet the requirements of vibration reliability and safety [1].

Rotary machines apply various elastic elements inbuilt between a machine rotor and its frame. They allow adjusting the rotor-frame system against resonant modes thus reducing the impact of exciting forces on a frame and eventually on the base. However, due to design constraints imposed on maximum rotor movements and constraints caused by high static stress it is not always possible to apply elastic elements with the required compliance.

## 2. Two-stage vibration isolation

Generally, the vibration insulation may include two stages with stiffness coefficients  $K_1$ ,  $K_2$ ,  $K_3$ ,  $K_4$ , packing pieces with masses  $M_1$ ,  $M_2$  separating vibration insulation stages, and anti-vibrators (vibration dampers) with masses  $m_{1b}$ ,  $m_{2b}$  set up to rotor speed and adjusted to packing pieces via elastic elements with stiffness coefficients  $K_{1b}$ ,  $K_{2b}$  [2].

The performance criterion of the considered vibration insulation may be the coefficient equal to the correlation of forces transmitted to the frame when a rotor is installed on rigid or elastic supports to the forces influencing the frame when two stages of vibration insulation with anti-vibrators are used.

The oil wedge caused by pin movement in a bearing may be considered as the first stage of vibration insulation in sleeve bearings. This makes the design of anti-vibration node simpler. However, the forces



caused by the oil layer are generally non-linear. It links oscillations in vertical and horizontal planes and may lead to loss of rotor rotation stability. Let us apply the linearized solutions and consider oscillations in the vertical plane. The Lagrange's equation of second kind is used for motion equations.

The expressions for kinetic  $T$  ( $T$ ) and potential  $P$  ( $\Pi$ ) energies of the considered model oscillating in the vertical plane are as follows:

$$2T = m_p \dot{y}_p^2 + I_p \dot{\varphi}_p^2 + M_1 \dot{y}_{np1}^2 + M_2 \dot{y}_{np2}^2 + m_{1b} \dot{y}_{1b}^2 + m_{2b} \dot{y}_{2b}^2 + m_k \dot{y}_k^2 + I_k \dot{\varphi}_k^2; \quad (1)$$

$$2\Pi = K_p (y_p - y)^2 + K_1 (y - y_{np1} + l_1 \varphi_p)^2 + K_2 (y - y_{np2} - l_2 \varphi_p)^2 + \\ + K_{1b} (y_{1b} - y_{np1})^2 + K_{2b} (y_{2b} - y_{np2})^2 + K_3 (y_{np1} - y_k - l_3 \varphi_k)^2 + \\ + K_4 (y_{np2} - y_k + l_4 \varphi_k)^2 + K_m (y_k + l_3 \varphi_k)^2 + K_m (y_k - l_4 \varphi_k)^2; \quad (2)$$

where  $m_p$ ,  $m_k$  – masses of a rotor and a frame respectively;  $y_p$ ,  $y$ ,  $y_{np1}$ ,  $y_{np2}$ ,  $y_{1b}$ ,  $y_{2b}$ ,  $y_k$  – displacement of centers of masses of a rotor, rotor trunnion, packing pieces, anti-vibrators, and frame respectively;  $\varphi_p$ ,  $\varphi_k$  – rotation angles of a rotor and a frame respectively;  $I_p$ ,  $I_k$  – inertia of a rotor and a frame respectively in relation to their centers of masses  $s_p$ ,  $s_k$ ;  $K_p$ ,  $K_m$  – stiffness coefficients of a rotor within the center of masses, external depreciation.

The dissipation function  $F$  ( $\Phi$ ) is defined by the following expression:

$$2\Phi = C_p (\dot{y}_p - \dot{y})^2 + C_{1b} (\dot{y}_{1b} - \dot{y}_{np1})^2 + C_{2b} (\dot{y}_{2b} - \dot{y}_{np2})^2 + C_m (\dot{y}_k + l_3 \dot{\varphi}_k)^2 + C_m (\dot{y}_k - l_4 \dot{\varphi}_k)^2, \quad (3)$$

where  $C_p$ ,  $C_{1b}$ ,  $C_{2b}$ ,  $C_m$  – damping factors of a rotor, 'anti-vibrator - packing pieces' elastic linkage, external depreciation;  $\dot{y}_i = d y_i / dt$ ;  $\dot{\varphi}_i = d \varphi_i / dt$ .

Using the Lagrange's equation of second kind the expressions (1)-(3) allow receiving motion equations of the considered system under the influence of unbalanced inertia force of a rotor. At the same time it is possible to consider the following circumstances. Depreciation stages are ensured by steel rings, a rotor – by a solid body, therefore damping is not considered.

Generally, external depreciation includes rubber-metal shock-absorbers thus damping shall be considered. Elastic linkage of anti-vibrators is mainly ensured by springs thus damping is insignificant and does not impact the frequency range where the considered vibration insulation is efficient.

### 3. 1D vibration isolation model

To assess the efficiency of vibration insulation let us consider the one-dimensional model describing the vertical forced oscillations influenced by rotor imbalance with angular speed  $\omega$  [3]. The corresponding system of equations is as follows:

$$\left. \begin{aligned} m_1 \ddot{y}_1 + Q_1 (y_1 - y_2) &= m_1 e \omega^2 \sin \omega t; \\ m_2 \ddot{y}_2 + Q_1 (y_2 - y_1) + Q_2 (y_2 - y_3) + Q_4 (y_2 - y_4) + C_4 (\dot{y}_2 - \dot{y}_4) &= 0; \\ m_3 \ddot{y}_3 + Q_2 (y_3 - y_2) + Q_3 y_3 + C_4 \dot{y}_3 &= 0; \\ m_4 \ddot{y}_4 + Q_4 (y_4 - y_2) + C_4 (\dot{y}_4 - \dot{y}_2) &= 0; \end{aligned} \right\} \quad (4)$$

where  $e$  – rotor eccentricity;  $m_1 = m_p$ ;  $m_2 = m_{np1} + m_{np2}$ ;  $m_3 = m_k$ ;  $m_4 = m_{1b} + m_{2b}$ ;  $Q_1 = K_3 + K_4$ ;  $Q_2 = K_p (K_1 + K_2) / (K_1 + K_2 + K_p)$ ;  $Q_3 = K_m$ ;  $Q_4 = K_{1b} + K_{2b}$ ;  $C_3 = C_m$ ;  $C_4 = C_{1b} + C_{2b}$ ;  $y_1 = y_p$ ;  $y_2 = y_{np}$ ;  $y_3 = y_k$ ;  $y_4 = y_m$ .

The solution of the system (4) is as follows:

$$y_i = a_i \sin \omega t + b_i \cos \omega t, \quad (5)$$

where  $a_i, b_i$  – arbitrary constants.

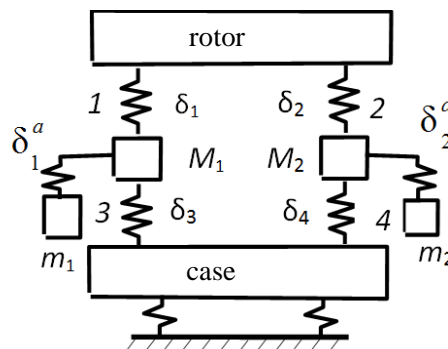
Plugging (5) in (4) we get the system of eight algebraic equations. For specific parameters of a machine the obtained solutions allow defining the vibration insulation coefficient  $\beta$  as a function of rotor angular speed

$$\beta = X / P = f(\omega),$$

where  $X = y_3(K_3 - m_k \omega^2)$  – amplitude of force impacting the frame in case of vibration insulation;  $P$  – amplitude of unbalanced inertial force of a rotor.

#### 4. Internal elastic inertial vibration protection

The internal elastic inertial vibration protection (IEIVP) inbuilt between a rotor and a frame (Fig. 1) is intended to reduce the vibration level on rotary machine arms caused by unbalanced forces of a rotor operating at low frequencies. Generally, the IEIVP may include two depreciation stages with compliance  $\delta_1, \delta_2, \delta_3, \delta_4$  of packing pieces with masses  $M_1, M_2$  separating depreciation stages, and anti-vibrators (vibration dampers) with masses  $m_1, m_2$ , set up to rotor speed and adjusted to packing pieces via elastic elements with compliances  $\delta_1^a, \delta_2^a$  [4].



**Figure 1.** Scheme of internal elastic inertial vibration protection

This dynamic model represents a system with concentrated and distributed masses and elastic inertia-free linkage exposed to deterministic excitement.

The performance criterion of the considered vibration insulation may be the coefficient  $\beta$  equal to the correlation of forces  $\bar{X}$  transmitted to the frame when a rotor is installed on rigid or elastic supports to the forces influencing the frame when two stages of vibration insulation with anti-vibrators are used.

The method of dynamic compliance is used to define forces influencing the frame [3]. The dynamic compliance of a system in any point is understood as the displacement amplitude caused by a single harmonious force or torque applied in the same or a different point. When this method is used the complex subsystem is divided into simple subsystems. The influence of linkage is replaced with reactions within these links. The studied dynamic model is split into four subsystems: a rotor, IEIVP blocks, a damped frame. The influence of subsystems on each other is defined by reactions  $X_1, X_2, X_3, X_4$  exerted in corresponding points. By defining the conditions, under which the relative movements of subsystems in

points of division are absent, we get the system of equations within the method of dynamic compliance. In its matrix perception this system is as follows

$$\begin{bmatrix} e_{\Sigma 1} & e_{p12} & -e_1 & 0 \\ e_{p21} & e_{\Sigma 2} & 0 & -e_2 \\ -e_1 & 0 & e_{\Sigma 3} & e_{\kappa 34} \\ 0 & -e_2 & e_{\kappa 43} & e_{\Sigma 4} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} e_{c1}^{(1)} \dots e_{cn}^{(1)} \\ e_{c1}^{(2)} \dots e_{cn}^{(2)} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_j \\ P_n \end{bmatrix}$$

where  $e_{\Sigma 1} = e_1 + e_{p1} + \delta_1$ ;  $e_{\Sigma 2} = e_2 + e_{p2} + \delta_2$ ;  $e_{\Sigma 3} = e_1 + e_{\kappa 3} + \delta_3$ ;  $e_{\Sigma 4} = e_2 + e_{\kappa 4} + \delta_4$ ;  $e_{p1}$ ,  $e_{p2}$ ,  $e_{p12}$  – dynamic compliances of a free rotor in fixed points;  $e_{cj}^{(1)}$ ,  $e_{cj}^{(2)}$  – dynamic compliances of a free rotor in fixed points characterizing the impact of exciting forces  $P_j$  applied in  $j$  rotor points;  $e_1$ ,  $e_2$  – dynamic compliances of free vibration blocks referred to as the packing pieces with elastically attached anti-vibrators;  $e_{\kappa 3}$ ,  $e_{\kappa 4}$ ,  $e_{\kappa 34}$  – dynamic compliances of the frame in points of connection with IEIVP blocks.

The linkage between a rotor and a frame is defined by the following matrix:

$$E_n = \begin{bmatrix} -e_1 & 0 \\ 0 & -e_2 \end{bmatrix}$$

The values of matrix elements may be obtained through forced oscillations of vibration blocks under the influence of a single harmonious force applied to a packing piece [5, 7, 8]

$$e_i = \frac{\omega_a^2 / \omega^2 - 1}{\omega^2 m_i [v_i - (1 + v_i) \omega_a^2 / \omega^2]},$$

where  $i = 1, 2$ ;  $\omega_a = 1 / \sqrt{m_i \delta_i^a}$  – frequency of anti-vibrators setup;  $\omega$  – frequency of exciting force;  $v_i = M_i / m_i$ .

Taking into account the equality  $\omega_a = \omega$  the dynamic compliances  $e_i = 0$ .

The system is split up into two independent subsystems: a rotor on elastic support (subsystem containing exciting forces) and a damped frame. At the same time,  $X_3 = X_4 = 0$  irrespective of the value of exciting forces influencing a rotor.

To define the influence of parameters of a rotor, a frame, and IEIVP blocks on effective bandwidth determined by  $|\beta| > 1$  there is a need to analyze force expressions at  $\omega \neq \omega_a$ . It is critical to define the required compliances of depreciation stages.

The limited compliances are generally defined as follows:

1. Allowable rotor subsidence depending on machine design features.
2. Allowable static stress in elastic elements of depreciation stages (elastic elements shall have enough strength to sustain the weight of a rotor and resist potential overloads).

Thus, it is advisable to solve the problem on the influence of compliance of depreciation stages on effective bandwidth in case of constraints determined by durability of elastic elements

$$\delta_1 \leq [\delta_1], \quad \delta_2 \leq [\delta_2], \quad \delta_3 \leq [\delta_3], \quad \delta_4 \leq [\delta_4], \quad (6)$$

and constraints determined by allowable rotor subsidence

$$\delta_1 + \delta_3 = \theta_1, \quad \delta_2 + \delta_4 = \theta_2, \quad (7)$$

where  $\theta_1$ ,  $\theta_2$  – target compliances.

At the same time, the selection of compliance values is stated as the problem of optimal distribution of target compliances of depreciation stages. In this case  $X_3$  and  $X_4$  may be considered as functions of two parameters

$$X_3 = f(\delta_1, \delta_2); \quad X_4 = f(\delta_1, \delta_2). \quad (8)$$

The minimum of functions (8) in case of constraints (1) and (2) is accepted as the criterion of optimality.

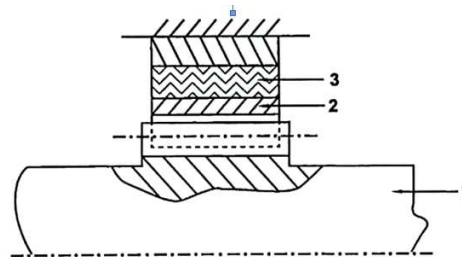
### 5. Wave ultrasonic motor

In recent decades the new kinds of drive mechanisms called vibration motors or ultrasonic motors are being widely developed. Vibration motors are based on the conversion of high-frequency elastic oscillations of solid or flexible bodies into directional one-dimensional motion of a mobile unit – a rotor [6].

The study of vibration motors showed their high efficiency when they are used as power units of manipulators and robots for miniature products. There are vibration motors that utilize piezoelectric elements applied as part of an electromechanical transducer.

The diagram (Fig. 2) shows the wave ultrasonic motor. The ridge surface of a rotor (1) is covered by a deformable gear-wheel (2), which is rigidly connected to a bimorph piezoceramic ring (3) with sector electrodes. The engine operates as follows. When voltage is supplied to electrodes of a bimorph piezoelectric ring the deformable ring is compressed in one direction and expands in perpendicular direction assuming a shape of an ellipse.

The teeth on the ends of a minor axis of the ellipse will fully engage rotor teeth, and the teeth located on the ends of the major axis will fully leave the rotor teeth. The direction of deformations of a deformable ring will change synchronously with voltage supplied to electrodes of a bimorph ring, and the teeth contact points will always run across a circle, and the rotor will rotate with a speed corresponding to the difference of teeth quantity of a deformable ring and a rotor.



**Figure 2.** Wave ultrasonic motor

The proposed setup may be used for rotary drives of compact devices. The main objective is to increase the motor torque and durability. The technical result of the suggested invention is the reduction of friction losses and increase in motor efficiency. The specified technical result is achieved due to a gap between a deformable ring and a rotor, and fine teeth cut on surfaces of a ring and a rotor.

### 6. Conclusions

The considered two-stage vibration insulation of rotor machines is efficient at low frequencies. It is advised for use in single-mode machines that ensure imbalance growth while in operation. The use of such vibration insulation makes it possible to apply simpler external depreciation thus taking into account its efficiency at medium and high frequencies.

It should be noted that the internal elastic inertial vibration protection is particularly efficient for single-mode machines ensuring the imbalance growth. The IEIVP makes it possible to apply simpler external depreciation taking into account its efficiency at medium and high frequencies.

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