

# Universal Mathematical Model of a Hydraulic Loader Crane

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**Abstract.** The paper presents the modeling technique of working processes of loader cranes installed on mobile machines using the developed universal mathematical model that considers complex interaction between the elements of a dynamic system ‘Working Body – Loader Crane – Basic Machine – Base Surface and Substructure – Control System – Environment’. The model considers the developed feedbacks between subsystems. The paper proposes the technique of imitating modeling of a loader crane loading during the entire operation period using the Monte-Carlo method and the universal mathematical model. The application of the developed approaches is shown on the example of a task concerning the general buckling of a mobile machine ACT-4-A equipped with a loader crane in the conditions of ground subsidence under a caterpillar.

## 1. Introduction

Hydraulic loader cranes are widely used for handling operations in construction industry, extractive industries, forest and metal-working industry, agriculture, marine and railway transport, defense industry complex. Loader cranes are installed on a variety of basic machines. Hydraulic loader cranes installed on trucks gained the maximum popularity [1-3].

The famous works [4-12] consider single questions of design and modeling of loader cranes operation. However, the issues of complex interaction of loader cranes with other elements of transport and technological facility are not given enough consideration [3, 8]. At present, there is low focus on the interaction between a loader crane, a basic machine and a base itself during the design and planning of process operations. This leads to the increase in the number of accidents caused by the general buckling of a mobile machine, ground subsidence under outrigger loading, and the falling cargo [3].

The paper presents approaches to modeling of working processes of loader cranes using the developed universal mathematical model that considers complex interaction between the elements of a dynamic system ‘Working Body – Loader Crane – Basic Machine – Base Surface and Substructure – Control System – Environment’.

## 2. Structure of the universal mathematical model

The studied dynamic system consists of several interconnected subsystems (Fig. 1).

Subsystem 0 unites external environmental factors (wind loading, temperature action, impulse force).

Subsystem I includes elements located under the basic machine thus creating support reaction forces. The influence of elements of the subsystem I on the loader crane considers the following: mathematical model of a base surface (macro - and microroughnesses models) and mathematical model of the deformed



support structure (with the movement of a basic machine or operation of a loader crane the base surface constantly changes). There are the following types of deformed support structures: deformed base for land vehicles (soil, asphalt road), railway track (track panel and ballast), liquid (for floating vehicles).

Subsystem II includes the elements of a basic machine (without the elements of a loader crane). The basic elements of a subsystem include a case, a pendant, a propeller, an engine with transmission system, outriggers. The influence of outriggers is considered during the operation of a loader crane, while it is not considered with the movement of a basic machine along the base surface. There are the following types of basic machines: wheeled or caterpillar chassis, railway cars [3], floating vehicles [13].

Subsystem III includes the elements of a loader crane: booms and rotating units, hydraulic cylinders, hinges. Besides, this subsystem includes MC damages obtained during operation (parts breakdown, pitch play).

Subsystem IV includes either a cargo with a load-handling device fixed on a rigid or flexible pendant, or a specialized working body (a clamshell, a debranching head, a mill, a pile driver).

Subsystem V includes the elements of a control system. In the simplest case scenario, when a loader crane is controlled manually, this subsystem may be neglected [3, 9].

### 3. Submodels of the universal mathematical model

The universal mathematical model includes several submodels that consider the dynamics of every component of the studied system (Fig. 1).

The submodel of a loader crane represents a set of motion equations regarding the articulated boom links. Depending on the modeling purpose the links may be considered as both absolutely rigid and deformable bodies [3].

During preliminary dynamic calculations or comparative analysis of various loader cranes their links may be considered as absolutely rigid. In this case the dynamics of a loader crane is described by the matrix equation:

$$[H(q)]\{\ddot{q}\} + \{C(q, \dot{q}, F)\} = \{\tau\},$$

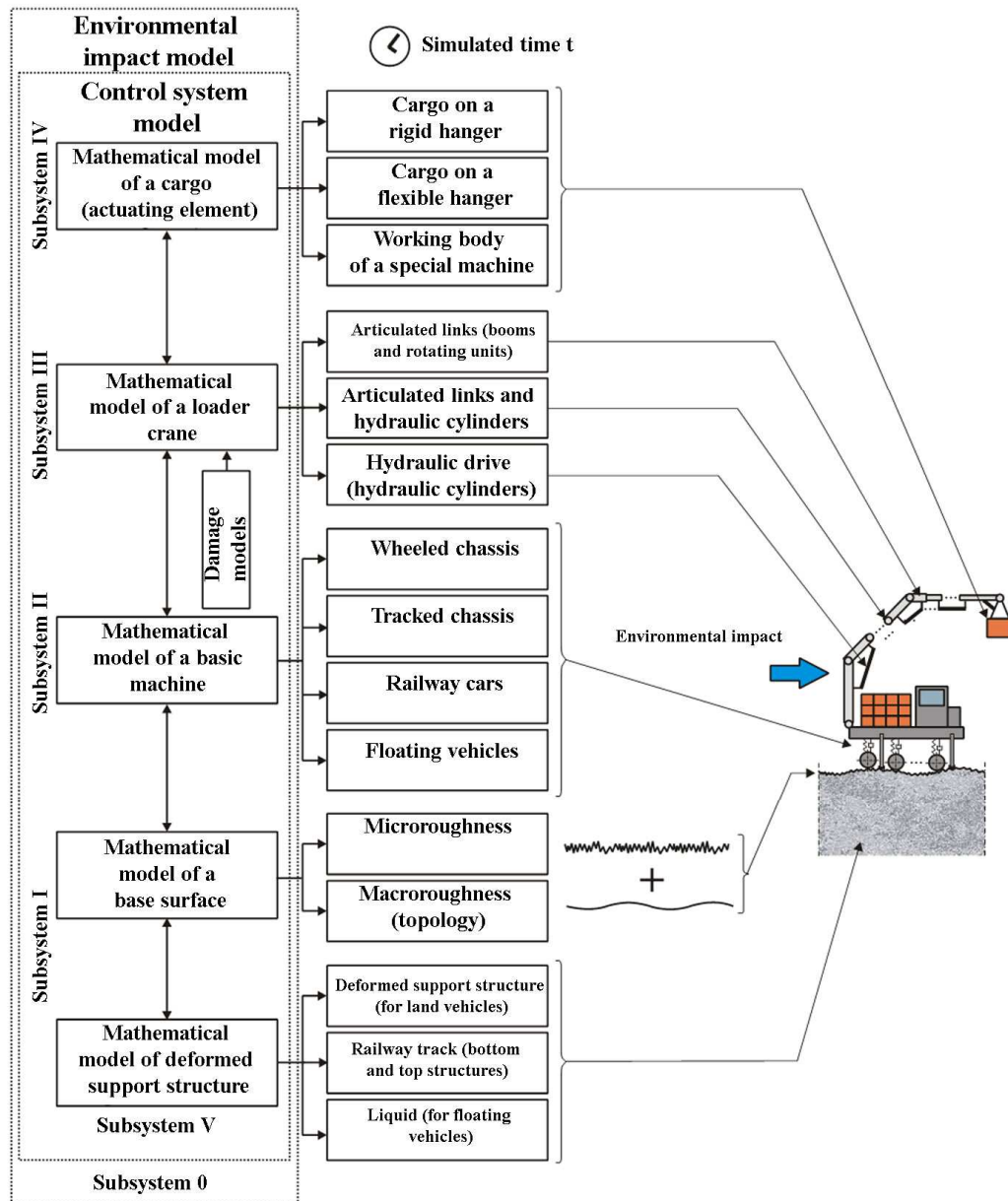
where  $\{\tau\}$  – vector of joint forces within hinges;  $[H(q)]$  – inertia matrix, which components depend on joint variables  $\{q\}$ ;  $\{\ddot{q}\}$  – vector of generalized speed;  $\{C(q, \dot{q}, F)\}$  – vector depending on joint variables, generalized speed and external impacts  $F$  [3, 9, 10].

When calculating the fatigue resistance, stability, and analyzing the resonant phenomena there is a need to consider the elastic compliance of links. In this case, the motion equations are based on the method of Lagrange multipliers:

$$M_{3i}\ddot{q}_i + C_q^T \lambda = Q_{e3i} - Q_{v3i} - Q_{f3i},$$

where  $C_q$  – matrix of link equations;  $\lambda$  – Lagrange multipliers;  $Q_{3i}$  – vector of external forces;  $Q_{v3i}$  – vector of forces depending on squared velocity;  $Q_{f3i}$  – vector of elastic forces [3, 8].

Elastic forces are expressed through components of the elasticity matrix calculated through finite element analysis [10].



**Figure 1.** Structure of the studied dynamic system

The submodel of hydraulic loader crane targeted at its inclusion into the structure of a complex mathematical model of multi-purpose MTTM.

The cargo submodel on a load-handling device is as follows:

$$m_i \ddot{x}_i + c_i \dot{x}_i + \beta_i x_i = f_i,$$

where  $x_i$ ,  $\dot{x}_i$ ,  $\ddot{x}_i$  – movements, speeds and accelerations of masses  $m_i$ ,  $c_i$  and  $\beta_i$  – rigidity and viscosity of elements,  $f_i$  – operating forces (propulsive force of a hoist, rope tension, cargo weight).

The submodel of a hydraulic drive represents the system of differential and algebraic equations calculated in  $j$ -points of a hydraulic system:

$$\begin{cases} \dot{Q} = F(m, A, f, \eta), \\ F(p_j, Q_j, R_j, A) = 0, \end{cases}$$

where  $p_j$  – pressure;  $Q_j$  – volume flow;  $F$  – algebraic expressions;  $R_j$  – hydraulic resistance;  $m$ ,  $A$  – inertial and geometrical parameters of a hydraulic cylinder;  $f$  – forces of a hydraulic cylinder;  $\eta$  – mechanical efficiency.

This submodel allows defining the force created by a hydraulic cylinder on a driven loader crane link. Since it is more convenient to present the kinematic scheme of a loader crane as an open-chain arm [7], then the forces of hydraulic cylinders are included into external influences  $F$ . During concurrent operation of several hydraulic cylinders via additional algebraic equations, the redistribution of a working liquid flow between them is considered. The forces of hydraulic cylinders are defined depending on pressure, rod area and friction forces [14]. A simple dry friction model [15] or a Shtribek model [16] is used to estimate the friction forces.

There is a great variety of basic machines ensuring the installation of handling systems. The propeller type exerts the greatest impact on the dynamics of the basic machine. The motion equations of the system in general are given below. The first three equations are the most critical for the sprung mass of the basic machine.

$$\begin{aligned}
 m_0 \ddot{z}_M - \sum_i \sum_j \sum_k [F_{kij} + F_{aji}] &= P_z; \\
 J_x \ddot{\phi}_x - \sum_i \sum_j \sum_k [F_{kij} s_\phi R_{xkij} + F_{aij} s_\phi R_{xaij}] &= M_x + P_y B; \\
 J_y \ddot{\phi}_y - \sum_i \sum_j \sum_k [F_{kij} s_\phi R_{ykij} + F_{aij} s_\phi R_{yaij}] &= M_y - P_z O_1 Z + P_x B; \\
 m_0 \ddot{x}_M - \sum_i \sum_j \sum_k [F_{xkij} + F_{xaji}] &= P_x; \\
 m_0 \ddot{y}_M - \sum_i \sum_j \sum_k [F_{ykij} + F_{yaji}] &= P_y; \\
 J_z \ddot{\phi}_z - \sum_i \sum_j \sum_k [F_{xkij} y_{Rkij} + F_{xaji} y_{Rakij} + F_{ykij} x_{Rkij} + F_{yaji} x_{Rakij}] &= M_z + P_y O_1 Z; \\
 \dots \\
 m_{kij} \ddot{x}_{kij} + (c_{nji} + c_{kij}) (x_{kij} - z_M - s_\phi \phi_y R_{ykij} - s_\phi \phi_x R_{xkij} + \xi_{kij}(t)) + \\
 + (\alpha_{nji} + \alpha_{kij}) (\dot{x}_{kij} - \dot{z}_M - s_\phi \dot{\phi}_y R_{ykij} - s_\phi \dot{\phi}_x R_{xkij}) &= 0; \\
 \dots \\
 m_{aij} \ddot{x}_{aij} + (c_{aji} + c_{naij}) (x_{aij} - z_M - s_\phi \phi_y R_{yaij} - s_\phi \phi_x R_{xaij} + \xi_{aij}(t)) + \\
 + (\alpha_{aji} + \alpha_{naij}) (\dot{x}_{aij} - \dot{z}_M - s_\phi \dot{\phi}_y R_{yaij} - s_\phi \dot{\phi}_x R_{xaij}) &= 0;
 \end{aligned}$$

where  $c_{k1i}$ ,  $\alpha_{k1i}$  – coefficient of elasticity and viscosity of mass suspension element  $m_{k1i}$ ;  $c_{k2i}$ ,  $\alpha_{k2i}$  – coefficient of elasticity and viscosity of mass suspension element  $m_{k2i}$ ;  $c_{n1i}$ ,  $\alpha_{n1i}$  – coefficient of elasticity and viscosity of a basic machine frame to consider the influence of mass suspension element  $m_{k1i}$ ;  $c_{n2i}$ ,  $\alpha_{n2i}$  – coefficient of elasticity and viscosity of a basic machine frame to consider the influence of mass suspension element  $m_{k2i}$ ;  $c_{aij}$ ,  $c_{naij}$ ,  $\alpha_{aij}$ ,  $\alpha_{naij}$  – coefficients of elasticity and viscosity of outriggers and a frame of a basic machine to consider the influence of outriggers;  $c_M$ ,  $\alpha_M$  – coefficient of elasticity and viscosity of a basic machine frame to consider the influence of a loader crane;  $s_\phi$  – multiplying factor (if the force of a basic element leads to positive rotation  $s_\phi=1$ , to negative  $-s_\phi=-1$ );  $R_{ykij}$ ,  $R_{yakij}$  – distance from the center of gravity to the basic element attachment point along  $y$  axis;  $R_{xkij}$ ,  $R_{xakij}$  – distance from the center of gravity to the basic element attachment point along  $x$  axis;  $F_{xkij}$ ,  $F_{xaji}$ ,  $F_{ykij}$ ,  $F_{yaji}$  – elastic forces at shifts along axes  $x$  and  $y$  calculated similar to forces  $F_{kij}$  and  $F_{aji}$ , but with the corresponding coefficients of rigidity and dissipation;  $x_R$  and  $y_R$  – components of  $R$  vectors;  $B$  –  $G$  point height over the center of gravity. The parameters are defined either using the analytical method, or via the finite element analysis. The handling system influences the basic chassis

through six forces:  $P_x, P_y, P_z$  forces and  $M_x, M_y, M_z$  moments.

The submodel of a base surface includes the model of macroroughness and microroughness. The macroroughness is presented as the superposition of linear or spline functions. The microroughness of a base surface is defined by the Monte-Carlo method based on experimental data concerning the spectral density [3].

The submodel of the deformed base surface is aimed to define its deformation  $\varepsilon_z$  under the support element via the nonlinear theory of elastic-viscous-plastic materials. Thus, the dependence for one of the chassis is built: caterpillar or wheeled chassis with weak or strong pumping. This dependence looks as follows:

$$\varepsilon_z = \varepsilon_z(t, E, K_z, \sigma_z, t_k),$$

where  $t$  – time;  $E$  – instantaneous deformation modulus;  $K_z$  – creep rate function;  $\sigma_z$  – tension (pressure) in a contact point of a support element and the base support;  $t_k$  – final soil deformation time under the base support [3].

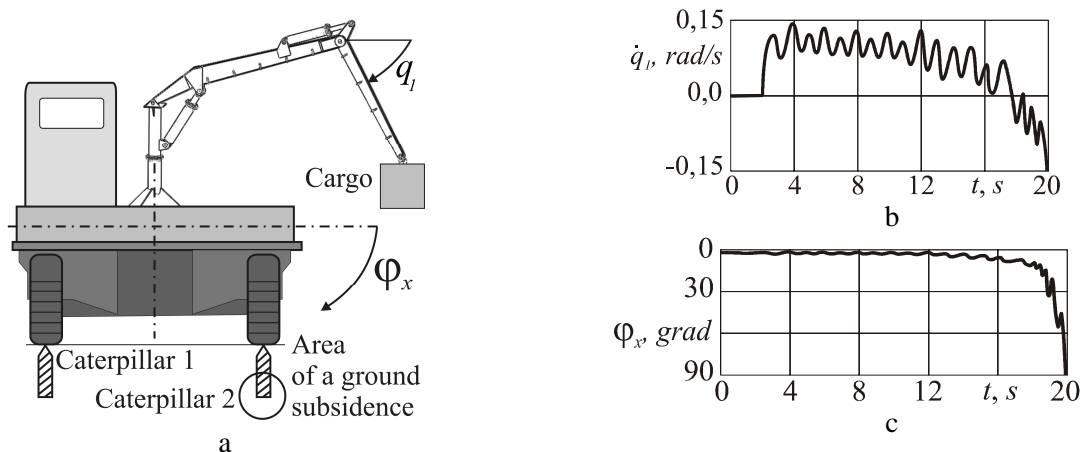
#### 4. Estimation of dynamic loading of a loader crane

The strength calculation of a loader crane requires defining its direct loading and the corresponding tension in dangerous points. For this purpose, the exact values of random loading factors are defined for each operating cycle of a loader crane during the simulation modeling. Then, the dynamic processes within a loader crane are simulated using the developed universal dynamic model. The simulation result represents a piecewise process of changing the loading parameters. The piecewise simulation corresponding to various combinations of loading factors forms the basis for the total implementation of a process of changing the loading parameters. This process is repeated thus leading to final probabilistic implementation.

#### 5. Application of the universal model using the analysis of ground subsidence under outriggers with subsequent general buckling

One of the objectives of the universal model of the studied system was the possibility to define the general buckling taking into account the interference of a loader crane, a basic machine and a base surface.

The ACT-4-A machine behavior is modelled under ground subsidence conditions (Fig. 2). The cargo is lifted by a three-link hydraulic loader crane with its arm turned by  $90^\circ$  in relation to a longitudinal axis of the machine.



**Figure 2.** General buckling of the ACT-4-A machine under ground subsidence:  
a – scheme of the studied situation, b – generalized speed of the last link  
of a loader crane, c – flank angle of the basic machine

After the basic machine is installed on a base support the soil is consolidated under both caterpillars. The cargo is lifted at  $t=2$  s thus leading to additional consolidation of soil under caterpillars. The total consolidation of soil reaches 3.1 cm. The ground subsidence is observed under one of the caterpillars in 9.5 s after the cargo is lifted. This leads to unloading of one caterpillar and roll of the basic machine within the loader crane plane. The ground subsidence under the loaded caterpillar, bearing the main load, is sharply increasing at  $t=18$  s. With some time lag amounting to 0.3...0.5 s the basic machine begins to tilt sharply. The stability angle for this machine makes  $62.6^\circ$ . Taking into account the ground deformation already at  $\varphi_x=55^\circ$  the tilt angle of a machine is sharply increasing thus leading to its rollover (general buckling).

## 6. Conclusions

The designed universal mathematical model may be used for modeling of various operating modes of a machine equipped with a loader crane. This may include the following: movement of separate links of a loader crane, joint movement of loader crane links, movement of a basic machine with cargo in case with motionless loader crane, movement of a basic machine with cargo in case with simultaneous movement of loader crane links, ground subsidence under the basic machine.

## 7. Acknowledgments

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