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Problems of heat and mass transfer in the snow-ice cover

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Abstract. The snow-ice cover is considered as a three-phase continuous medium consisting of water, air, and ice. The mathematical model is based on the equations of mass conservation for each phase, the two-phase filtration equation for water and air in a movable porous ice skeleton, the rheological equation for porosity, the balance of forces and the equation of energy conservation for the ice-water-air system. The paper presents the results on analytic and numerical solutions of the problems under consideration.

1. Introduction

The urgency of the study of heat and mass transfer in multiphase media is due to the fact that numerous problems that arise, along with technological processes also in ecology and nature use lead to the need to model the processes of interpenetrating motion of continuous media. The constructed in this case mathematical models, as a rule, are nonclassical, and require the development of new approaches to their justification and numerical modeling.

A large class of filtration problems in a deformable porous medium with phase transitions is the problem of heat and mass transfer in a melting snow-ice cover [1]. The predominant part of the runoff of the northern rivers is formed due to the melting of the seasonal snow cover. Snowmelt conditions have a decisive influence not only on the amount of thawed water entering the reservoirs, but also on their quality. In addition, the amount of snow cover (snowfall) affects the freezing of the surface layer of soils and, consequently, its absorbency and determines the relationship between slope and ground runoff. Therefore, modeling the state of the snow cover and salt transfer during the snow melting period is of great importance in the development of methods for calculating and forecasting the hydrographers of spring flood and water quality in reservoir-receivers. There is a large number of works on salt-mass transfer in melting snow, in which the data of observations and empirical dependences are used [2]. Most empirical models are one-dimensional and do not allow to calculate the fluid filtration rate, and models that calculate the fluid filtration rate usually do not take into account phase transitions or are suitable only for specific modes of water movement in the snow cover, they also do not provide the necessary information on the velocity field and the saturation of the fluid phase necessary for the assessment of the flow of pollutants.

Thus, for a reliable prediction of pollutant discharge, it is necessary to know the velocity field and the saturation of the water phase, i.e., it is proposed to use complex models describing the joint movement of pollutants and water in the snow cover, taking into account different boundary conditions, phase transitions and sublimation process. These models will allow calculating the non-stationary movement of pollutants within the snow cover and assess the surface and underground runoff of substances. They must take into account a number of important factors, including variable snow cover porosity, phase transitions, specificity of boundary conditions (in particular, the presence of frozen or not frozen ground). The fundamentals of the theory of the motion of water and air in melting snow are laid in the works of S.C. Colbeck [3] and his followers. However, snow in these works, although considered as a multiphase medium, deformation of ice and phase transitions were not taken into account.

The purpose of this work is: to build a model on the joint movement of water and air in melting snow, taking into account phase transitions and deformation of ice; the statement of the problem of ablation of deformed ice cover; statement of the problem of distribution of water flow of melting snow between ground and surface waters; the construction of a model of groundwater movement in contact with frozen ground.

2. Mathematical models of the snow cover

Due to the high demand for data on snow cover, for practical purposes, there is a significant number of works devoted to field observations (see, for example, [4], [5]). For estimating calculations, simple balance models (see [6], [7]) and more complex models are used [3].

Following [8, 9, 10] we will consider melting snow as a continuous medium consisting of water ($i = 1$), air ($i = 2$) and ice ($i = 3$) constituting a solid porous skeleton. The filtration of water and air in a porous ice skeleton is described by the equations of mass conservation for each of the phases, taking into account phase transitions, the equations of two-phase filtration and the heat balance equation for a three-phase medium

$$\frac{\partial \rho_i}{\partial t} + div(\rho_i \vec{u}_i) = \sum_{j=1}^{3} I_{ji}, \quad i = 1, 2, 3, \quad I_{ji} = -I_{ij}, \quad \sum_{i,j=1}^{3} I_{ij} = 0, \tag{1}$$

$$\phi(\vec{u}_i - \vec{u}_3) = -K_0 \frac{k_{0i}}{\mu_i}(\nabla p_i + \rho_i^0 \vec{g}), \quad i = 1, 2, \quad p_2 - p_1 = p_c(s_1, \theta), \quad \sum_{i=1}^{2} s_i = 1, \tag{2}$$

$$(\sum_{i=1}^{3} \rho_i^0 c_i \alpha_i)\frac{\partial \theta}{\partial t} + (\sum_{i=1}^{3} \rho_i^0 c_i \alpha_i \vec{u}_i)\nabla\theta = div(\lambda_c \nabla\theta) - L_s I_{23} + L_i I_{13}. \tag{3}$$

Here $\vec{u}_i$ is the speed of the i phase; ρ_i is the reduced density associated with the true density ρ_i^0 and the volume concentration α_i by the relation $\rho_i = \alpha_i \rho_i^0$ (the condition $\sum_{i=1}^{3} \alpha_i = 1$ is a consequence of the definition of ρ_i); I_{ji} is the intensity of the mass transition from j into the i component per unit volume per unit time; ϕ is the porosity of the snow; s_1, s_2 - saturation of water and air ($\alpha_1 = \phi s_1$, $\alpha_2 = \phi s_2$, $\alpha_3 = 1 - \phi$); $K_0(\phi)$ is the filtration tensor; k_{0i} - phase permeabilities ($k_{0i} = k_{0i}(s_i) \geq 0$, $k_{0i}|_{s_i=0} = 0$); μ_i - dynamic viscosity; p_i is the phase pressure; p_c is the capillary pressure, $\vec{g}$ - acceleration vector of gravity; θ is the temperature of the medium ($\theta_i = \theta$, $i = 1, 2, 3$), $c_i = const > 0$ - the heat capacity of the i phase at constant volume; $L_i = const > 0$ - specific heat of melting of ice; $L_s = const > 0$ - specific heat of ice sublimation; λ_c is the thermal conductivity of snow.

The system of equations (1) - (3)with respect to the characteristics $\vec{u}_i$, p_i, s_i and θ of immiscible liquids moving in a non-deformable porous medium is closed either by the assumption of incompressibility of liquids, i.e. $\rho_i^0 = const$, or by the condition $\rho_i^0 = \rho_i^0(p_i)$. The obtained mathematical model in the case of a stationary porous medium ($\vec{u}_3 = 0$) and for a given porosity ϕ is called the Musket-Leverett model.

The principal point is to take into account the compressibility of the porous medium. Following [11], we supplement the system (1) - (3)

$$div\vec{u}_3 = -\frac{1}{\xi(\phi)}p_e - \beta_t(\phi)(\frac{\partial p_e}{\partial t} + \vec{u}_3 \cdot \nabla p_e), \tag{4}$$

$$\rho_{tot}\vec{g} + div((1-\phi)\eta(\frac{\partial \vec{u}_3}{\partial \vec{x}} + (\frac{\partial \vec{u}_3}{\partial \vec{x}})^*)) - \nabla p_{tot} = 0, \tag{5}$$

where $p_e = p_{tot} - p_f$ is the effective pressure, $p_{tot} = \phi p_f + (1-\phi)p_3$ is the total pressure, $p_f = s_1p_1 + s_2p_2$, p_3 are, respectively, the pressures of the liquid and solid phases, $\rho_{tot} = (1-\phi)\rho_3^0 + \phi(s_1\rho_1^0 + s_2\rho_2^0)$ is the total density; η is the viscosity of a solid skeleton, $\xi(\phi)$ and $\beta_t(\phi)$ are the coefficients volumetric viscosity and volume compressibility of a porous medium are given functions.

After determining the saturation of water and air s_i, temperature θ and the filtration rates $\vec{v}_i = \phi s_i(\vec{u}_i - \vec{u}_3)$ we can consider the problem of the motion of a conservative impurity due to the transfer of the water phase and diffusion. This process describes the convective diffusion equation [12]:

$$R + \frac{\partial}{\partial t}(\phi s_1 \sigma) + div(\sigma\vec{v}_1 - D\nabla\sigma) = 0. \tag{6}$$

Here σ is the impurity concentration, $\vec{v}_1$ is the water filtration rate, R is the source that takes into account the possible deposition of the impurity. The following dependencies are used: $D = \eta_1 + \lambda_0|\vec{v}_1|$, $\eta_1 = const > 0$ is the coefficient of molecular diffusion, $\lambda_0 = const > 0$ is the dispersion parameter; $R = -\Gamma s_1(\sigma_* - \sigma)$, $\Gamma = const > 0$, $\sigma_* = const \in [0,1]$. System (1) - (5) is very complex and has been studied only in individual cases [13, 14, 15].

2.1. Self-similar solution

On the basis of [13], a self-similar solution of the traveling wave type for the problem of salt transfer in melting snow is constructed under the following assumptions:

$$\vec{u}_3 = 0, \quad I_{13} = I_{13}(\theta), \quad I_{12} = 0, \quad I_{23} = 0, \quad L_s = 0, \quad \rho_i^0 = const\ . \tag{7}$$

By (7), the continuity equation for the solid phase implies $\frac{\partial \rho_3^0(1-\phi)}{\partial t} = I_{31}(\theta)$. In particular, we can assume that porosity is a function of temperature.

The following dependence is accepted: $\alpha_3(\theta) = 0, \theta \geq \theta^+$ (ice melting temperature); $\alpha_3(\theta) = 1 - \phi^- - \phi_1(\theta - \theta_1), \theta_1 \leq \theta \leq \theta^+; \alpha_3(\theta) = 1 - \phi^-, \theta \leq \theta_1$. Here $0 < \theta^- < \theta_1 < \theta^+, \phi^- = \phi(\theta^-), \phi_1 = (1-\phi^-)/(\theta^+ - \theta_1)$ – specified parameters. Also assumed $K_0 = const > 0, \lambda_c = a_c + b_c\rho_c^2, \rho_c = \sum_{i=1}^3 \rho_i^0\alpha_i, (a_c, b_c) = const > 0, \vec{g} = (0, 0, -g)$; the unknown functions depend only on the variable $\xi = x_3 - ct \in (-\infty, 0), c = const < 0$.

The system of equations (1) - (5) reduces to a system of equations for $s(\xi) \equiv s_1(\xi)$ and $\theta(\xi)$ of the form

$$\lambda_c\frac{d\theta}{d\xi} = f_1(\theta), \quad a_0(s) = \frac{ds}{d\xi} = f_2(s, \theta),$$

and s, θ satisfy the following conditions $\theta(0) = \theta^+, s(0) = s^+, (\theta, s, \frac{d\theta}{d\xi})_{\xi\to-\infty} = 0$. Here functions $f_1(\theta), f_2(s,\theta), a_0(s)(a_0(0) = a_0(1) = 0)$ can easily be recalculated through the initial parameters of the source system [13]. The solution of the formulated problem exists and in particular has properties: $0 \leq s(\xi) \leq 1$, there exists a point $\xi_* \in (-\infty, 0)$ such that $s(\xi) = 0$ for all $\xi \leq \xi_*$.

In the self-similar case, the following problem arises for finding the impurity concentration σ:

$$
\begin{gathered}
-\Gamma s(\sigma_* - \sigma) + \tfrac{d}{d\xi}(|c|\tfrac{\rho_3^0}{\rho_1^0}(\phi - \phi^-)\sigma - D\tfrac{d\sigma}{d\xi}) = 0, \\
\sigma|_{\xi\to-\infty} = 0, \quad \tfrac{\partial\sigma}{\partial\xi}|_{\xi\to-\infty} = 0, \quad \sigma(0) = \sigma^+ \in (0,1].
\end{gathered} \tag{8}
$$

Since $s(\xi) = 0$ for $\xi \le \xi_*$, then $\sigma(\xi) = 0$, $\xi \le \xi_*$. Thus for $\sigma(\xi)$ we consider the problem

$$
\begin{gathered}
\tfrac{d}{d\xi}(D\tfrac{d\sigma}{d\xi} - |c|r\sigma) + \Gamma s(\sigma_* - \sigma) = 0, \quad \xi_* < \xi < 0, \\
\sigma(\xi_*) = 0, \quad \sigma(0) = \sigma^+ \in (0,1], \quad \sigma_* \in [0,1].
\end{gathered} \tag{9}
$$

Here $r = 0$, $D = \eta_1 + \lambda_0|c|\phi^- s$ for $\xi \in [\xi_*, \xi_1]$; $r = \frac{\rho_3^0}{\rho_1^0}(\phi - \phi^-)$, $D = \eta_1 + \lambda_0|v_1|$, $v_1 = |c|(\phi s - r)$ for $\xi \in [\xi_1, 0]$.

The solution of problem (8), (9) exists and satisfies inequality $0 \le s \le 1$. The numerical investigations of problem (8), (9) were carried out in [16].

2.2. The numerical study of the two-dimensional problem

Based on the hypotheses (7) of paragraph 2.1, the two-dimensional problem of the motion of water and air in melting snow is considered. The system (1) - (5) reduces to a system of three equations with respect to the saturation of the water phase s, the temperature θ and the reduced pressure p [1]:

$$
\begin{gathered}
\frac{\partial}{\partial t}(\rho_1^0\phi s) = div(\rho_1^0(K_0 a\nabla s + K_1\nabla p + \vec{f_0})) + \frac{\partial}{\partial t}(\rho_3^0\phi(1-s)), \\
(\sum_{i=1}^{3}\rho_i^0 c_i\alpha_i)\frac{\partial\theta}{\partial t} + (\sum_{i=1}^{2}\rho_i^0 c_i\vec{v}_i)\nabla\theta = div(\lambda_c\nabla\theta) + \nu\frac{\partial\rho_3}{\partial t}, \\
div(K\nabla p + \vec{f}) = -\frac{\partial}{\partial t}((1 - \frac{\rho_3^0}{\rho_1^0})\phi),
\end{gathered}
$$

where ν – specific heat of melting ice,

$$
\begin{gathered}
p = p_1 - \int_s^1 \frac{\partial p_c}{\partial\xi}\frac{\bar{k}_{02}}{k}d\xi, \quad k = \bar{k}_{01} + \bar{k}_{02}, \bar{k}_{0i} = \frac{k_{0i}}{\mu_i}, \\
K = kk_0, K_i = K_0\bar{k}_{0i}, \quad a = -\frac{\partial p_c}{\partial s}\frac{\bar{k}_{01}\bar{k}_{02}}{k} \ge 0, \\
\vec{f} = \sum_{i=1}^{2}(K_i\rho_i^0\vec{g}), \quad \vec{f_0} = K_1\rho_1^0\vec{g}.
\end{gathered}
$$

The solution of this system is sought in the region $\Omega = 0 \le x_1 \le L, 0 \le x_2 \le H$ under the following conditions

$$
\begin{gathered}
s(x_1, x_2, 0) = s_0(x_1, x_2), \quad \theta(x_1, x_2, 0) = \theta_0(x_1, x_2), \\
s(x_1, 0, t) = 0, \quad \theta(x_1, 0, t) = \theta^-, \quad s(x_1, H, t) = 0, \quad \theta(x_1, H, t) = \theta^+, \\
p(x_1, H, t) = p_H(x_1, t), \quad p(x_1, 0, t) = p_0(x_1, t), \\
\frac{\partial s}{\partial x} = \frac{\partial\theta}{\partial x} = \frac{\partial p}{\partial x} = 0, \quad x_1 = 0, x_1 = L.
\end{gathered}
$$

The dependence of porosity on temperature is described in paragraph 2.1. For the numerical solution of this initial-boundary value problem, the variable direction method is used. After finding the temperature θ, the filtration rate $\vec{v}_1$ and the saturation s on the basis of equation (6), we can consider the problem of the motion of a conservative impurity in melting snow.

2.3. The problem of sublimation of the snow-ice cover

In addition to thawing, a high role in the balance of the snow cover is played by the sublimation process. Under certain conditions, a significant mass of snow passes into the gas phase, bypassing the liquid phase. The sublimation process is significantly accelerated by the effect of the wind impaction the snow. A mathematical study of the windless sublimation of snow was carried out in work [17] in which, in analogy with [11], a self-similar solution is constructed with allowance for the phase transition of ice into moist air. The formula for the intensity of the phase transition I_{32} (ice-moist air) is obtained in work [18] under wind-induced conditions:

$$I_{32}(x,t) = \frac{dm}{dt}\frac{c(x,t)}{\frac{4}{3}\pi r^3}, \quad \frac{dm}{dt} = \frac{2\pi r(\frac{\rho}{\rho_n(\theta)} - 1)}{\frac{L_s}{K\theta Nu}(\frac{L_s M}{R\theta} - 1) + \frac{1}{D\rho_n(\theta)Sh}}. \tag{10}$$

Here $c(x,t)$ is the concentration of ice, dm/dt is the rate of change in mass, $r(m)$ is the radius of the particle, ρ is the air density, θ is the absolute temperature, $\rho_n(\theta)$ is the saturated water vapor density, L_s is the heat of sublimation of ice, M is the molecular mass of water, D is the diffusion coefficient, R is the universal gas constant, K is the molecular thermal conductivity in the atmosphere, Nu is the Nusselt number, Sh is the Sherwood number.

Formula (10) is actively used to describe the processes of sublimation in the snow cover [19], [20]. As an example, let us consider the problem of the motion of moist air in a mobile porous ice skeleton. The system (1) - (5) in the case under consideration takes the form

$$\frac{\partial \phi \rho_2^0}{\partial t} + div(\phi \rho_2^0 \vec{u}_2) = -I_{32}, \tag{11}$$

$$\frac{\partial (1-\phi)\rho_3^0}{\partial t} + div((1-\phi)\rho_3^0 \vec{u}_3) = I_{32}, \tag{12}$$

$$\phi(\vec{u}_2 - \vec{u}_3) = -K_0 \frac{k}{\mu}(\nabla p_2 + \rho_2^0 \vec{g}), \quad p_2 = R\theta\rho_2^0, \tag{13}$$

$$(\sum_{i=2}^{3} \rho_i^0 c_i \alpha_i)\frac{\partial \theta}{\partial t} + (\sum_{i=2}^{3} c_i \rho_i^0 \alpha_i \vec{u}_i)\nabla\theta = div(\lambda_c \nabla\theta) - L_s I_{32}. \tag{14}$$

Here we use the notations adopted for the system (1) - (5) (taking into account the absence of the liquid phase: $\alpha_2 = \phi, \alpha_3 = 1-\phi, p_{tot} = \phi p_2 + (1-\phi)p_3, p_e = p_{tot} - p_2$). In case $I_{32} = 0$ and $\theta = const$ the system (11) - (14) was used in [15]. In case $I_{32} \neq 0$, a self-similar solution of the isothermal motion of a compressible gas in a deformable porous medium is constructed, and it is established that $0 \le \rho_2 \le \rho_n, 0 \le \phi \le 1$ [21].

2.4. Distribution of water flow of melting snow between groundwater and surface water

The slope current of surface and ground waters, caused by intensive snowmelt, is considered. Such flows are characteristic of hilly regions. Consider a two-dimensional (in the vertical plane x, z) flow of groundwater to the river bank in a layer of soil between a horizontal water column and a land surface inclined to the river. The current is caused by the melting of snow, the intensity of which is greater than or equal to the filtration coefficient of the upper layer of the soil, as a result of which a slope runoff over the surface of the earth can also be formed. The complex aspects of freezing of such water are not yet considered, i.e. the case of positive surface temperature is modeled. The underground part of the current is modeled by the equation of water content:

$$div(K\nabla\Phi) = \frac{\partial W}{\partial t},$$

where the pressure $\Phi = p - z$ (the z axis is directed downwards), p is the capillary pressure. At full saturation of the pores $p \geq 0$ (it is assumed that the atmospheric pressure is 0), coefficient of moisture conductivity $K = K_s$, volume moisture $W = W_s$. For incomplete saturation, when $p < 0$, it is assumed that the dependences of K and W on p can be described by the expressions [22]

$$K = K_s \left(\frac{W - W_r}{W_s - W_r} \right)^n, \quad W = W_r + (W_s + W_r) \exp(p/p_0).$$

Here K_s is the filtration coefficient (at full saturation); W_s is the total saturation humidity, W_r is the residual water content; p_0 is the conditional height of capillary uplift; $n \approx 3.5$. The slope runoff on the surface of the earth Γ in the presence of a water layer ($p \geq 0$) is modeled by the diffusion wave equation [23] in the following form:

$$\frac{\beta}{\alpha} \frac{\partial}{\partial l} \left(p^{5/3} \frac{\partial \Phi}{\partial l} \right) + R - Q = \frac{\partial p}{\partial t},$$

where p, Φ are the sought in points of Γ; R is the specified snowmelt intensity; Q is the flow inside the domain; γ is the inverse coefficient of the roughness coefficient, $\beta = \cos^2 \alpha tg^{1/2} \alpha$, where α is the slope angle of the earth's surface; $l = x/\cos\alpha$ is a coordinate along Γ. The boundary conditions for W and ϕ are described in [22].

2.5. Movement of groundwaters in contact with frozen ground

If the snowmelt passes in the conditions of the frozen ground, the problem of distributing the water flow of melting snow between ground and surface waters becomes much more complicated. In particular, there is a need to describe the movement of groundwater in the aquifer, which is in contact with the frozen ground.

Following [24], frozen ground will be considered as a thermoelastic porous medium, the porous space of which fills the ice. For the stress tensor of the frozen soil skeleton P^{si}, the Dumel-Neumann law is adopted. For the stress tensor P of a frozen ground, it is natural to take $P = (1 - \phi)P^{si} + \phi P^i$, where ϕ is the porosity, P^i is the stress tensor ice.

In the region of groundwater movement, which have a positive temperature, the system of equations (1) - (5) is considered. The boundary between the frozen ground and the groundwater area is determined by the phase diagram of the frozen soil and by the strong discontinuity equation.

3. Conclusion

An analysis of mathematical models of the melting snow-ice cover, taking into account phase transitions and deformation of the ice skeleton under different rheological ratios, is carried out.

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