

# Proxy Pseudo3D model: the optimum of speed and accuracy in hydraulic fracturing simulation

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**Abstract.** The paper describes an efficient approach to simulating hydraulic fracturing in Pseudo3D setting for reservoirs with a low permeable matrix. By reformulating the problem, the authors succeeded in reducing it to two ordinary first-order differential equations instead of a nonlinear second-order partial differential equation, which fundamentally affects the computational complexity. Also, this approach allows a more correct computation of a fracture front and fracturing fluids interface versus a traditional Cell-Based Pseudo3D model. Nowadays the main limitation of the developed software module is the absence of matrix leak-off.

## 1. Introduction

The paper describes the development of an effective method to solve the fracturing simulation problem in a Pseudo3D setting versus a traditional Cell-Based Pseudo3D model.

Hydraulic fracturing (frac) is the main approach to extract hydrocarbons from low-permeable reservoirs. The design and implementation of these operations is supported by calculations based on physical and mathematical models of fracture propagation. Nowadays there are a lot of models of hydraulic fracturing which are roughly divided [1] into four main types: Analytical 2D, Pseudo3D, Planar3D, and Full3D. A Pseudo3D model is considered to be the most popular class of models for practical calculations. This model evolved from a PKN model (Perkins-Kern-Nordgren) [2-3] developed in the 1960-70s. The need to develop the Pseudo3D model is explained by the inability to apply the PKN model for realistic cases because it leaves out the effect of distribution of vertical rock stresses and vertical rock properties on the hydraulic fracture geometry. Although the Pseudo3D model was introduced back in the 1980s [4-9], its development still continues [10-14].

Despite the fact that the Pseudo3D model is the fastest among the models used for practical calculations, the speedup task while maintaining satisfactory accuracy remains critical, and that is a focus of this work. The need for higher speed becomes even more demanding when solving problems of frac design optimization with multiple serial runs and a search of various treatment strategies for a huge number of wells. It can also be recommended to implement a Pseudo3D proxy model in cases where the tasks of simultaneous propagation of a significant number of hydraulic fractures are to be addressed, for example, fracture network propagation in unconventional reservoirs or in other fractured structures.

Besides our previous paper [14] also focuses on building a simplified semi-analytical Pseudo3D model, the main idea being the cross-linking of analytical solutions in a simplified setting: a three-layer model and single hydraulic fracturing fluid. In contrast, this paper describes multilayered



lithology with a more complex fracture height distribution lengthwise, including the effect of injecting a number of fracturing fluids with differing rheologies.

## 2. Formulation of Pseudo3D Model

The basic assumptions of the Pseudo3D model are as follows:

- The fracture propagates vertically
- The fracture length by far exceeds its height
- The fluid flow along the fracture is quasi-one-dimensional and horizontally-oriented
- The vertical propagation of the fracture is determined by the ratio of fracture pressure and the contrast of the minimum horizontal stress.
- Leak-off process is controlled by the Carter leak-off law.

All these assumptions are further expressed in the equations describing the three basic physical processes of hydraulic fracture development: rock destruction, rock deformation, and fluid flow. The strongest assumption related to a significant excess of length over height allows to consider a vertical mechanical problem using a 2D piecewise-homogeneous material and a 1D horizontal flow of fluid.

Let us introduce the following symbols. We assume that a hydraulic fracture intersects  $n$  geological layers, and  $i = 0, \dots, (n-1)$  is the layer number upward;  $\sigma_i$  is the minimum rock stress in the  $i$ -th layer,  $K_{IC\_i}$  is the fracture toughness of the  $i$ -th layer;  $h_i$  is the distance from the lower tip of the fracture to the upper boundary of the  $i$ -th layer at  $i = 0, \dots, (n-2)$ ;  $h_{n-1} = h$ , where  $h$  is the fracture height;  $h_{cp}$  is the distance from the lower tip of the fracture to the middle of the fracture initiation interval corresponding to the minimum  $\sigma_i$  value.

### 2.1. Vertical Fracture Propagation

The vertical fracture propagation is determined by the two-dimensional brittle fracture mechanism at each horizontal point independently. The Irwin criterion is used here which states the equality of a stress intensity factor  $K_I$ , the fracture toughness  $K_{IC}$  of an interlayer intersected by the tip of the fracture.  $K_{I\_top}$  is a stress intensity factor for the upper tip of the fracture,  $K_{I\_bot}$  is a stress intensity factor for the lower tip of the fracture. The formula for the stress intensity factor in the two-dimensional case produces the following:

$$K_{I\_bot} = K_{IC\_0}, K_{I\_top} = K_{IC\_n-1}, \quad (1)$$

$$K_{I\_top} = \sqrt{\frac{\pi h}{2}} \left[ p_{cp} - \sigma_n + \rho_f g \left( h_{cp} - \frac{3}{4} h \right) \right] + \sqrt{\frac{2}{\pi h}} \sum_{i=0}^{n-2} (\sigma_{i+1} - \sigma_i) \left[ \frac{h}{2} \cos^{-1} \left( \frac{h-2h_i}{h} \right) - \sqrt{h_i(h-h_i)} \right], \quad (2)$$

$$K_{I\_bot} = \sqrt{\frac{\pi h}{2}} \left[ p_{cp} - \sigma_n + \rho_f g \left( h_{cp} - \frac{1}{4} h \right) \right] + \sqrt{\frac{2}{\pi h}} \sum_{i=0}^{n-2} (\sigma_{i+1} - \sigma_i) \left[ \frac{h}{2} \cos^{-1} \left( \frac{h-2h_i}{h} \right) + \sqrt{h_i(h-h_i)} \right], \quad (3)$$

where  $p_{cp}$  is the fluid pressure inside the fracture at the middle of the initiation interval,  $\rho_f$  is the fracturing fluid density, and  $g$  is the gravitational acceleration.

### 2.2. Rock Deformation

The deformation is governed by a linear elasticity theory described by Hooke's law. The 2D theory of elasticity demonstrates the following solution [11-12] for the fracture opening  $w(z)$ :

$$w(z) = \frac{4}{E'} \left[ p_{cp} - \sigma_n + \rho_f g \left( h_{cp} - \frac{h}{4} - \frac{z}{2} \right) \right] \sqrt{z(h-z)} + \frac{4}{\pi E'} \sum_{i=0}^{n-2} (\sigma_{i+1} - \sigma_i) \left[ (h_i - z) \cosh^{-1} \frac{z \left( \frac{h-2h_i}{h} \right) + h_i}{|z-h_i|} + \sqrt{z(h-z)} \arccos \left( \frac{h-2h_i}{h} \right) \right], \quad (4)$$

where  $E' = E/(1-\nu^2)$  is the plane strain modulus averaged by the fracture height,  $E$  is the Young modulus, and  $\nu$  is the Poisson ratio.

### 2.3. Flow in Hydraulic Fracture

Since the fracture opening is much smaller than the fracture height and length, the flow in a fracture can be described by the lubrication theory equation. For the case of a Newtonian fluid with viscosity  $\mu$ :

$$\vec{q} = -\vec{\nabla} p \frac{w^3}{12\mu}, \quad (5)$$

where  $w$  is fracture opening at a point, and  $\vec{q}$  is a thickness-averaged fluid flow vector.

### 3. The traditional approach of solving the Pseudo3D model. The governing equation.

As with the PKN model, the equation of continuity (the law of local volume balance) is traditionally used in the Pseudo3D model as the basic equation.

$$\frac{1}{H} \frac{\partial Q}{\partial x} + \frac{\partial \bar{w}}{\partial t} + \frac{C_L}{\sqrt{t-t_0(x)}} = 0, \quad (6)$$

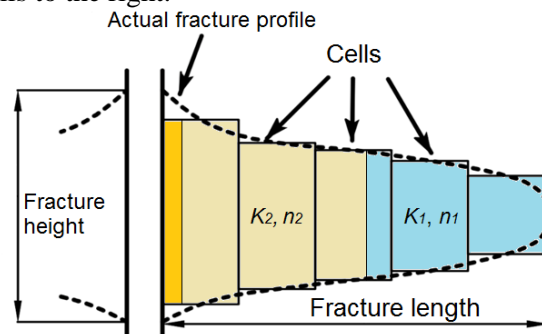
where the  $Q = \int_h q(z)dz$  is the average volume flow,  $H$  is the height of the permeable (leak-off) interval,  $\bar{w}$  is the average-height opening,  $t_0(x)$  is the time at which the front edge of the fracture will reach the  $x$  coordinate,  $C_L$  is the Carter leak-off coefficient. Thus, the classical approach of Pseudo3D model solves a system of equations (1-6).

### 4. The Proposed Proxy Pseudo3D Model

#### 4.1. The idea of the proposed approach is to address the shortcomings of the traditional approach.

The traditional Cell-based Pseudo3D approach focuses on the numerical model solution characterized by the following features.

**4.1.1. Disadvantages of the numerical grid in the Cell-based Pseudo3D model.** The fracture geometry is divided into cells of fixed size lengthwise (see Figure 1). The fracture horizontal propagation is modeled by adding new cells to the right.



**Figure 1.** Cell-based Pseudo3D (several fluids).

The accuracy of tracking the front edge of the fracture represents the first drawback of the traditional approach. The fact is that the fracture length increases differently at each time step and can not be a multiple of a fixed cell size, which, with a sufficiently coarse grid, can lead to a significant accumulation of numerical error along the fracture length.

**4.1.2. Tracking the front between fracturing fluids in Cell-based Pseudo3D model.** If a number of fracturing fluids (see Figure 1) and a proppant are present, there is a challenge similar to that related the fracture front: the fluids interface does not coincide with computational nodes at the boundaries of cells. In that case, in order to keep the material balance between the injected volumes, very fine-pitch grids should be used when running the models. There is another approach which uses a saturation parameter, but this requires extra equations to be solved.

*4.1.3. The sequence of the Cell-based Pseudo3D equation.* Since the flow rate  $Q$  is a pressure derivative in the  $x$  coordinate, after substituting expression (5) into (6), the classical approach will produce the final second-order partial-derivative equation. It must be solved iteratively at each step in time.

Next, an approach to obtain a first-order equation which is solved by shock capturing method will be described.

#### *4.2. The system of equations of the suggested model*

The reason for solution complexity in the Cell-based Pseudo3D model is due to the coordinate system which is related to perforation interval at the bottom hole. Therefore, the authors of this paper suggest to address the problem in a fracture-related coordinate system, namely the tip-related system.

The fracturing fluid rheology can be of Newtonian type - in case of water-based frac (SlickWater technology). However, the fracturing fluid is typically a polymer system and is described by a power-law model

$$\tau = K\dot{\gamma}^n, \quad (7)$$

where  $K$  is the consistence index,  $n$  is a non-Newtonian behavior factor (the fluid is Newtonian at  $n = 1$ ). If proppant is present,  $K = K_f(1 - c/c^*)^{-2.5}$ , where  $K_f$  is the consistence index of pure fluid without proppant,  $c$  is the volume concentration of proppant,  $c^*$  is the critical concentration (usually  $c^* = 0.64$ ).

For a power-law fluid, the lubrication layer equation is as follows:

$$q|q|^{n-1} = -\frac{1}{\phi K} w^{2n+1} \frac{\partial p}{\partial x}, \quad (8)$$

$$\phi = 2^{2n+1}(2n+1)^n n^{-n}. \quad (9)$$

We assume that the abscissa axis points from the tip of the fracture to the well, the  $x = 0$  coordinate determines the tip of the fracture. Then the total flow through a fracture cross-section  $Q$  from (8) will be:

$$q(z) = \left( \frac{1}{\phi K} w(z)^{2n+1} \frac{\partial p}{\partial x} \right)^{1/n}, \quad (10)$$

$$Q = \int_h q(z) dz = \left( \frac{1}{\phi K} \frac{\partial p}{\partial x} \right)^{1/n} \int_h w^{(2n+1)/n} q(z) dz, \quad (11)$$

the fracture shape is related to the friction pressure loss through the following expression:

$$\frac{\partial p}{\partial x} = \phi K Q^n \left( \int_h w^{\frac{2n+1}{n}} dz \right)^{-n}. \quad (12)$$

Here the height and width of the fracture are the pressure functions  $h=h(p)$ ,  $w=w(p,z)$ , calculated from the system (1-5).

If we assume that there are no leak-offs, then the fracture volume  $V$  at time point  $t$  is equal to the volume of the injected fluid

$$V = Qt. \quad (13)$$

The fracture half-length  $L(t)$  is determined by the fracture volume  $V(t)$ :

$$V(t) = \int_0^{L(t)} \int_{h(p)} w(p,z) dz dx. \quad (14)$$

Thus, equations (1-5), (9), and (11-14) form a closed system of equations for solving the Pseudo3D model for the case with no leak-off.

#### *4.3. Numerical Algorithm*

The numerical algorithm consists of a number of consecutive stages:

1. A preliminary computation of a geomechanical problem: the height  $h = h(p)$  and the fracture opening profile  $w(z, p)$  as a function of fluid pressure  $p$  are determined from equations (1-4). And the minimum pressure  $\text{Min } p = p_0$  corresponds to the value at which the fracture height is equal to the height of the initiation interval  $h(p_0) = h_{\text{init}}$ .

2. A preliminary computation of the integrals (15-16) using the computed opening profiles determined at the first stage of the numerical algorithm:

$$J(p) = \int_{h(p)} w(p, z) dz, \quad (15)$$

$$F_n(p) = \int_{h(p)} (w(p, z))^{\frac{2n+1}{n}} dz. \quad (16)$$

3. Then we used a Cauchy problem for the first-order ordinary differential equation

$$\frac{dp}{dx}(x) = \phi K Q^n (F_n(p))^{-n}, p(0) = p_0 \quad (17)$$

to determine the  $p(x)$ .

4. In order to determine the complete fracture geometry, we only need to determine its boundary  $L(t)$ . For this,  $V = Qt$  is used to reformulate (14) into a Cauchy problem for a first-order ODE:

$$\frac{dV}{dx}(x) = J(p(x)), V(0) = 0. \quad (18)$$

Taking into account that the inverse function  $L^{-1}(x) = V(x)$  is found, the fracture front  $x = L(V)$  can be determined.

Thus, the solution of the system of equations (1-5), (9), (11-14) was reduced to the pre-computation (15-16) and the solutions of two Cauchy problems for first-order ODEs (17-18) which are solved by any standard method through a number of operations  $O(N)$ , where  $N$  is the number of sampling points along the  $x$  coordinate

#### 4.4. Generalization for the case of several fracturing fluids

Let us consider the case when several fluids are injected during fracturing operations. This does not affect the preliminary geomechanical computation. Let us consider how the form of the Cauchy problem (17), (18) will change.

We assume that  $M$  fracturing fluids with rheological parameters  $K_m, n_m, m=1, \dots, M$ , are consistently injected into a hydraulic fracture starting from the time  $t = 0$ ;  $t_m$  is the end time of injection fluid  $m$  with a volume of a single wing  $V_m = Qt_m$ ; then  $p(V), L(V)$  are piecewise smooth functions consisting of smooth functions  $p_m(V), L_m(V)$  on the intervals  $V_{(m-1)} < V < V_m$ . Thus, the Cauchy problem domain to determine the pressure (17) is divided into  $M$  intervals:

$$\frac{dp_m}{dx}(x) = \phi_m K_m Q^{n_m} (F_{n_m}(p_m(x)))^{-n_m}, L_{m-1}(V_{m-1}) < x < L_m(V_m), \quad (19)$$

$$p_m(L_{m-1}(V_{m-1})) = p_{m-1}(L_{m-1}(V_{m-1})), p(L_0) = p(0) = p_0, m = 1, \dots, M, \quad (20)$$

$$\phi_m = 2^{2n_m+1} (2n_m + 1)^{n_m} n_m^{-n_m}. \quad (21)$$

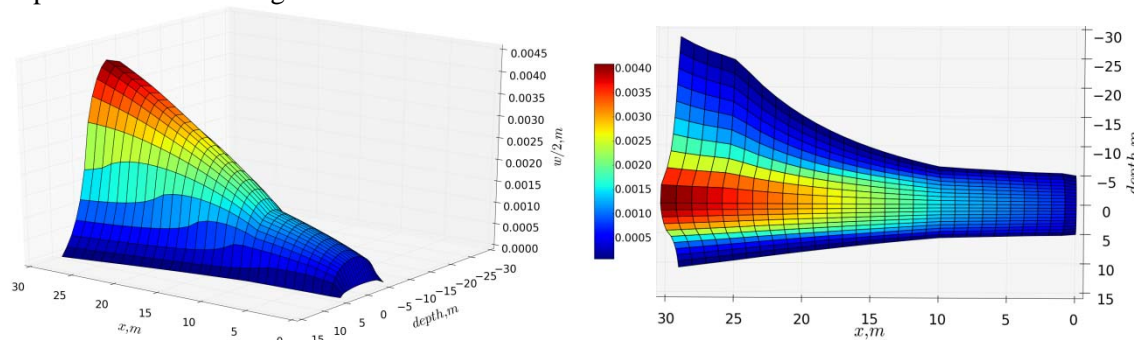
Moreover, the boundaries of the intervals  $L_m(V_m)$  are determined from the solution of the Cauchy problem (18) which in the case of several fracturing fluids has the form:

$$\frac{dV_m}{dx}(x) = J(p_m(x)), L_{m-1} < x < L_m, \quad (22)$$

$$V_m(L_{m-1}) = V_{m-1}(L_{m-1}), V(0) = 0, m = 1, \dots, M. \quad (23)$$

Thus, in the case of  $M$  fracturing fluids, we have two Cauchy problems (19-21) and (22), which must be solved in sequence for each of the intervals  $V_{(m-1)} < V < V_m, m=1, \dots, M$ . Moreover, the overall complexity of the problem solution remains the same  $O(N)$ , where  $N$  is the number of sampling points at the  $x$  coordinate for solving the Cauchy problems (19-21) and (22-23). In this case, the boundaries

of various fluids  $L_m(V_m)$  are determined quite accurately. Figure 2 below shows a computation example for two fracturing fluids.



**Figure 2.** The example of Proxy Pseudo 3D calculations: 2 fluids, 4 layers.

## 5. Summary

In this paper, the authors described an efficient approach to calculate the geometry of a hydraulic fracture in a Proxy Pseudo3D setting. By reformulating the problem, the authors succeeded in reducing it to two ordinary first-order differential equations instead of a nonlinear second-order partial derivative equation, which fundamentally affects the computational complexity. Also, this method allows a more correct computation of the fracture front and fracturing fluids interface versus the traditional Cell-Based Pseudo3D model. Thus, this model meets the requirements of practically meaningful computations and significantly beats its counterparts in running speed, which is critically important for serial calculations. The developed proxy model can also be effectively used as an integral part of the fracture network model in fractured reservoirs, which will be a focus of further studies.

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