Simulation of fracture of elasto-platic solids with cracks under conditions of out-of plane deformation

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Abstract. The paper considers a tearing mode crack (mode III fracture) propagation in elasto-plastic materials having ultimate strain. The fracture process of such material has been described in the modified Leonov-Panasyuk-Dugdale model with using an extra parameter such as the plastic zone width (the pre-fracture zone width. The two-parameter criterion of quasi-brittle fracture for mode III cracks in elasto-plastic material has been derived for conditions of the low-scale yielding in the presence of a stress field singularity of the field in the vicinity of a crack. The deformation fracture criterion has been deduced for the initial crack tip, whereas the force criterion has been deduced for tangential stresses with account for their averaging at the tip of a model crack. The lengths of initial and model cracks differ by the length of the pre-fracture zone. The sequential analysis of application possibility of the proposed strength criterion has been performed when determining fracture loads for solids with tearing mode cracks. Fracture diagrams of quasi-brittle fracture under conditions of antiplane deformation have been plotted for a band with an edge crack.

1. Introduction

In studies [1–3], the two-parameter (coupled) discrete-integral fracture criterion has been proposed. This criterion may be used when constructing fracture diagrams of plane specimens with opening mode cracks (mode I fracture). In the plane representing stress versus crack length, curves are plotted, which divide this plane into three subareas corresponding to the absence of fracture, damage accumulation in a pre-fracture zone, and fragmentation of a specimen. Fitting constants applied for analytical description of fracture diagrams of materials in the presence of cracks is by the way of approximation of the classical stress-strain diagram of starting material and the critical stress intensity factor (SIF).

The coupled fracture criterion is applied in the present paper for determination of critical loads for an edge mode III crack in a band made from elasto-plasic material.

2. Quasi-brittle fracture diagrams tearing mode crack

The proposed model [1–3] uses the non-classical scheme of material failure.

Suppose that a $(\tau-\gamma)$-diagram is obtained during laboratory experiment when testing macro-specimen. We take the simplest approximation of the real $(\tau-\gamma)$-diagram of material under study when this diagram is approximated by a two-link broken line. When approximating, original material is substituted for ideally plastic material having the ultimate strain. Given in figure 1, *a* are the original $(\tau-\gamma)$ - diagram (curve 1) and its two-link approximation (curve 2). Parameters of this approximation are selected so that the areas under the curves should be coincident. The approximation parameters are:

G is shear modulus, τ_Y is shear yield point of the material and constant stresses acting according to the Leonov-Panasyuk-Dugdal (LPD) model [4, 5], γ_0 is the maximum elastic strain of material $(\tau_Y = G\gamma_0)$ and γ_1 is the maximum material strain. The approximation of the ($\tau - \gamma$)-diagram within the segment $\gamma_0 < \gamma < \gamma_1$ can be treated as ideal plasticity.

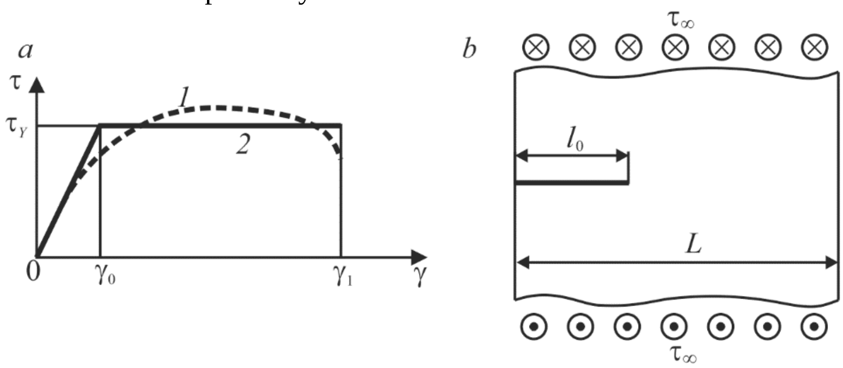

Figure 1. The original diagrams of material strain (curve 1) and its two-link approximation (curve 2) (*a*); the loading scheme of a band with edge tearing mode crack (*b*).

Suppose that r for material with the regular structure is the grain diameter, more exactly, it is the effective diameter of breakdown structures [3]. The Neuber-Novozhlov approach [6, 7] allows solutions having the singular term with integrable singularity to be used for structured media.

Consider an edge tearing mode crack in a band with the width L (Figure 1, *b*). Let this plane tearing mode crack be extended rectilinearly. Except for the length l_0 of a real crack-cut, we introduce into consideration the length of a model crack-cut. The length of the model crack-cut is $l = l_0 + \Delta$, the pre-fracture zone of length Δ being located on the real crack continuation. The fracture problem has two linear scales: if a grain diameter r is governed by the material structure, then the second linear scale is produced by the system itself. This second linear scale is the pre-fracture zone length Δ that changes in accordance with how the following values change: 1) the real crack length and 2) the load intensity. Emphasize that critical pre-fracture zone length Δ^* under single loading is the well-defined parameter ($l^* = l_0 + \Delta^*$ is the critical macro-crack length).

When plotting diagrams of quasi-brittle fracture, sufficient fracture criteria are used [1–3] when short, long macro-cracks and those of medium length are considered. The sufficient (coupled) criterion can be represented in the form of two relations for short macro-cracks and macro-cracks of medium length

$$\frac{1}{r}\int_0^r \tau_{yz}(x,0)dx = \tau_Y , \quad 2w(-\Delta^*) = \delta^* . \tag{1}$$

Here $\tau_{yz}(x,0)$ are shear stresses on the continuation of cracks; $Oxyz$ is the Cartesian coordinate system provided that the coordinate origin is coincident with the model crack tip in the modified LPD model [4, 5], the axis Ox is directed along the crack plane; $2w = 2w(x)$ is the displacement of the model macro-crack flanks ($x < 0$); δ^* is the critical displacement of flanks of this crack, and Δ^* is the critical pre-fracture zone length (critical values obtained via the sufficient and necessary fracture criteria are marked with superscripts * and ^{0}). Attention should be given to the fact that the proposed criterion (1) and (2) is coupled. Figure 2 demonstrates compressive stresses acting in the LPD model on the crack continuation (*a*) and the approximation of a plastic zone by a rectangular pre-fracture zone (*b*). Under conditions of antiplane deformation, by “compressive” are meant stresses opposite in sign to stresses τ_∞, acting at remote boundaries. The pre-fracture zone occupies only a part of the plastic zone.

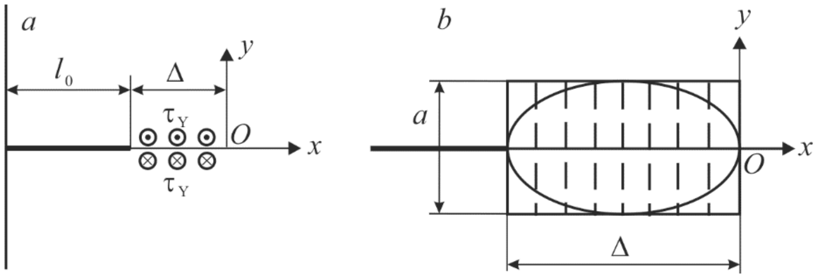

Figure 2. Compressive stresses acting in the LPD model on the continuation of a crack (*a*); approximation of the plastic zone (ellipse) by the rectangular pre-fracture zone (*b*).

The field of shear stresses $\tau_{yz}(x,0)$ on the model crack continuation $x>0$ may be represented as a sum of two terms

$$\tau_{yz}(x,0)=\frac{K_{\mathrm{III}}}{\sqrt{2\pi x}}+\tau_{nom}\,. \qquad (2)$$

Here $\tau_{nom}=Y_r\tau_\infty$ are nominal stresses, otherwise the regular part of the stress field in the vicinity of a model crack, $Y_r=Y_r(l/L)$ is the correction factor to be determined, $K_{\mathrm{III}}=K_{\mathrm{III}\infty}+K_{\mathrm{III}\Delta}>0$ is the total stress intensity factor (SIF); $K_{\mathrm{III}\infty}>0$ is the SIF generated by test conditions; $K_{\mathrm{III}\Delta}<0$ is the SIF generated by the given constant stresses τ_Y, acting in the pre-fracture zone. The first and second summands in relation (2) are the singular and regular part of the solution, respectively. The first equality (1) in the coupled criterion keeps track of stresses acting on the continuation of a model crack and their attainment of yield point τ_Y after averaging, and the second equality of this criterion describes blunting of a real macro-crack tip.

We pass to assessment of singular terms of stress fields for edge cracks. Inasmuch as deformation of material under conditions of the low-scale yield is studied, then for specimens with sharp edge cracks, the following relations [8] are obtained for the total SIF $K_{\mathrm{III}}=K_{\mathrm{III}\infty}+K_{\mathrm{III}\Delta}>0$:

$$K_{\mathrm{III}\infty}=Y_s\tau_\infty\sqrt{\pi l}\,,\quad Y_s=\sqrt{\frac{2L}{\pi l}\,\mathrm{tg}\frac{\pi l}{2L}}\,,\quad K_{\mathrm{III}\Delta}=-\tau_Y\sqrt{\pi l}\left[1-\frac{2}{\pi}\arcsin\left(1-\frac{\Delta}{l}\right)\right]. \qquad (3)$$

When considering quasi-brittle fracture under conditions of low-scale yield with account for inequality $\Delta^*/l_0\ll 1$ to the accuracy of higher order of infinitesimals for the summand $\arcsin\left(1-\Delta/l\right)$ in relation (3) the representation $\arcsin\left(1-\Delta/l\right)\approx\pi/2-\sqrt{2\Delta/l}$ is valid for $\Delta/l\ll 1$. Finally, the simplified expression for SIF $K_{\mathrm{III}\Delta}$ has the form

$$K_{\mathrm{III}\Delta}=-2\tau_Y\sqrt{\frac{2\Delta}{\pi}}\,. \qquad (4)$$

The correction factor $Y_r=\dfrac{1}{1-l/L}-Y_s\sqrt{\dfrac{2l/L}{1-l/L}}$, that accounts for a band width L, is deduced from equilibrium condition of stresses applied to remote boundaries and stresses acting on the ligament $L-l$. As $l\to L$, then $\tau_{nom}\to\infty$ are nominal stresses that reflect the rise in stresses with vanishing the net cross-section under constant loading. Both coefficients Y_s and Y_r for an edge tearing mode crack in a semi-infinite solid are identically equal to unity.

When there exists the singular term of the solution under conditions of the low-scale yield, the model crack opening $2w$ in a band is given as

$$2w(-x,0)=\frac{4K_{\mathrm{III}}}{G}\sqrt{\frac{-x}{2\pi}}\,. \tag{5}$$

The critical crack opening of a model crack δ^* in relation (1) depends on a plasticity margin $\gamma_1-\gamma_0$ of studied material and the plastic zone width a at the tip of a real crack. This critical opening of model cracks δ^* calculated for specimens with a sharp crack is given in such a way

$$\delta^*=(\gamma_1-\gamma_0)a\,. \tag{6}$$

Let the pre-fracture zone width a in relation (6) be proportional to the plastic zone diameter for specimens with a sharp crack manufactured from a homogeneous material [9]

$$a=\chi\frac{1}{\pi}\left(\frac{K_{\mathrm{III}\infty 0}}{\tau_Y}\right)^2. \tag{7}$$

Here $\chi<1$ is some correction factor for determination of which it is necessary to use the data of the numerical or laboratory experiments. Emphasize that the plastic zone width in relation (7) depends on the length l_0 of an initial crack, i.e., for SIF $K_{\mathrm{III}\infty 0}=K_{\mathrm{III}\infty 0}\left(l_0/r, l_0/L\right)$ representation (3) is used. The model crack length l and parameter l/L in this representation are substituted by the initial crack length l_0 and parameter l_0/L, respectively. The critical opening of the model crack δ^* in relation (6) is fitted so that the material at the real crack tip fails when γ_1 is the maximum material strain.

Let us obtain estimates of the critical material state at the crack tip. All necessary analytical expressions are available in relations (2), (3), and (5) – (7) to make use of sufficient (coupled) criterion (1). After appropriate transformations, the original equalities of criterion (1) change into approximate equalities for a band with a crack:

$$K_{\mathrm{III}\infty}\left(\tau_\infty^*, l^*, \frac{l^*}{L}\right)+K_{\mathrm{III}\Delta}\left(\tau_\infty^*, l^*, \Delta^*\right)=\left(\tau_Y-Y_r\tau_\infty^*\right)\sqrt{\frac{\pi r}{2}}\,, \tag{8}$$

$$\frac{4}{G}\left[K_{\mathrm{III}\infty}\left(\tau_\infty^*, l^*, \frac{l^*}{L}\right)+K_{\mathrm{III}\Delta}(\tau_\infty^*, l^*, \Delta^*)\right]\sqrt{\frac{\Delta^*}{2\pi}}=\chi\left(\gamma_1-\gamma_0\right)\frac{2}{\pi}\left(\frac{K_{\mathrm{III}\infty}\left(\tau_\infty^*, l_0, \frac{l_0}{L}\right)}{\tau_Y}\right)^2. \tag{9}$$

Draw attention to the fact that the initial crack length l_0 enters into equation (9). Since the quasi-brittle approximation is studied, then this length l_0 is substituted by the model crack length l^*, taking into consideration the inequality $\Delta^*/l_0\ll 1$. After transformation, we get the system of equations

$$\overline{\tau}_\infty^* Y_s\sqrt{\frac{2l^*}{r}}-\frac{2\sqrt{2}}{\pi}\sqrt{\frac{2l^*}{r}}\sqrt{\frac{\Delta^*}{l^*}}=1-Y_r\overline{\tau}_\infty^*, \tag{10}$$

$$\frac{\sqrt{2}\tau_Y}{G}\left[\overline{\tau}_\infty^* Y_s-\frac{2\sqrt{2}}{\pi}\sqrt{\frac{\Delta^*}{l^*}}\right]\sqrt{\frac{\Delta^*}{l^*}}=\chi\left(\gamma_1-\gamma_0\right)(Y_s\overline{\tau}_\infty^*)^2\,. \tag{11}$$

Here $\overline{\tau}_\infty^*=\tau_\infty^*/\tau_Y$ are critical dimensionless stresses in a band via the sufficient fracture criterion. After opening square brackets in the left-hand side of equation (11), the term with the factor Δ^*/l^* appears, which can be dropped as a value of the higher order of infinitesimals in comparison with $\sqrt{\Delta^*/l^*}$ in virtue of the inequality $\Delta^* / l_0 \ll 1$. As a result, only terms with factors $\sqrt{\Delta^* / l^*}$ remain in the system of equations (10) and (11). After solving the obtained system of equations, taking into account the equality $\tau_Y / G = \gamma_0$, we find analytical expressions of dimensionless critical stresses $\overline{\tau}_\infty^*$ and the dimensionless length $\overline{\Delta}^* = \Delta^* / l^*$ of the pre-fracture zone:

$$\overline{\tau}_\infty^* = \left[Y_r + \left(1 - \frac{2}{\pi} \chi \overline{\gamma}_{\mathrm{III}} \right) Y_s \sqrt{2\overline{l}^*} \right]^{-1}, \tag{12}$$

$$\overline{\Delta}^* = \frac{1}{2} \left(\chi \overline{\gamma}_{\mathrm{III}} Y_s \overline{\tau}_\infty^* \right)^2 \tag{13}$$

Here $\overline{l}^* = l^* / r^*$ is the critical dimensionless crack length, $\overline{\gamma}_{\mathrm{III}} = (\gamma_1 - \gamma_0) / \gamma_0$ is the parameter characterizing plasticity margin in longitudinal shear. There is the restriction $2\chi\overline{\gamma}_{\mathrm{III}} < \pi$, wherein the quasi-brittle fracture exists under condition of the low-scale yielding of a homogeneous material in the pre-fracture zone.

Within the limit for $\gamma_1 \to \gamma_0$ the following formula, corresponding to the sufficient fracture criterion, can be derived from relation (13)

$$\overline{\tau}_\infty^0 = \left(Y_r + Y_s \sqrt{2\overline{l}_0} \right)^{-1}. \tag{14}$$

Here $\overline{\tau}_\infty^0 = \tau_\infty^0 / \tau_Y$ is the dimensionless load via the sufficient criterion and $\overline{l}_0 = l_0 / r_0$ is the dimensionless crack length. Relation (14) describes brittle fracture of materials. It is obvious that $\overline{\tau}_\infty^0 < \overline{\tau}_\infty^*$ for $l_0 < l^*$.

If the critical SIF $K_{\mathrm{III}c}$ and the classical $(\tau - \gamma)$ diagram (more precisely its approximation) are obtained in two laboratory experiments, then, using three parameters $r_0, \tau_Y, \chi\overline{\gamma}_{\mathrm{III}}$ or $r^*, \tau_Y, \chi\overline{\gamma}_{\mathrm{III}}$ it is possible to construct two critical curves $\overline{\tau}_\infty^0 = \overline{\tau}_\infty^0 \left(\overline{l}_0, \overline{L}_0 \right)$ and $\overline{\tau}_\infty^* = \overline{\tau}_\infty^* \left(\overline{l}^*, \overline{L}^* \right)$ within the wide range of variation in crack lengths. These curves depend on the crack lengths $\overline{l}_0$, $\overline{l}^*$ as well as on the width of a band $\overline{L}_0 = L / r_0$ and $\overline{L}^* = L / r^*$. Let the plane $\left(\overline{l}_0, \overline{\tau}_\infty^0 \right)$ be compatible with the plane $\left(\overline{l}^*, \overline{\tau}_\infty^* \right)$. On the combined plane "crack length versus stresses" $\left(\overline{l}, \overline{\tau}_\infty \right)$, where $\overline{l} = l / r$ and $\overline{\tau}_\infty = \tau_\infty / \tau_Y$, we construct diagrams of quasi-brittle fracture for the specimen type being considered. The critical curves on the plane $\left(\overline{l}, \overline{\tau}_\infty \right)$ depend on the relation $\overline{L} = L / r$. Let the load intensity $\overline{\tau}_\infty$. be given. Then the diagram of the quasi-brittle fracture allows one to estimate the state of a solid with a crack. Two critical curves $\overline{\tau}_\infty^0$ and $\overline{\tau}_\infty^*$ divide the plane $\left(\overline{l}, \overline{\tau}_\infty \right)$ into three subareas: the subarea $\overline{\tau}_\infty < \overline{\tau}_\infty^0$, where there is no fracture; ; the subarea $\overline{\tau}_\infty^0 < \overline{\tau}_\infty < \overline{\tau}_\infty^*$, where damage accumulation takes place in material of the pre-fracture zone; the subarea $\overline{\tau}_\infty > \overline{\tau}_\infty^*$, where the specimen falls under monotonic loading.

Figure 3 demonstrates the dimensionless critical stresses $\overline{\tau}_\infty^0 = \overline{\tau}_\infty^0 \left(\overline{l}_0, \overline{L}_0 \right)$ (curves 1 – 5) and $\overline{\tau}_\infty^* = \overline{\tau}_\infty^* \left(\overline{l}^*, \overline{L}^* \right)$ (curves 1' – 5') for specimens with a sharp crack in the log-log coordinates When the concrete implementation of calculation was conducted, the parameters $\overline{L}_0 = 200, 400, 800, 1600, \infty$ for curves 1, 2, 3, 4, 5 and $\overline{L}^* = 200, 400, 800, 1600, \infty$ for curves 1', 2', 3', 4', 5', respectively, were chosen.

The pairs of curves 1 – 1', 2 – 2', 3 – 3', 4 – 4' and 5 – 5' are diagrams of quasi-brittle fracture for the type of a studied specimen manufactured from a homogeneous material.

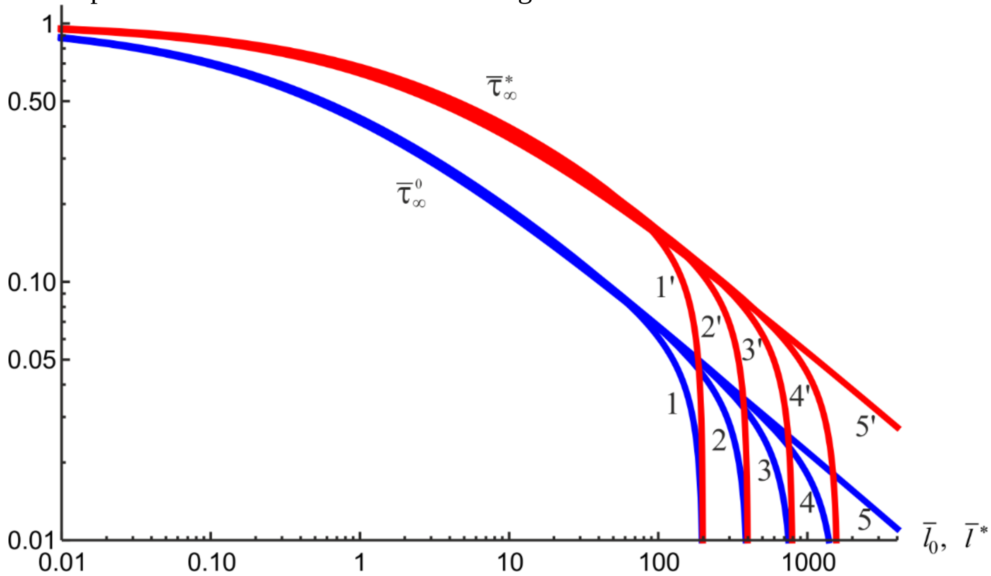

Figure 3. Diagrams of quasi-brittle fracture.

3. Conclusion

The quasi-brittle fracture diagram of an edge crack of the longitudinal shear in a band of the finite width was constructed in the plane "load versus crack length". The diagram consists of two curves, which divide the plane into domains. In the first domain, there is no fracture. Damage accumulation under repeated loading occurs in the second domain. Finally, fragmentation of the specimen under monotonic loading occurs in the third domain. The constants for the analytical description of the fracture diagrams of quasi-brittle materials with cracks were chosen making use of the approximation of the classical stress–strain diagram for the original material and the critical stress intensity factor. The derived structural relations (12) – (14) may be useful for prediction of a critical fracture loading and estimation of the pre-fracture zone length when specimens are loaded with respect to the mode III (antiplane strain loading) in structured materials. These relations express the value of critical loading and the length of a pre-fracture zone in terms of the crack length making use of the following four parameters: r is the characteristic linear parameter of material structure, γ_0 and γ_1 are parameters of the stress-strain diagram, χ is the correction coefficient. The first three parameters are found as a result of a laboratory experiment, and the last parameter is found during computer simulation.

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