Compression waves in porous media containing gas hydrate

A A Gubaidullin[1,2], O Yu Boldyreva[1] and D N Dudko[1]

[1]Tyumen Branch of Khristianovich Institute of Theoretical and Applied Mechanics SB RAS, Tyumen, Russia
[2]Tyumen State University, Tyumen, Russia

E-mail: gubai@tmn.ru

Abstract. Wave processes in a porous medium containing water, gas and gas hydrate are numerically investigated. The two-velocity model of porous medium is used. The nonlinearity of the oscillations of the bubbles is taken into account in the equation of gas state and the equation for bubble radius. The transmission of compression of step waves from the water into hydrate-containing porous medium is investigated. Beyond the line of phase equilibrium the association of gas hydrate is possible. The influence of the medium parameters and the intensity of the incident wave on the evolution of waves in this porous medium is studied. When entering the porous medium, the initial pulse is divided into fast (deformation) and slow (filtration) waves. In a slow wave damped oscillations associated with pulsations of bubbles are observed. The increase in the initial equilibrium pressure in the medium leads to an increase in the velocities of the deformation and filtration waves. With the initial volume content of the gas in the pore space unchanged, an increase in the initial hydrate content leads to a small increase in the velocities and amplitudes of fast and slow waves. At constant initial volume content of hydrate, the increase of gas content leads to a noticeable decrease of slow wave speed and decrease of amplitudes of both fast and slow waves. It was found that the accounting of the processes of formation of gas hydrate in a porous medium at the considered time intervals does not have a noticeable effect on the character of wave propagation.

1. Introduction

Currently, the scientific literature presents the results of experimental studies on the association and dissociation of hydrates in porous media [1] and the association of hydrate in water with bubbles of hydrate-forming gas during the passage of shock waves [2]. The theoretical study of the process of hydrate association in the propagation of a shock wave in water with bubbles of a hydrate-forming gas was carried out in [3]. The pore space of hydrate-containing porous media usually includes water and gas bubbles, so the presence of gas bubbles should be taken into account in the study of wave processes in such media. Wave propagation in porous media saturated with liquid and gas bubbles was studied experimentally in [4-7]. The numerical study of wave processes in partially saturated porous media was carried out in [8].

In this paper, wave processes in a porous medium, whose pore space contains water, gas hydrate and hydrate-forming gas in the form of bubbles, are numerically investigated. The nonlinearity of bubble oscillations is taken into account in the gas state equation and the Rayleigh-Plesset equation for a gas bubble in a porous medium.

2. Basic Equations

To study wave propagation in a porous medium containing gas hydrate and a bubbly liquid, two velocity, two stress tensors model of saturated porous medium is used [9, 10]. The pore space is filled with three-phase mixture of hydrate and liquid with gas bubbles, bubble size is calculated using the equation of Rayleigh-Plesset for a gas bubble in a porous medium [4].

The equations of mass and momentum balance are in the form:

$$\frac{\partial \rho_l}{\partial t} + \nabla^i\left(\rho_l v_l^i\right) = 4\pi a_b^2 n_b j_l, \qquad \frac{\partial \rho_g}{\partial t} + \nabla^i\left(\rho_g v_l^i\right) = 4\pi a_b^2 n_b j_g,$$

$$\frac{\partial \rho_h}{\partial t} + \nabla^i\left(\rho_h v_l^i\right) = 4\pi a_b^2 n_b j_h, \qquad \frac{\partial \rho_s}{\partial t} + \nabla^i\left(\rho_s v_s^i\right) = 0, \qquad \frac{\partial n_b}{\partial t} + \nabla^i\left(n_b v_l^i\right) = 0, \qquad (1)$$

$$\rho_{l+g+h}\frac{d_l v_l^i}{dt} = -\alpha_{l+g+h}\nabla^i p_l - F^i, \qquad \rho_s\frac{d_s v_s^i}{dt} = -\alpha_s \nabla^i p_l + \nabla^j \sigma_{s*}^{ji} + F^i, \qquad (2)$$

where ρ_j, v_j, α_j are the density, velocity, volumetric content of the j - th phase, $v_h = v_g = v_l$, the subscripts $j = s, l, g, l+g+h$ relate to the skeleton of the porous medium, liquid, gas or mixture of liquid and bubbles and hydrate; σ_{s*}, p_l are effective stress in the skeleton and the pressure in the liquid, respectively, a_b is bubble radius, n_b is the number of bubbles per unit volume.

Interphase force interaction is a sum of viscous friction force F_μ and added masses force F_m:

$$F = F_m + F_\mu, \qquad F_m = \frac{1}{2}\eta_m \alpha_s \rho_{l+g+h}\left(\frac{d_l v_l}{dt} - \frac{d_s v_s}{dt}\right), \qquad F_\mu = \eta_\mu \alpha_s \alpha_{l+g+h}\mu_l a_{s*}^{-2}\left(v_l - v_s\right), \qquad (3)$$

where a_{s*} is the characteristic size of the grains of the skeleton, μ_l is viscosity of the fluid, η_m, η_μ are the coefficients of interaction between phases dependent on the structure of the pore space.

The temperatures of the skeleton, water and hydrate are assumed constant and equal to the initial temperature T_0. The heat equation of inflow for gas phase is taken in the form [10]:

$$\rho_g c_{gV}\frac{d_l T_g}{dt} = \alpha_g p_g \frac{1}{\rho_g^\circ}\frac{d_l \rho_g^\circ}{dt} - 4\pi a_b^2 n_b q_{gl} - 4\pi a_b^2 n_b j_g c_{gV}\left(T_g - T_0\right), \qquad (4)$$

$$q_{gl} = \mathrm{Nu}\,\lambda_g \frac{T_g - T_0}{2a_b}. \qquad \mathrm{Nu} = \begin{cases} 10, \mathrm{Pe} < 10^2 \\ \sqrt{\mathrm{Pe}}, \mathrm{Pe} \geq 10^2 \end{cases}, \qquad \mathrm{Pe} = 12\left(\gamma_g - 1\right)\frac{T_0}{\left|T_g - T_0\right|}\frac{a_b\left|\dot{a}_b\right|}{\nu_g}, \qquad (5)$$

where λ_g, ν_g are thermal conductivity and thermal diffusivity.

If the pressure and (or) temperature deviate from the phase equilibrium line "gas + water $\leftrightarrow$ gas hydrate" given by the equation

$$\frac{p_s}{p_{s0}} = \exp\left(\frac{T_s - T_{s0}}{T_*}\right), \qquad (6)$$

then phase transitions are possible. In the compression wave the association of gas hydrate may occur at the gas bubble surface. The intensity of the phase association is given by the following equation [9, 10]:

$$j_g = -\frac{\beta\left(p_g - p_s(T_0)\right)}{\sqrt{2\pi R_g T_0}}, \qquad j_h = -j_g / G, \qquad j_l = -j_h - j_g. \qquad (7)$$

Here G is mass fraction of gas in gas hydrate, β is empirical coefficient characterizing the intensity of hydrate association.

The skeleton of the porous medium is assumed to be elastic with effective moduli of elasticity λ_{s*}, μ_{s*}:

$$\sigma_{s*}^{ij} = \alpha_s \left(\lambda_{s*} \delta^{ij} \varepsilon_s^{mm} + 2\mu_{s*} \varepsilon_s^{ij} + \gamma_{s*} \delta^{ij} p_l \right), \quad \gamma_{s*} = \frac{\lambda_{s*} + 2/3\,\mu_{s*}}{K_s}, \tag{8}$$

$$\frac{d_s \varepsilon_s^{ij}}{dt} = \frac{1}{2} \left(\nabla^i v_s^j + \nabla^j v_s^i \right),$$

where ε_s is strain of the skeleton, K_s is the bulk elastic modulus of the skeleton material, γ_{s*} is the ratio of bulk elastic moduli of the skeleton and its material.

The state equations for solid and liquid phases are given in the acoustic approximation:

$$p_s - p_{s0} = C_s^2 \left(\rho_s^\circ - \rho_{s0}^\circ \right), \quad p_l - p_{l0} = C_l^2 \left(\rho_l^\circ - \rho_{l0}^\circ \right), \quad p_h - p_{h0} = C_h^2 \left(\rho_h^\circ - \rho_{h0}^\circ \right) \tag{9}$$

here subscript 0 indicates the unperturbed value, ρ_j°, C_j, p_j are the true density, the sound velocity and the true pressure in the material of the j - th phase.

The behavior of gas in the bubbles follows the Mendeleev – Clapeyron equation:

$$p_g = \rho_g^\circ R_g T_g, \tag{10}$$

bubble radius a_b changes according to the equation of Rayleigh-Plesset for the bubble in a porous medium [4]:

$$\frac{da_b}{dt} = w_R + w_A + \frac{j_g}{\rho_{l0}^\circ}, \qquad w_A = \frac{p_g - p_l - \dfrac{2\Sigma}{a_b}}{\rho_{l0}^\circ C_l x_g^{1/3}}, \tag{11}$$

$$\rho_{l0}^\circ \left(\dot{w}_R a_b + \frac{3}{2} w_R^2 \right) = p_g - p_l - \frac{2\Sigma}{a_b} - 4\mu_l \frac{w_R}{a_b} \left(1 + \frac{1}{4} \alpha_s \eta_\mu \left(\frac{a_b}{a_{s*}} \right)^2 \right)$$

here x_g is the gas volume fraction in the bubbly liquid, Σ is the coefficient of surface tension at the liquid and gas boundary, w_A takes into account the compressibility of the liquid [11].

To close the system of equations, the relations between true and effective densities and volume fractions, and the relations between true pressures p_l, p_s, p_h in the phases and the effective pressure p_{s*} in the skeleton are used
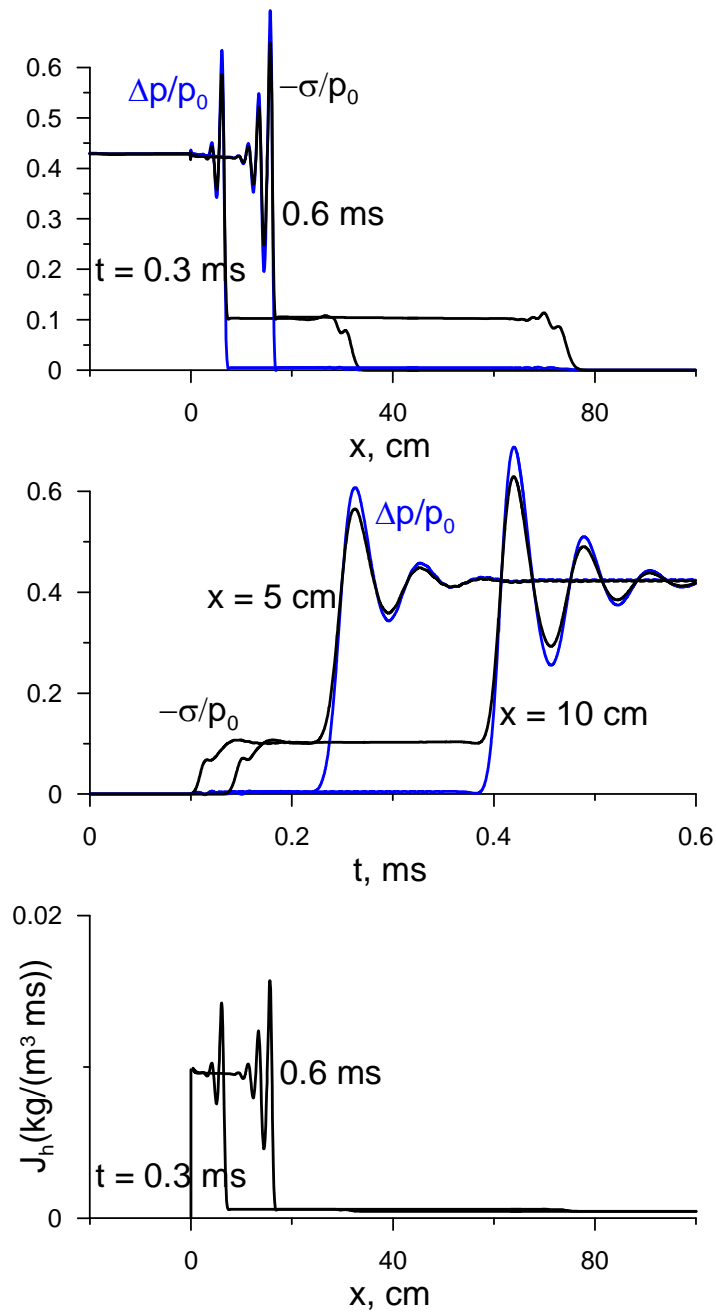
$$\rho_j = \alpha_j \rho_j^\circ, \quad \rho_{l+g+h} = \alpha_l \rho_l^\circ + \alpha_g \rho_g^\circ + \alpha_h \rho_h^\circ, \quad \alpha_s + \alpha_l + \alpha_g + \alpha_h = 1, \tag{12}$$

$$\alpha_g = \frac{4}{3} \pi a_b^3 n_b, \qquad p_{s*} = \alpha_s (p_s - p_l), \qquad p_{s*} = -\frac{1}{3} \sigma_{s*}^{mm}.$$

3. Propagation of stepwise wave
Methodology for calculating the motion of a porous medium containing liquid with gas bubbles based on the method of Lax–Wendroff was developed, and the numerical study of pressure wave propagation in such medium was carried out. Perturbation in the porous medium ($x>0$) is generated by a wave incident from the pure liquid ($x<0$).

We have investigated the influence of medium parameters and of the initial pulse on the wave evolution in a porous medium containing the water, gas and gas hydrate.

Figure 1. The pressure in the liquid, the total stress and the intensity of hydrate association in the step wave of the amplitude equal to 0.5· p_0 from a liquid into a porous medium at the distances of 5 cm and 10 cm from the boundary and at the moments 0.3 and 0.6 ms.

Figure 1 shows the variation of the pressure in the liquid p, the total stress $\sigma = \sigma_{s*} - p$ and the intensity of hydrate association $J_h = 4\pi a_b^2 n_b j_h$ in the step wave of the amplitude equal to 0.5· p_0 passing from the water into the porous medium containing water with freon bubbles and freon hydrate inclusions. The skeleton material is quartz. The initial pressure and temperature are p_0 =20 bar, T_0 = 294 K; these pressure and temperature are on the line of phase equilibrium (6). The initial volume

fractions are $\alpha_s = 0.6$, $\alpha_l = 0.352$, $\alpha_g = 0.008$, $\alpha_h = 0.04$; grain size $a_* = 1$ mm, the initial bubble radius $a_b = 0.5$ mm. The parameters of Equations (6, 7) for freon and freon hydrate $p_{s0} = 0.42$ bar, $T_{s0} = 274$ K, $T_* = 5.2$ K, $\beta = 3 \cdot 10^{-5}$ $G = 0.3$ are taken from [3]. The other parameters of the porous medium are $\lambda_{s*} = \mu_{s*} = 2 \cdot 10^9$ Pa, $\eta_m = 1$, $\eta_\mu = 100$.

When entering a porous medium initial pulse splits into fast (deformational) and slow (filtrational) compression waves. In the fast wave, the pressure amplitude in the liquid is close to zero, i.e. the compression is observed only by the skeleton of the porous medium, there is a monotonic increase in the total stress in the medium. In the slow wave, damped oscillations of the pore pressure associated with the pulsation of gas bubbles are observed. If you increase the amplitude of the original incident wave, pressure oscillations in the slow wave become more noticeable. In a medium with a higher equilibrium pressure, there is a higher propagation velocity of both fast (deformational) and slow (filtrational) waves.

The effect of phase transitions on the propagation of compression waves in a porous medium containing gas hydrate was also studied. With increasing pressure above the phase equilibrium line, hydrate formation occurs at the surface of the bubbles. The calculations show that the process of hydrate formation during the compression wave passage is rather slow and does not have a noticeable effect on the wave propagation in the medium at the considered time intervals.

References

[1]   Tohidi B, Anderson R, Masoudi A, Arjmandi J, Burgass R and Yang J 2003 *Russian Chemical Journal* **XLVTI(3)** 49–58

[2]   Dontsov V E, Nakoryakov V E and Chernov A A 2007 *Journal of Applied Mechanics and Technical Physics* **48(3)** 346–60

[3]   Shagapov V Sh, Lepihin S A and Chiglintsev I A 2010 *Thermophys. Aeromech.* **17(2)** 229–41

[4]   Dontsov V E, Kuznetsov V V and Nakoryakov V E 1987 *Izv. USSR Academy of Sciences, MZhG* **4** 85–92

[5]   Grinten J G M van der 1987 *An Experimental Study of Shock-Induced Wave Propagation in Dry, Water-Saturated, and Partially Saturated Porous Media,* Ph.D. Thesis (Proefschrift Eindhoven)

[6]   Smeulders D M J 1992 *On Wave Propagation in Saturated and Partially Saturated Porous Media,* Ph.D. Thesis (Proefschrift Eindhoven)

[7]   Smeulders D M J and van Dongen M E H 1997 *J. Fluid Mech.* **343** 351–73

[8]   Dunin S Z, Mikhailov D N and Nikolaevskii V N 2006 *J. Appl. Math. Mech.* **70(2)** 251–63

[9]   Nigmatulin R I 1978 *Fundamentals of Mechanics of Heterogeneous Media* (Moscow: Nauka)

[10]   Nigmatulin R I 1990 *Dynamics of Multiphase Media. Part 1* (New York: Hemisphere Publ)

[11]   Nigmatulin R I, Shagapov V S and Vakhitova N K 1989 *Dokl. Akad. Nauk SSSR* **304** 1077–81