

Design optimization of hydraulic fracturing

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Abstract. The approach to the problem of hydraulic fracturing design optimization is proposed which combines three modules: a fracture geometry module, an oil production module and an economic module. The approach is based on the genetic algorithm — the universal multivariate and multiobjective optimization method. The optimization process involves subsequent determination of the fracture geometry, calculation of the oil production depending on the number of fractures, and description of the investment–return cash flows. Iterated by the genetic algorithm for different set of input parameters, this approach allows determination of the optimal design of a multiply fractured well for a given set of geological and economical information. The results show that the proposed method is plausible and can be useful for achieving projected objectives.

1. Introduction

One of the most efficient methods for enhancement of oil recovery is the hydraulic fracturing of reservoirs. Modern technologies imply creation of multiply fractured horizontal wells (MFHW) in low-permeable reservoirs. In light of the high cost of this method, the problem of optimization of the MFHW design is of great interest.

It is necessary to distinguish between physical optimization, when the criterion of success is to achieve the maximum production rate, and the economic one — when the maximum production should be reached under certain economic constraints, for example, the limitations to the total costs of the hydraulic fracturing jobs.

The concept of the Unified Fracture Design (UFD) is proposed in [1], where the size of the fracture is determined by the proppant mass pumped into the reservoir. A linear relationship is assumed between the well flow rate and the pressure drawdown, whereas the proportionality coefficient is called the Productivity Index (PI). It was found that for any proppant mass pumped into the reservoir, there is a unique fracture geometry corresponding to the maximum PI. All other combinations of lengths and widths of the fracture would result in lower PI values. This is called the physical optimization [1]. In [2], an advanced model describing the fracture width profile was added to the physical optimization. This addition has improved the accuracy of the net pressure and the fracture geometry calculation.

The procedure linking physical and economic optimization for a gas reservoir is offered in [3]. This optimization algorithm can be summarized as follows: for the fixed proppant mass, authors



are looking for the fracture geometry (length and propped width) providing the maximum of PI. Then, by variation of the proppant mass, authors select the fracture giving the maximum of the Net Present Value (NPV). However, it remains unclear, how to determine the fracturing job schedule that would lead to the desired fracturing geometry.

In this article, a different approach is considered: for a given set of fracturing parameters, we use a two-dimensional fracture propagation model for predicting fracture geometry. Then, we couple the geometry with a productivity module to predict the MFHW flow rate. And finally, we use the economic module for computation of NPV. The optimization is performed by the Genetic Algorithm (GA) — Non-dominated Sorting Genetic Algorithm-II (NSGA-II) proposed in [4].

The problem close to our is considered in [5, 6], where authors reduce the multi-objectives optimization problem to single-objective one.

2. The optimization algorithm

Formally, any optimization problem can be stated as: find values of parameters x_1, \dots, x_n to maximize or minimize objective functions

$$(f_1(\mathbf{x}), \dots, f_k(\mathbf{x})), \quad \mathbf{x} = (x_1, \dots, x_n), \quad (1)$$

subject to bounds

$$\mathbf{x} : x_{L,i} \leq x_i \leq x_{U,i}.$$

Here \mathbf{x} represents the vector of free optimization parameters; $x_{L,i}$, $x_{U,i}$ are lower and upper bounds; n is the number of free optimization parameters. We will consider only minimization problem. Maximization of a function $f(\mathbf{x})$ can be achieved by minimizing $-f(\mathbf{x})$.

In the article, the real-coded GA consisting in the direct representation of the actual values of variables is used. The optimization procedure starts with generating the initial population in a random way by using the uniform distribution in each variable. Each set of optimization parameters is named as the individual. The size of the population is N .

It is said that individual $\mathbf{x1}$ is better than $\mathbf{x2}$ for the i -th objective function if $f_i(\mathbf{x1}) < f_i(\mathbf{x2})$. It is said that the individual $\mathbf{x1}$ dominates individual $\mathbf{x2}$ by Pareto, if $\mathbf{x1}$ is not worse than $\mathbf{x2}$ by all objective functions and at least by one criterion is better than $\mathbf{x2}$. On the other hand, if $\mathbf{x1}$ is better than $\mathbf{x2}$ for one criterion and $\mathbf{x2}$ is better than $\mathbf{x1}$ for another one, then it is said that individuals $\mathbf{x1}$ and $\mathbf{x2}$ are non-dominant. The set of non-dominant individuals is called Pareto optimal set or Pareto front.

Since in general case, it is impossible to find \mathbf{x} minimizing all objective functions simultaneously, the solution of multiobjective optimization problem is the Pareto front.

Once the initial population is created the population is sorted on the non-domination criterion. The non-dominant set in the current population is called the first front, whereas individuals in it become the unit rank value. Individuals in the second front being dominated by the individuals of the first front only, are assigned the rank value of 2 and so on. In addition to the rank, a crowding distance [7] is calculated for each individual. The crowding distance is a measure of how close an individual is to its neighbours. Large average crowding distance results in better diversity in the population.

“Parents” are selected from the population by using binary tournament selection on the base of the rank and the crowding distance. An individual is selected if its rank is smaller than the other or if the crowding distance is greater than the other. The selected “parents” generate “children” by the crossover and mutation operators. In this article, we use the arithmetical crossover and the mutation of a polynomial type [8].

After the operations of crossover and mutation, an intermediate population of size $2N$ is created by gathering the “parent” and “child” populations. The joint population is sorted

again based on the rank value and the crowding distance, and the new “parent” population is obtained by keeping N best individuals.

3. Problem definition

The problem of hydraulic fracture optimization can be formulated as follows. For a given reservoir one must select a set of fracturing parameters, such as the viscosity of fracturing fluids, the injection rate of the fluid, the injection time and the proppant concentration, such that a hydraulic fracture geometry (propped width, length and permeability) is obtained to provide the design objectives: maximum of cumulative well production (Q_{tot}) and Net Present Value (NPV) at minimum treatment costs (C_{HF}):

$$Q_{tot} \rightarrow \max, \quad NPV \rightarrow \max, \quad C_{HF} \rightarrow \min. \quad (2)$$

In practice, the most valuable customer information is the proppant mass required for the hydraulic fracturing [1, 3], therefore, we choose

- (i) M_p (proppant mass for single fracturing) and
- (ii) N_f (number of fractures)

as free optimization parameters under the following bound constraints:

- (i) $4000 \text{ [kg]} \leq M_p \leq 20000 \text{ [kg]}$,
- (ii) $3 \leq N_f \leq 10$.

The injection rate q_i , the proppant concentration C_p (C_p is the volume concentration) and the effective viscosity of the fracturing fluid μ_0 are considered to be given. By the proppant concentration we mean an average value for the whole injection process.

To calculate the objective functions, it is necessary to include the hydraulic fracture model for obtaining fracture geometry, the post fracturing production model and the economical model.

3.1. Fracture geometry

The fracture geometry is defined by length, width and permeability of a fracture. The height of fracture is assumed to be equal the height of the reservoir. Concerning the properties of the reservoir, standard assumptions of the linear theory are made and the proppant slurry is considered as Newtonian fluid with an effective viscosity defined by the rheological parameters. In this paper, the Khristianovich – Girtsma – de Klerk (KGD) model with Carter’s equation for fluid leak-off, is used to predict the fracture geometry. In [9] a fast algorithm for calculating the fracture parameters is proposed. In order to apply this algorithm it is necessary to define the injection time t_i and the viscosity of proppant slurry μ_f .

If the injection rate is constant, the time required to pump a given proppant mass with a given concentration of the slurry is calculated as

$$t_i = \frac{M_p}{\rho_p(1 - m_p)q_i C_p}, \quad (3)$$

where ρ_p is the proppant density, and m_p is the proppant pack porosity.

Once the injection time is determined, the total volume of the injected slurry is found: $V_i = q_i \cdot t_i$.

Using the proppant mass and the proppant volume concentration, we obtain the volume of “pure” fluid as

$$V_F = \frac{M_p}{\rho_p(1 - m_p)} \cdot \frac{(1 - C_p)}{C_p} = V_i \cdot (1 - C_p).$$

The viscosity of the proppant slurry is determined by the viscosity of the carrier fluid and the proppant concentration [10]:

$$\mu_f(C_p) = \mu_0 \left(1 - \frac{C_p}{C_{p*}}\right)^{-2.5}, \quad (4)$$

where C_{p*} is the maximum of the proppant concentration. Note, that formula (4) holds at concentrations $C_p \leq 0.5$.

Following [9], we find the fracture parameters: the fracture half-length; the fracture width and the fluid efficiency η . The latter is determined as the ratio of the fracture volume V_{Frac} to the volume of the total injected slurry V_i ($\eta = V_{Frac}/V_i$).

The proppant concentration in the propped fracture is considered to be equal to the maximum value, hence, the propped width of fracture is $w_F = (wC_p)/(\eta C_{p*})$.

3.2. Calculation of the post-fracture production rate

We consider a single-phase fluid flow in a rectangular reservoir Ω with a fractured horizontal well located in the centre of the reservoir. The fractures are assumed to be identical and placed uniformly and symmetrically along the well.

The express assessment of the production rate of the horizontal well with multiple fractures is proposed in [11]. According to [11], the flow rate Q is a sum of inflows of internal Q_{in} and external Q_d fractures:

$$Q_{in} = \frac{2kHL}{b\mu_n(R-l)} \left(p_p - \frac{p_0}{2} \left(\frac{1+2a}{1+a} \right) - \frac{p_z}{2} \left(\frac{1}{1+a} \right) \right), \quad (5)$$

$$Q_d = \frac{4kHl(p_p - p_z)}{b\mu_n(R-l)}, \quad (6)$$

$$Q = Q_{in} + Q_d. \quad (7)$$

Here k is the reservoir permeability, H is the reservoir height, L is the well length, b is the oil volumetric factor, μ_n is the viscosity of oil, R is the drainage radius, l is the fracture half-length, p_p is the reservoir pressure, p_z is the bottomhole pressure and p_0 is some intermediate pressure.

The parameter a is determined as:

$$a = \frac{2(k/k_f)l^2}{wL} (N_f - 1) = \frac{2l(N_f - 1)}{LF_{cd}}. \quad (8)$$

Here w is the fracture width, k_f is the permeability of the propped fracture, N_f is the number of fractures, $F_{cd} = (k_f w)/(kl)$ is the dimensionless fracture conductivity.

The intermediate pressure p_0 is defined as:

$$p_0 = \frac{p_p(1+a) - (0.5 - \bar{b})p_z}{0.5 + \bar{b} + a}, \quad \bar{b} = \frac{4(N_f - 1)^2 l(R-l)}{L^2}. \quad (9)$$

Considering Q as the initial production rate, we assume that the post-fracture production rate declines exponentially [12]:

$$Q_t = Qe^{-\alpha t}, \quad (10)$$

where t is time, Q_t is the production rate in time t , α is an empirical decline rate.

The cumulative production Q_{tot} to the time t is given by: $Q_{tot} = \int Q_t dt = \int Q_t \exp(-\alpha t) dt = (Q - Q_t)/\alpha$. The cumulative yearly production $N_{p,t}$ for the t -year is $N_{p,t} = (Q_{t-1} - Q_t)/\alpha$.

3.3. Economic criteria

The economic criterion NPV, commonly used for estimating the hydraulic fracturing efficiency, is calculated as follows [3, 13]:

$$NPV = \sum_{t=1}^{T_{max}} \frac{\Pi_t - A_t}{(1 + D)^t} - C_{HF}. \quad (11)$$

Here T_{max} is the number of years for which a revenue is calculated, t is the current year, Π_t is the cash inflow at t -th year, A_t is current expenses, D is the discount rate, C_{HF} is the cost of hydraulic fracturing with N_f fractures.

The cash inflow Π_t in the t -th year is obtained as the product of the cumulative oil production $N_{p,t}$ and an average oil price. We propose to estimate the fracturing cost C_{HF} as follows:

$$C_{HF} = N_f(FC + Pr_p \cdot M_p + V_F \cdot Pr_F) + AC, \quad (12)$$

where M_p is the proppant mass [kg], Pr_p is the proppant price [\$/kg], Pr_F is the fluid price [\$/m³], V_F is the volume of the fluid [m³], N_f is the number of fractures, FC is the cost of equipment, e.g. pumping [\$], AC is the fixed and miscellaneous costs [\$].

Current expenses A_t include operational costs, taxes, costs of oil treatment and transportation. The treatment and transportation costs are proportional to the volume of the oil produced and can be included into the oil price, the taxes are proportional to the revenue of the t -th year, operational costs do not depend on the volume of the oil produced, so we consider them to be constant.

4. Examples and results

A horizontal well located in the centre of the oil reservoir is used to illustrate an application of the proposed approach. Multi-objective optimization problem for three objective functions

$$-Q_{tot} \rightarrow \min, -NPV \rightarrow \min, C_{HF} \rightarrow \min$$

is solved. We use a population of size 100 and run NSGA II for the 100 generations. The parameters characterizing the reservoir and wellbore properties are presented in Table 1, which also contains fracture mechanics data for obtaining the fracture geometry. The fracture permeability is assumed to be determined only by the proppant pack. The economic data are taken as follows : the oil price is 80 [\$/m³]; the carrier fluid price is 80 [\$/m³]; the proppant price is 2.2 [\$/kg]; the discount rate D is 0.2; taxes are 0.24; fixed costs are $AC=10000$ [\$]; equipment costs are $FC = 1000$ [\$]. The oil price includes the oil treatment and transportation costs. We calculate the NPV and the cumulative production for 3 and 5 years and for various reservoir permeability $k = 1$ mD and $k = 10$ mD. It is clear that the NPV depends on the cumulative production but the dependence is non-linear.

Figure 1 shows Pareto front of 100-th generation for reservoir permeability $k = 1$ mD. It represents the dependencies of NPV and cumulative production on fracturing costs.

As one can see, the maximum of NPV does not coincide with the maximum of cumulative production. Figure 2 shows parameters of fracturing design providing the NPV values close to the maximum and the total inflow values close to the maximum. In order to receive high NPV, the number of fractures from 6 to 9 and lengths up to 100m should be taken. The larger the number of fractures one takes, the smaller should be their length. On the contrary, to obtain the maximum of the cumulative production, one should take the fractures with lengths from 120 to 140 m independently on the number of fractures. Pareto fronts for various T_{max} and reservoir permeability $k = 1$ mD are presented in Figure 3. It can be seen that the NPV values do not differ much, but the total inflows do.

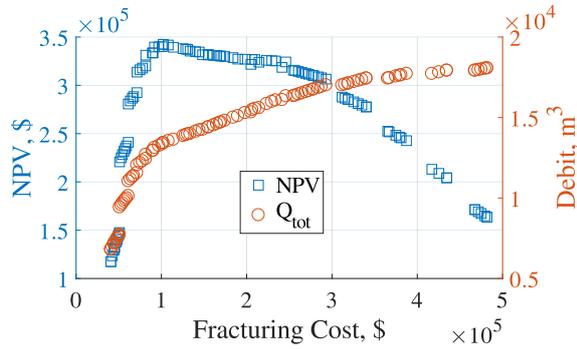


Figure 1. Pareto front for reservoir permeability $k = 1$ mD.

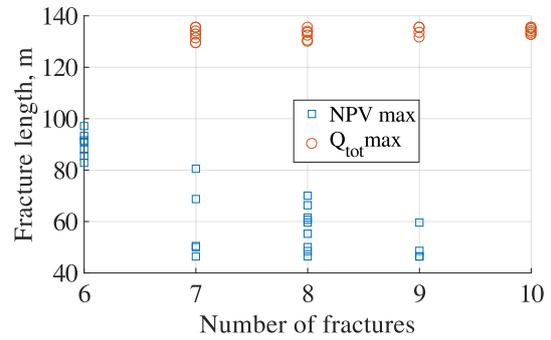
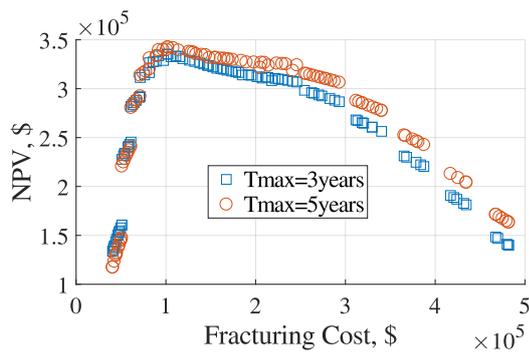


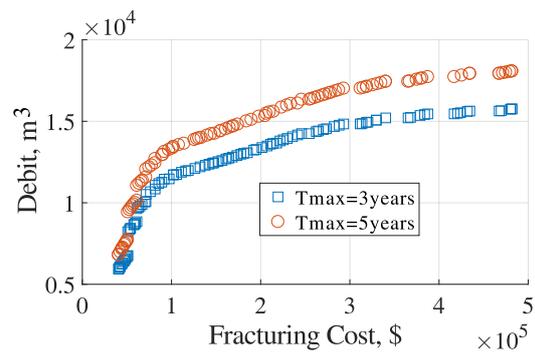
Figure 2. Parameters plane.

Table 1. Reservoir and well parameters.

Parameter	Value	Parameter	Value
reservoir height	$H = 20$ [m]	oil volumetric factor	$b = 1.189$
oil viscosity	$\mu_n = 2 \cdot 10^{-3}$ [Pa s]	drainage radius	$R = 400$ [m]
fracture permeability	$k_f = 30$ [D]	bottomhole pressure	$p_z = 5$ [MPa]
well length	$L = 600$ [m]	reservoir pressure	$p_p = 15$ [MPa]
annual decline rate	$\alpha_a = 0.587$ [year ⁻¹]	Poisson ratio	$\nu = 0.25$
leak-off coefficient	$C_l = 10^{-5}$ [m/s ^{0.5}]	Young's modulus	$E = 3.5 \cdot 10^{10}$ [Pa]
fracture toughness	$K_I = 3 \cdot 10^6$ [Pa m ^{0.5}]	proppant density	$\rho_p = 2645$ [kg/m ³]
proppant concentration	$C_p = 0.3$	injection rate	$q_i = 1$ [m ³ /s]
maximum proppant concentration	$C_{p*} = 0.65$	fracturing fluid viscosity	$\mu_0 = 0.1$ [Pa s]



(a)



(b)

Figure 3. Pareto fronts NPV (a) and total inflow (b) for various values of T_{max} and $k = 1$ mD.

Pareto front for the reservoir with higher permeability $k = 10$ mD is shown in Figure 4. For this case, the dependence of NPV on cumulative production is almost linear. It means that the fracture geometry providing the maximum of NPV at the same time provides the maximum of cumulative production. Pareto fronts for various T_{max} are presented in Figure 5.

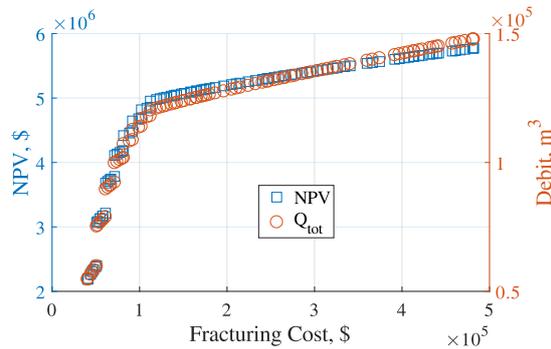


Figure 4. Pareto front for the reservoir permeability $k = 10\text{mD}$.

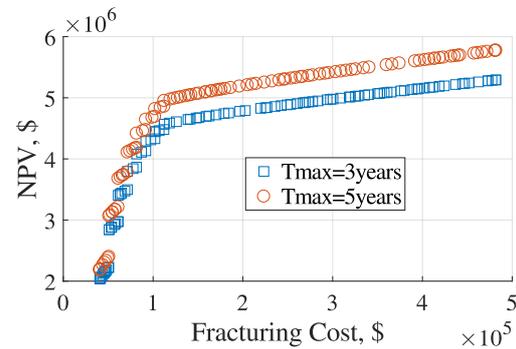


Figure 5. Pareto fronts for various values T_{max} and $k = 10\text{mD}$.

5. Conclusion

The approach to determine an optimization design of a multiply-fractured horizontal well is proposed. The numerical tests demonstrate reasonable results on a synthetic set of variables close to real. Although the results are qualitative, they are useful for hydraulic fracturing design and for choosing wells best suited for the fracturing.

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