

# Spatiotemporal variability of runoff using fractal dimension in the Weihe River Basin

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**Abstract.** The paper studies the characteristics of runoff which can provide the basis for rational development and utilization of water resources in the Weihe River Basin (WRB). In this paper, sixteen typical hydrological stations in the WRB are selected, including six stations in the mainstream, eight in the south bank and two in the north bank of WRB. The ArcGIS and the Hawth's Analysis Tools are used to calculate the fractal dimensions of the hydrological stations. The variations of annual and monthly runoff for stations are investigated using the fractal dimension during 1956-2015 in the WRB. The differences of fractal dimensions between the mainstream and the main tributaries located at the south bank and the north bank are compared. The results show that the runoff fractal dimension varies for different stations and months. In the WRB, the maximum monthly runoff fractal dimensions of stations are mainly in the flood season (June to October) and the minimum are mainly in the dry season (December to March). The fractal dimensions of stations in the mainstream are smaller than those in the south bank, and the fractal dimensions in the north bank are smaller than those in the south bank of the WRB. Especially, in the mainstream of the WRB, the fractal dimensions of stations in the midstream are the largest, followed by downstream and upstream. In the north bank of the WRB, the fractal dimension of Zhangjiashan station in the Jinghe River Basin is larger than Zhuangtou station in the Beiluohe River.

## 1. Introduction

As the "Mother River" of local people, Weihe River Basin (WRB) plays an important strategic significance of the development in Shaanxi province, China. With the climate change and socio-economic development, the water resources in the WRB have declined and the water demand of human society has increased since the 1980s. Therefore, the water resources in the WRB have become a common concern of the public. This paper investigates the spatial and temporal variability of the runoff to provide a basis for the rational development and utilization of water resources in the WRB.

In recent years, fractal theory has been widely used in the field of hydrology research [1-7]. Previous researchers used the fractal theory to analyze the runoff time series. For example, Fang *et al* presented two new approaches to dividing flood sub-seasons by using the fractal theory [8]. Wang *et al* studied the fractal dimension of runoff series based on continuous wavelet transform [9]. Xu *et al* analyzed long-term trends and fractal features of annual runoff in the Tarim River [10]. At present, the application of fractal theory in the analysis of runoff characteristics is still in the development stage. Most of researches study the characteristics of annual runoff changes. However, few studies were



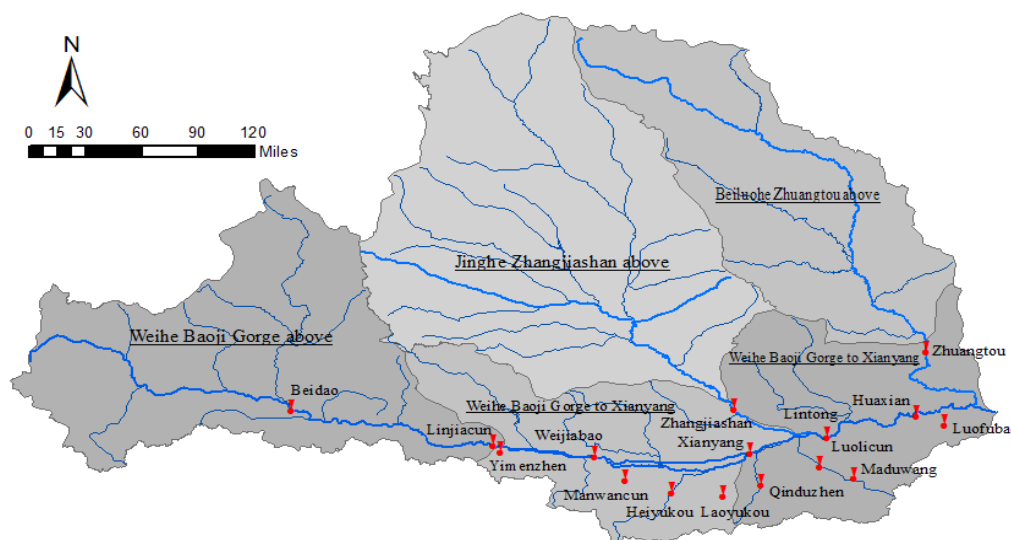
found to analyze the monthly runoff changes and the runoff time series are relatively short. This paper focuses on the monthly runoff in the WRB for the period of 1956-2015 using the fractal theory.

The fractal dimension is used to quantitatively describe the complexity of the runoff process. This paper studies the variation characteristics of runoff in the WRB using the fractal theory. The monthly runoff fractal dimensions of each station in the WRB are analyzed, and the relationships between the mainstream, the south bank, and the north bank are comparatively analyzed, to provide a basis for the rational development and utilization of water resources in the WRB.

## 2. Materials and methodology

### 2.1. Study area

The WRB is the largest tributary of the Yellow River, it lies between 103.5 E–110.5 E and 33.5 N–37.5 N. It originated from Niaoshu Mountain in Gansu province, mainly flows through this Tianshui city in Gansu province, Baoji, Xianyang, Xi'an, Weinan cities in Shaanxi province, to Tongguan County in Weinan city sinks into the Yellow River. The Loess Plateau and Qinling Mountains are respectively located in the northern and southern banks of the WRB. It has a total length of 818 km and a total catchment area of 134766 km<sup>2</sup>, of which Shaanxi Province accounted for 49.8% of the total basin area. The annual average runoff volume of WRB is 7.57 billion cubic meters and that in Shaanxi province is 5.38 billion cubic meters. In the WRB, above Baoji Gorge is considered as the upstream, Baoji Gorge to Xianyang is the midstream, Xianyang to Tongguan is the downstream. The geographical locations of the WRB and typical hydrological stations are shown in figure 1.



**Figure 1.** Geographical locations of WRB and hydrological stations.

### 2.2. Runoff data

The runoff data were obtained from Hydrological Data of the Yellow River Basin, Shaanxi Province River and Reservoir Administration and Shaanxi Water Resource Bulletin. The runoff data used in this paper has good periodicity, representativeness and consistency and can be used for the corresponding calculation and regularity research.

Taking into account the representative of hydrological stations and the reliability and completeness of the data, the runoff of 16 typical hydrological stations for the period of 1956-2015 are used in this paper. Six stations located in the mainstream. Ten stations located in the nine typical tributaries. Considering the existence of data missing at some hydrological stations in this paper, the correlation analysis (correlation coefficient greater than 0.95), area ratio and runoff deep correction method were

used to interpolate the missing data. The basic information of the above hydrological station is shown in table 1.

**Table 1.** The information of hydrological station in the WRB.

Code	River	Station	Longitude	Latitude	Distance to the mouth (km)	Catchment area (km <sup>2</sup> )
41100550	mainstream	Beidao	105.9	34.57	564	24871
41100600	mainstream	Linjiacun	107.05	34.38	388	30661
41100900	mainstream	Weijiabao	107.75	34.30	321	37012
41101100	mainstream	Xianyang	108.7	34.32	211	46827
41101300	mainstream	Lintong	109.2	34.43	157	97299
41101600	mainstream	Huaxian	109.77	34.58	73	106498
41201100	Jinghe	Zhangjiashan	108.36	34.38	58	43216
41300650	Beiluohe	Zhuangtou	109.5	35.02	130	25645
41104300	Qingjianghe	Yimenzhen	107.06	34.2	5.4	219
41106200	Tangyuhe	Manwancun	107.54	34.08	18	122
41107200	Heihe	Heiyukou	108.12	34.03	35	1481
41107300	Laohe	Laoyukou	108.32	34.01	40	347
41107500	Fenghe	Qinduzhen	108.46	34.06	36	566
41108300	Bahe	Maduwang	109.22	34.09	55	754
41108500	Bahe	Luolicun	109.09	34.14	30	1601
41110100	Luofuhe	Luofubao	109.57	34.48	16	122

### 2.3. Methodology

This paper uses the ArcGIS and the Hawth's Analysis Tools to calculate the fractal dimensions of the stations. Hawth's Analysis Tools is an extension toolbox of ArcGIS. The Line Metrics tool enables the calculation of the fractal dimension of runoff process lines. Compared with the box counting method, this method can simplify the calculation process of line fractal dimension and save manpower and time greatly. The fractal dimension ( $D$ ) is calculated as:

$$D = \frac{\lg n}{\lg n + \lg(d/L)} \quad (1)$$

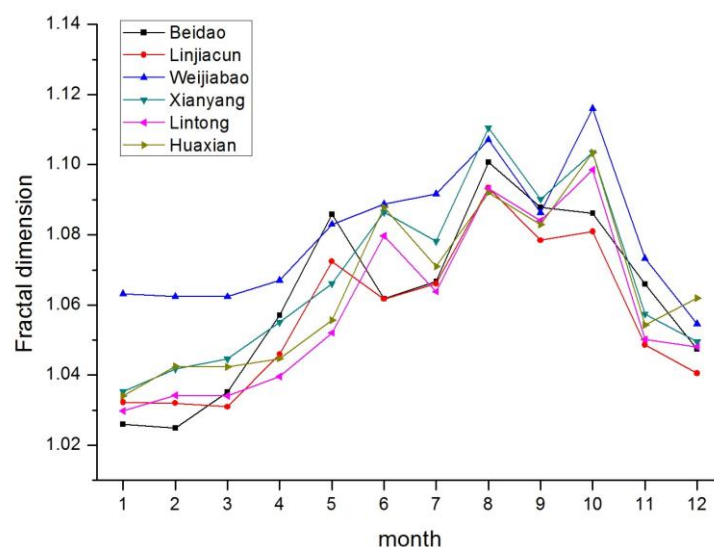
Where  $n$  is the number of line segments that make up the line,  $d$  is the distance between the start and end points of the line, and  $L$  is the total length of the line.

The size of the fractal dimension indicates the complexity of the runoff process line changes. The larger the fractal dimension, the more complicated the process changes.

## 3. Results and analysis

### 3.1. Analysis of monthly runoff fractal dimension in the mainstream of the WRB

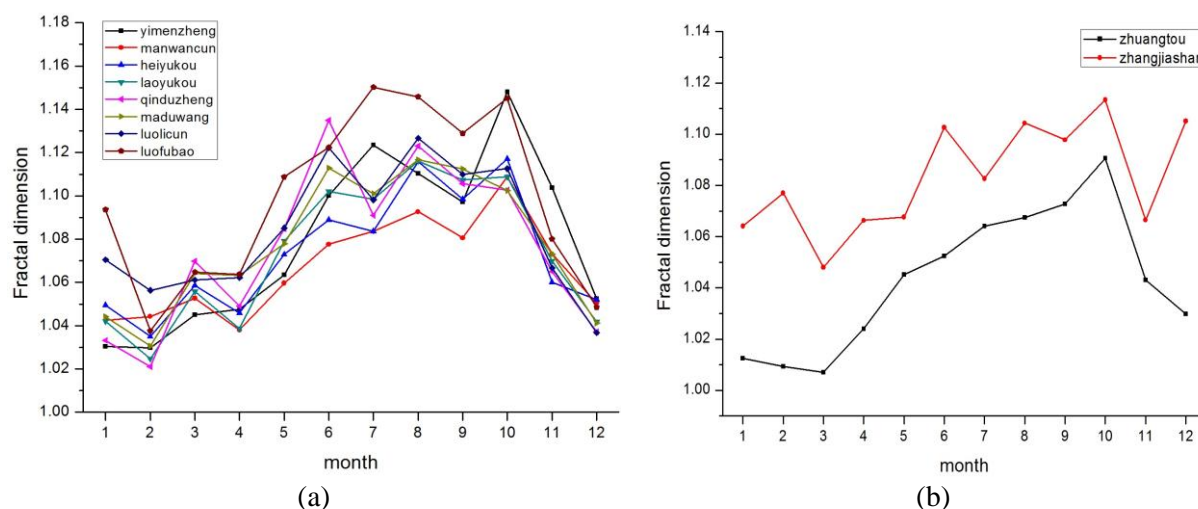
Six stations in the mainstream of the WRB are divided into the upstream, midstream and downstream, Beidao and Linjiacun stations in the upstream, Weijiabao and Xianyang stations in the midstream, Lintong and Huaxian stations in the downstream, according to the principle of watershed division in figure 1. Figure 2 shows that the variation of monthly runoff fractal dimensions for six stations are the same from July to November. The variation trend of the monthly runoff fractal dimensions for stations of the same basin in the upstream, midstream, and downstream of the WRB are the same. This is because stations in the same basin are close and the climate, topography, geology and soil conditions are similar. On the whole, in the mainstream of WRB, figure 2 shows that the monthly runoff fractal dimensions of stations in the upstream are smaller than those in the downstream, and the monthly runoff fractal dimensions of stations in the midstream are the largest.



**Figure 2.** Monthly runoff fractal dimension of the mainstream in the WRB.

### 3.2. Analysis of monthly runoff fractal dimension in the south bank of the WRB

Eight hydrological stations of seven typical tributaries in the south bank of the WRB were selected for analysis. Figure 3(a) shows that the variation of monthly runoff fractal dimensions of stations in the south bank of the WRB. The variation of monthly runoff fractal dimensions are the same from August to November. The variation trend of monthly runoff fractal dimensions for stations except Yimenzhen and Luofubao are the same in the south bank of the WRB. The variation of the monthly runoff fractal dimensions of the Yimen and Luofubao stations are the similar. This is mainly because Yimenzhen is the import station and Luofubao is the export station in the south bank of the WRB. Other stations in the south bank of the WRB are close and the conditions of climate, topography, geology, soil and forest coverage are similar (figure 1). In the south bank of the WRB, the maximum fractal dimension is Luofubao station and the minimum fractal dimension is Manwancun station.



**Figure 3.** Monthly runoff fractal dimensions of the stations in the WRB: a) south and b) north bank.

### 3.3. Analysis of monthly runoff fractal dimension in the north bank of the WRB

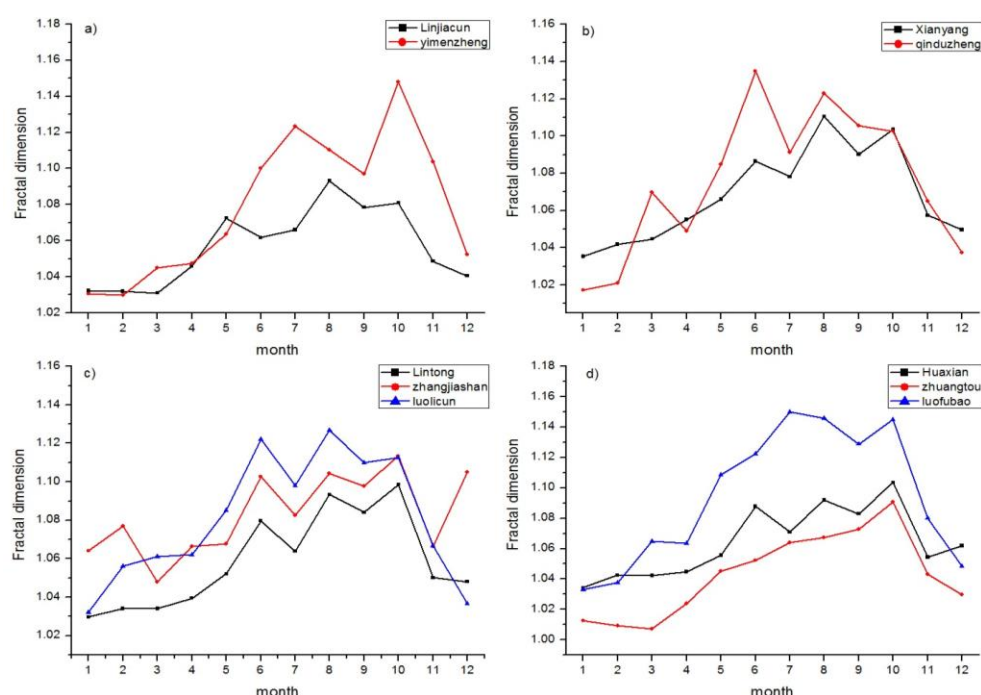
Figure 3(b) shows that the variation of monthly runoff fractal dimensions for two stations named Zhangjiashan and Zhuangtuo located in the north bank of the WRB are the same from September to

November. The results show that the monthly runoff fractal dimensions have increasing trends before October. In the north bank of the WRB, the fractal dimension of Zhangjiashan station is larger than that of Zhuantou station. The runoff variation of Zhangjiashan station is more complicated.

### 3.4. Comparative analysis of runoff fractal dimension in the WRB

Four stations in the mainstream, four stations in the south bank and two stations in the north bank of the WRB are divided into the upstream, midstream and downstream, as shown in figure 1.

Figure 4 shows that the fractal dimensions of stations in the south bank are larger than those in the mainstream of the WRB. The complexity of the runoff changes process for stations in the south bank are larger than those in the mainstream of the WRB. This is because the south bank has the simple structure, small basin area and poor regulation ability, but the mainstream has complex structure, large basin area, and strong regulation ability.



**Figure 4.** Comparison of monthly runoff fractal dimension in the WRB.

In the tributaries of the WRB, the fractal dimensions of stations in the south bank are larger than those in the north bank. In the downstream of WRB, the Lintong station of mainstream is smaller than the Zhangjiashan station in Jinghe River of the north bank while the Huaxian station of mainstream is larger than the Zhuangtuo station in Beiluo River of the north bank.

**Table 2.** Analysis of runoff fractal dimension in the WRB.

River basin	Fractal dimension range	Corresponding to the maximum		Corresponding to the minimum	
		Station	month	Station	month
Weihe	1.007~ 1.150	Luofubao	7	Zhuangtuo	3
Mainstream	1.025~ 1.116	Weijiabao	10	Beidao	2
The south bank	1.017~ 1.150	Luofubao	7	Qinduzhen	1
The north bank	1.007~ 1.113	Zhangjiashan	10	Zhuangtuo	3

Table 2 shows that the maximum monthly runoff fractal dimensions are concentrated in the flood season (June to October), and the minimum monthly runoff fractal dimensions are concentrated in the dry season (December to March) of the basin. It is consistent with the month distribution of the maximum and the minimum monthly runoff fractal dimensions of stations in the WRB. This is because the runoff in flood season is large and the runoff is affected by the upstream water and rainfall, so that the process of the change is complex, however, in the dry season, the runoff is small, the rainfall is small and there is no obvious change in the runoff.

The variations of the monthly runoff fractal dimensions for stations in the WRB except Luofubao, Yimenzheng and Zhuangtuo have similar trends from July to November. The difference of fractal dimensions for stations in the WRB are relatively small.

#### 4. Conclusion and discussion

- In the WRB, the largest monthly runoff fractal dimensions are concentrated in the flood season (June to October), and the smallest monthly runoff fractal dimensions are concentrated in the dry season (December to March). The maximum monthly runoff fractal dimension is detected in the Luofubao station and the minimum monthly runoff fractal dimension is found in the Zhuangtuo station.
- In the mainstream of the WRB, the runoff fractal dimensions of stations in the midstream are the largest, followed by downstream and upstream. In the north bank of the WRB, the runoff fractal dimension of Zhangjiashan station is larger than that of Zhuangtuo station. In case of downstream, the runoff fractal dimensions of stations in the south bank are larger than those in the north bank. In the WRB, the runoff fractal dimensions for stations in the south bank are larger than those in the mainstream.
- Only two stations of Jinghe River and Beiluohe River in the north bank of the WRB were analyzed in this paper. Our further study will focus on the monthly runoff fractal dimensions of other watersheds in the north bank of the WRB and their relationships with the mainstream and the south bank of the WRB.

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