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Numerical method for bed load discharge estimation

X H Wang[1], H Y Sui[1,2], W Cheng[1], L Y Gong[1] and L Yang[1]

[1]State Key Laboratory of Eco-hydraulics in Northwest Arid Region of China, Xi'an University of Technology, NO. 5, South Jinhua Road, Xi'an, Shaanxi, China

E-mail: 1419733009@qq.com

Abstract. This paper put forward a new method to estimate the bed load discharge through numerical model of bed load. Based on the concept of none-equilibrium bed load discharge, using numerical model and reverse problem solving approach through numerical simulation, this paper estimated the bed load transportation rate of the riverbed. A numerical model that can describe none-uniform and none equilibrium bed load transportation was developed in this paper. The model can simulate the hydraulic factors, Manning coefficient, bed load transportation rate, and the change of bed sediment graduation with the change of the time. Through the solution of the flow continuous equation, flow momentum equation, resistance equation, none-equilibrium bed load transportation equation and the sediment mass continuous equation, the deformation of the river bed is obtained. This model is applied to rivers with the following features: the gravel-cobble bed with sand, the little effect of the suspended sediment on the deformation of the river bed, the depth and width of none-uniform flow along the river. The model was verified by a set of flume experiment of none-equilibrium bed load discharge. The results indicate that the observed bed load transportation rate is consistent with the computational one. The results of the computing bed sediment gradation reflect the phenomenon that the scour and silting cause the larger of the bed sediment correctly. This model was applied to the estimation of bed load discharge of Changma Reservoir near the dam site, and estimate that the ratio of bed loads to suspended load 14%, which is very close to 16% --the estimation of the physical model experiment.

1. Introduction

The large gradient of river course in mountainous area often leads to great bed load discharge, which is a significant restrict factor of the planning and designing of conservancy projects. For reliable bed load discharge data are hardly possible through hydrological survey, many methods (devised by Graf [1-5] of bed load discharge estimation are developed through theoretical, laboratory-based and field-based approaches. One of the most widely used methods in project design can be stated as following procedures: A bed load formula is selected for a given river at first and the variables relevant to this formula are determined for each time step in three typical years (the raining year, the average year and the dry year). Each typical year bed load is then obtained by integrating each time step bed load in corresponding year. Finally, the annually averaged bed load is determined by analysis of means. Examining aforementioned methods, two major deficiencies can be identified: Firstly, the selection of a bed load formula is difficult. A number of bed load rate formulas, which are often based on different theories and data, are presented in various scientific literatures. If a specific river is considered, how to choose a suitable formula is an incompletely reversed problem. Secondly, the

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validity of bed load formulas is difficult to evaluate. There is often great difference between the average bed load discharges estimated from different formulas for a selected river, furthermore, there is no effective criterion up to now to assess which formula is more accurate. To solve these problems, a new method of the estimation for bed load discharge estimation was developed in this paper.

2. Establishment of the numerical model

Bed load is the main factor, determining the scour and silting of river course in mountainous area. When the rate of bed load transportation is lower than the equilibrium rate, the scour of the river bed occurs and when the rate of bed load transportation is higher than the equilibrium one, the silting occurs. The process of the scour and silting can be simulated by a numerical model of none equilibrium bed load discharge.

2.1. Basic control equation

The basic equations which control the water flow in mountainous river, can be described as following equation:

$$\frac{\partial Q}{\partial x}=0 \tag{1}$$

$$\frac{\partial\left(\frac{Q^2}{A}\right)}{\partial x}+gA\left(\frac{\partial Z}{\partial x}\right)+gAJ_e=0 \tag{2}$$

Where, Q is the flow discharge; A is the discharge area; Z is the elevation of water surface; g is the gravitational acceleration; x is the flowage and Je is energy gradient. The basic equation which describe the bed load movement are the sediment continuous equation and the none equilibrium bed load transportation equations by Bell and Sutherland [6] and Cheng [7]. They are shown as follows:

$$\frac{\gamma'\partial A_s}{\partial t}+\frac{\partial G}{\partial x}=0 \tag{3}$$

$$\frac{\partial G}{\partial x}=-K_G\left(G-G_*\right) \tag{4}$$

Where, As is the area of scour and silting, γ' is the dry density of sediment, G .G_* are the actual and equilibrium rate of bed load, K_G is the spatial log time coefficient; and x, t are the flowage and time. Supposing K_G is independent of flowage, the integral form of equation (4) is as follows:

$$G=\left\{1-\left[1-\left(\frac{G_0}{G_{0*}}\right)\times Exp\left(-K_G\left(x-x_0\right)\right)\right]\right\}G_* \tag{5}$$

Where, suffix "0" represents the upstream section. The value of KG differs by the spatial step, the scour and silting character and other factors, which were discussed by Bell R G and Sutherland [6]. To simplify the calculation, the initial value of K_G (x- x_0) in this paper is chosen as 0.5 and then adjusts it to the computational results. Equations (1)-(4) constitute the basic equations of river sediment movement. These equations are not enough for calculation and some complementary equations are needed.

2.2. Complementary equations

Using following equation by Qian and Wan [8], energy gradient can be related with Manning coefficient:

$$n = \frac{d_{50}^{1/6}}{A_n} \tag{6}$$

Where, d_{50} is the median diameter, An is an empirical coefficient (An =19-24), n is Manning coefficient ($J_e = U^2 \times n^2 \times R^{-4/3}$, where U is flow velocity and R is hydraulic radius). What should be pointed out is that in the equation (6) the sediment slope, which is very important to flow resistance, is not considered. But its form is simple and the coefficients of it can be changed to reflect other resistance factors, therefor the equation is used in this paper.

This paper chooses several equations frequently used in project for selection. These equations include Einstein-Equation, Alan-Equation, and so on. All these equations can be found from relative literatures [1,8].

2.3. Meyer-peter equation, stable solution conditions of the equation

The stable solution conditions of the flow sediment equations (1)-(4) include: the initial section form of river course and the composition of bed materials, the flow charge and the bed load transportation rate at upstream cross section, and the process of water elevation at downstream cross section. For the problems studied in this paper, water elevation of downstream boundary is usually unknown, and upstream bed load transportation rate is just the solution expected.

To an annually equilibrium river (neither gradation nor degradation occurs), the downstream water depth of studied river section can be considered approximately equal to the depth of uniform stream with movable bed. The solution of upstream bed load rate actually forms a reverse problem, which can only be resolved by means of trial-and-error method. The solution procedures are as follows: A bed load formula is randomly selected at first. Suppose the actual bed load rate of the upstream boundary is equal to the calculated one by this formula, the long-term deformation of the studied river section can be simulated by the numerical model presented in this paper. If the computing results of the accumulative quantity of the degradation or gradation are consistent with the measured ones which are approximately equal to zero for an annually equilibrium river reach, the process of upstream bed load transportation rate calculated by the model is the actual ones. And if the computational results are not consistent with the actual data, another bed load transportation rate equation is selected and the preceding steps are repeated until the computational results are consistent with the actual data.

2.4. Calculation methods

When the basic equations mentioned above are solved in mathematical model, the process of coming water from upstream can be generalized and simplified the gradient steady flow, and the river course can be classified into several time and length sections, water flow in each section nearly approach the steady flow. The non-coupling finite difference scheme is adopted in the computational method. Firstly, equations (1) and (4) are used to deduce the water surface profile thereby to obtain hydraulic elements in each section. Secondly, equation (4) is used to obtain the bed load transportation rate and graduation in each section from upstream to downstream. Thirdly the equation (3) is used to deduce the variation in sluicing and silting area of different sediment graduation. Lastly, Adjust the sediment graduation and section form, then return to hydraulic calculation for the next time section. Thus far, hydraulic calculation and sediment calculation can be carried out alternatively until the calculations in all time sections are completed.

3. Model verification

The numerical model developed in this paper was tested by the experimental data of bed load flume [9]. The length of the flume is 33 m, the breadth is 0.5 m, the height is 0.55 m, the bed gradient ratio is 517%, the inside wall is cement washed, the sand thickness of the flume bed is 0.06 m, the gradation curve is shown in figure 2, the median radius is 3.85 mm. Through experiment, the relation between unit discharge and unit sediment transportation rate can be obtained:

$$G_{bm} = 1.425 \times 10^5 \times q_m^{\ 2.93} \tag{7}$$

Where, G_{bm} is the unit sediment transportation rate and the dimensional unit is g/s.m, q_m is unit discharge, and the dimensional unit is 10^{-3} m^3/s.m. To know the effectiveness of equation (7) when it was applied to bed load transportation calculation during flood season, the process of silt-discharge shown as diagram I was put into the flume and the observed average bed load transportation rate near the outlet is 15.7 g/s. Based on the numerical model of bed load transportation developed in this paper numerical calculation about this experiment was carried out. In the calculation, the value of x---distance between two sections, supposed to be 2m, and the division of time section was consistent with figure 1. And the equation (7) was adopted as the formula of the bed load transportation rate. The relative coefficient value was: Kg= 0.23, An= 23. The computational result of the bed load transportation rate near the outlet of the flume is 15.8 g/s, which is very close to the actual observed ones. Figure 2 represents the comparison of the computational sediment gradation at the beginning and ending of the time section. It can be seen from figure 2 that radius of sediment at the ending of time sections is larger than that of the beginning, which reflect the phenomenon that the scour and the silting of the river bed lead to the larger sediment radius of the bed.

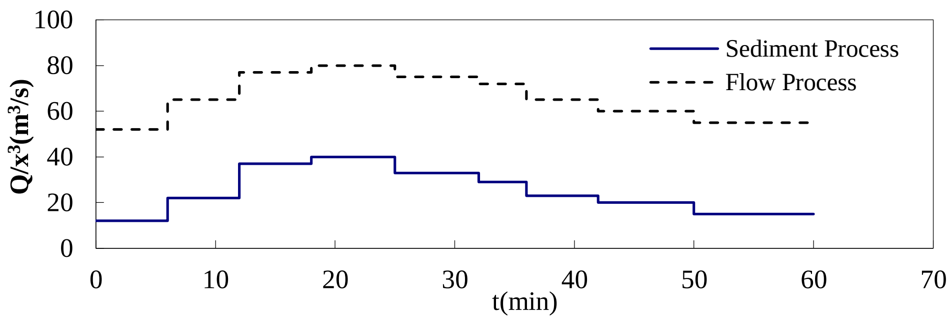

Figure 1. The flow and sediment process of the flume experiment.

Using the constant none-equilibrium model of bed load transportation developed in this paper, the study of the coming bed load discharge about Changma Reservoir was conducted. Changma Reservoir is located in Yumen municipality of Gansu province. The annual average flow near the dam site is 32.7 m^2/s; the sediment concentration is 3.49 kg/m^3. The median radius of the suspended load is 0.027 mm. The riverbed is composed of coarse sand and gravel, the median radius of which is 19.3 mm, and the maximum radius up to 80-150 mm. The average gradient ratio near the dam site is 0.68%.

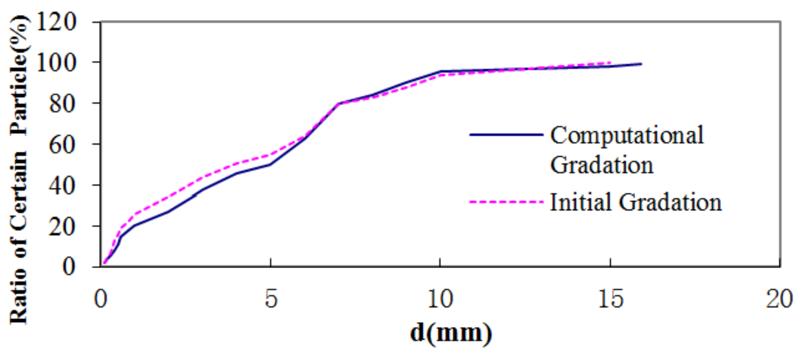

Figure 2. Comparison of the initial and computational gradation at the end of time section.

To estimate the bed load discharge flowing into the Changma Reservoir by numerical model, the river course about 16.2 km long near the dam site was studied and this course was divided into 31 sub-course. The computing time section series was from year 1954 to 1996, totally 43 years. The division of the computing time section was: During the flood season (from July to August), take each day as one time section, and in none-flood season, take 10 day as one computing time section. The intake flow boundary condition of the upstream was the daily averaged flow course of 43 years near the dam site. T he outlet of the downstream has not any hydrological data .According to the analysis of field exploration and observed data comparison, the longitudinal section of Changma river is stable, and the scour and silting of the river course is amulet equilibrium. Therefore, the uniform flow depth of the downstream outlet was taken as the boundary control condition. Because the sediment composition of the riverbed in the calculation river course is similar, the initial graduation of each section was taken the same value during the calculation. The relative coefficient value in the model was Kg = 0.01; An = 19. On condition that the initial data and coefficient value listed above are not change, seven different bed load transportation rate equation was selected to calculate the 43-year-long series bed load scour and silting, and the results were shown in table 1. It can be seen from table 1. That if the Einstein equation is used, the total quantity of the scour and silting is the minimum, and the according annually averaged bed load discharge is $5.11*10^8$ kg. According to these data, the ratio between annually bed load discharge and the suspended load discharge is 14%. It can be deduce from the model experiment conservancy project about the area of the Changma dam site [10] that the ratio between bed load and suspended load discharge is about 16%. The computational results is very close to that of the model experiment, which illustrate the feasible of the numerical method of the bed load discharge estimation in river course presented in this paper.

Table 1. The computational results of different bed load transportation equation.

Bed Load Transportation Equation	Annually Averaged Bed Load Discharge (1×10^6 t)	Total Quantity of Scour and Silting (1×10^6 t)	The Ratio of the Bed Load and Suspended load (%)
Uniform Einstein-Equation	51.1	12.6	14.2
None-uniform Einstein-Equation	14.1	-203.0	3.9
Mann-Peter Equation	55.5	115.0	15.5
Alan-Equation	53.9	81.0	15.0
Alex-White Equation	35.2	-94.3	9.8
Angellon-Equation	65.2	557.0	18.2
Samoph-Equation	22.6	-171.0	6.3

4. Conclusions

- This paper put forward a new method to estimate the bed load discharge through numerical model of bed load. Based on the concept of none-equilibrium bed load discharge, using numerical model and reverse problem solving approach through numerical simulation, this paper estimated the bed load transportation rate of the riverbed.
- A numerical model that can describe none-uniform and none equilibrium bed load transportation was developed in this paper. The model can simulate the hydraulic factors, Manning coefficient, bed load transportation rate, and the change of bed sediment graduation with the change of the time. Through the solution of the flow continuous equation, flow momentum equation, resistance equation, none-equilibrium bed load transportation equation and the sediment mass continuous equation, the deformation of the river bed is obtained. This model is applied to rivers with the following features: the gravel-cobble bed with sand, the

little effect of the suspended sediment on the deformation of the river bed, the depth and width of none-uniform flow along the river.

- The model was verified by a set of flume experiment of none-equilibrium bed load discharge. The results indicate that the observed bed load transportation rate is consistent with the computational one. The results of the computing bed sediment gradation reflect the phenomenon that the scour and silting cause the larger of the bed sediment correctly.
- This model was applied to the estimation of bed load discharge of Changma Reservoir near the dam site, and estimate that the ratio of bed load to suspended load id 14%, which is very close to 16% --the estimation of the physical model experiment.
- The model was just testified by flume experiment data, and the test is not carried out systematically, so, further test by actual data of natural river is expected.

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