Optimal combination design method of two hydrologic variables based on risk analysis

S P Gao[1], G Zhao[1] and X Yang[1,2]

[1]Hydraulic Research Institute of Jiangsu, 97# Nanhu road, Nanjing, China

E-mail: ydaxue@163.com

Abstract. The joint occurrence of hydrological variables may cause structural failure for a civil hydraulic engineering. However, the large values of these variables are not always occurring together. The assumption of two extreme hydrology events occurring simultaneously is unreasonable and uneconomic in the traditional engineering design method. For better choice of combined variables in design, the combined effect should be considered. So, based on conditional risk probability, a new optimal combination design method of two hydrologic variables was proposed. A case study from Shenzhen using the rainfall and the wind data showed that there was a low joint occurrence probability of high rainfall and high wind speed. For a given design risk probability of 1%, the 24 h rainfall with a design risk threshold of 457.7 mm and a wind speed of 27.4 m/s are the best combination, or the wind speed with a design risk threshold of 30.99 m/s and a 24 h rainfall of 396.5 mm are the best combination. The two best combination conditions should be met simultaneously for effectively improving safety, credibility and economy of a civil hydraulic structure design.

1. Introduction

For civil hydraulic engineering, most risks are caused by joint hydrological variables (e.g. flood and tide, wind wave and tide, rainfall and tide), but rarely single variable. If one considered them as independent variables, the risk probability of failure of structure can be calculated easily when one variable exceeds its own initial design value, but this would be an incorrect assumption without considering the combined effects of both. Therefore, combined occurrence probability of hydraulic variables is of interest in engineering design.

Many studies have focused on joint probability analysis of the relevant variables. To give the probabilities of both high and low extreme sea levels, Pugh and Vassie [1] proposed a joint probability method (*JPM*) by recombining the probability distributions of the tide and surge components of sea levels. Vassie *et al* [2], Pirazzoli and Tomasin [3] revised *JPM* to overcome its some deficiencies. Yue [4] obtained the joint distributions and the joint return periods of storm peaks and amounts which are mutually correlated based on the bivariate normal distribution. Erdem and Shi [5] analyzed the joint distributions of wind speed and wind direction by constructing different bivariate statistical models.

Copulas as an alternative to the commonly used can be used for joint frequency analysis. Salvatori and Michele [6] provided a general theoretical framework of copulas for studying the return periods of hydrological events. Lian *et al* [7] analyzed the joint probability of rainfall and tidal level by the copula function. They found the flood probability would outnumber the design probability, if the design standard only depended on the return period of rainfall. Yang *et al* [8] used different copulas to study the joint distributions of wind speed and significant wave height in Bohai Bay, China. Candela *et al* [9]

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applied copulas to analyze the correlation between the rainfall duration and intensity.

These existing research results will permit more accurate evaluation of the risk of civil hydraulic works and will significantly increase the validity and rationality of civil hydraulic engineering design. Although the probability-based design concept has been presented and used in many fields, it is still less for civil hydraulic engineering design. So, this paper will put forward the concept of conditional risk probability (*CRP*) and the optimal combination design method (*OCDM*) of two hydrologic variables. Taking Shenzhen city as an example, the paper will analyze the conditional risk probability and the best combination of the 24 h-rainfall and wind speed.

2. Optimal combination design method (OCDM)

2.1. Design risk probability and design risk threshold

From a view point of economy, the design of a civil hydraulic engineering should meet a design standard of usually 20, 50 or 100 year return period (*T*). For example, a 100 or 200 years return period is adopted in seawall design in Shenzhen of China. This means there is a 1% or 0.5% risk probability of failure of structure caused by higher water levels in excess of the initial design standard. The design risk probability (*DRP*) can be given as

$$DRT(T) = 1/T \times 100\% \quad (1)$$

where *T* is return period determined by a specified design standard. When assessing the risk threshold caused by extreme hydrologic variables respectively, structural failure occurs whenever variable *X* exceeds or is less than a certain critical threshold level determined by *DRP* (*T*).

- Structural failure occurs when X exceeds a certain critical threshold level. This yields:

$$1 - F(x_{T1}) = DRP(T) \quad (2)$$

where x_{T1} is the design risk threshold of variable *X*, and *F* is cumulative distribution function.

- Structural failure occurs when X is less than a certain critical threshold level. This yields:

$$F(x_{T2}) = DRP(T) \quad (3)$$

where x_{T2} is the design risk threshold of variable *X*.

2.2. Conditional risk probability (CRP)

The conditional risk probability can give some guidelines to help us determine the proper combination of two hydrologic variables with a specified design risk level *DRP* (*T*). Four formulas of *CRP* were given respectively, according to different combination design conditions of two variables (*X*, *Y*).

- CRP_I. Structural failure occurs when *Y* is less than a certain level. The conditional risk probability (CRP_I) of *X* and *Y*, given $X \le x_{T1}$, is

$$CRP_{I}(x_{T1}, y) = P(Y < y \mid X \le x_{T1}) = \frac{F(x_{T1}, y)}{F(x_{T1})} \quad (4)$$

where *y* is value of variable *Y*, and *P* is the probability function.

- CRP_{II}. Structural failure occurs when *Y* exceeds a certain level. The conditional risk probability (CRP_{II}) of *X* and *Y*, given $X \le x_{T1}$, is

$$CRP_{II}(x_{T1}, y) = P(Y > y \mid X \le x_{T1}) = \frac{F(x_{T1}) - F(x_{T1}, y)}{F(x_{T1})} \quad (5)$$

- CRP_{III}. Structural failure occurs when *Y* is less than a certain level. The conditional risk probability (CRP_{III}) of *X* and *Y*, given $X \ge x_{T2}$, is

$$CRP_{\mathrm{III}}(x_{T2}, y) = P(Y < y \mid X \geq x_{T2}) = \frac{F(y) - F(x_{T2}, y)}{1 - F(x_{T2})} \tag{6}$$

- CRP$_{\mathrm{IV}}$. Structural failure occurs when *Y* exceeds a certain level. The conditional risk probability (CRP_{IV}) of *X* and *Y*, given $X \geq x_{T2}$, is

$$CRP_{\mathrm{IV}}(x_{T2}, y) = P(Y > y \mid X \geq x_{T2}) = \frac{1 - F(x_{T2}) - F(y) + F(x_{T2}, y)}{1 - F(x_{T2})} \tag{7}$$

2.3. *Optimal combination design method of two hydrologic variables*

For a given design risk threshold (x_T), CRP_{I} and CRP_{III} increase with *y* while CRP_{II} and CRP_{IV} decrease with *y*. A schematic presentation of assumptive $CRP_{\mathrm{I}} \sim CRP_{\mathrm{IV}}$ curves is shown in figure 1. We can get the optimal value of y_T by measuring the coordinates of the critical point shown in figure 1 or by solving:

$$CRP_{\mathrm{I}}(x_T, y_T) = 1 - F(x_T) \Rightarrow F(x_T, y_T) = F(x_T)[1 - F(x_T)] \tag{8}$$

$$CRP_{\mathrm{II}}(x_T, y_T) = 1 - F(x_T) \Rightarrow F(x_T, y_T) = F^2(x_T) \tag{9}$$

$$CRP_{\mathrm{III}}(x_T, y_T) = F(x_T) \Rightarrow F(y_T) - F(x_T, y_T) = F(x_T)[1 - F(x_T)] \tag{10}$$

$$CRP_{\mathrm{IV}}(x_T, y_T) = F(x_T) \Rightarrow F(y_T) - F(x_T, y_T) = [1 - F(x_T)]^2 \tag{11}$$

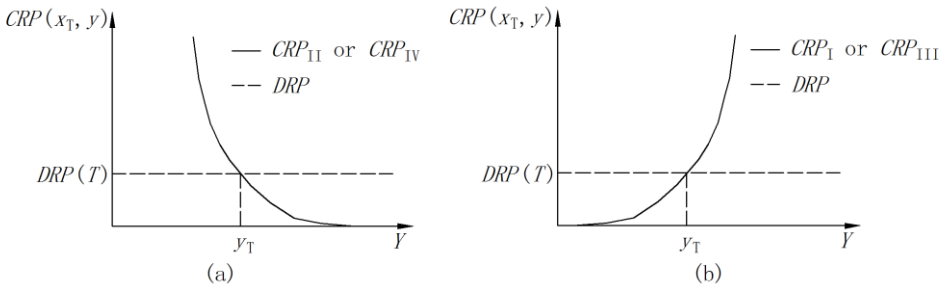


Figure 1. Illustration of the $CRP_{\mathrm{I}} \sim CRP_{\mathrm{IV}}$ with given *T* and x_T.

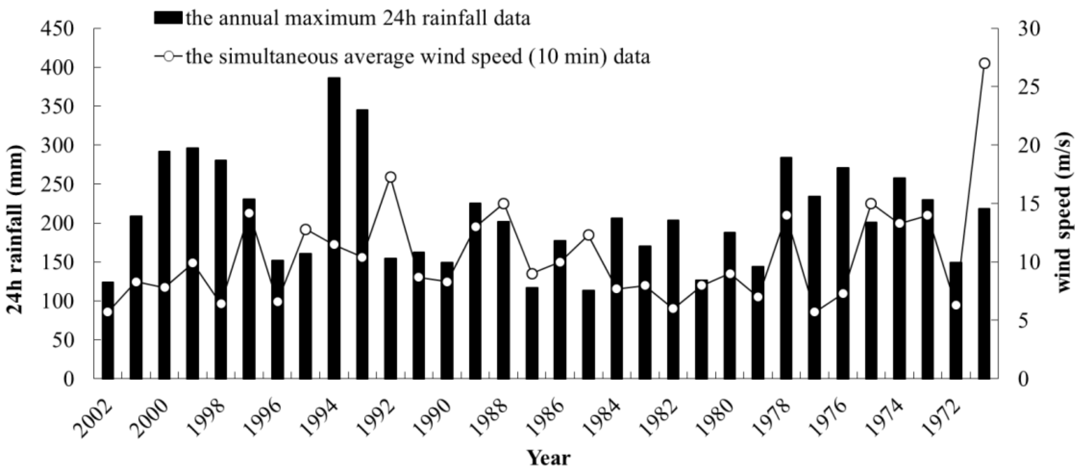


Figure 2. The annual maximum 24 h rainfall data and its simultaneous average wind speed data.

3. Application and results

3.1. Study area and data

Shenzhen neighbors Hong Kong and has a subtropical marine climate. As an example, the annual maximum 24 h rainfall data with the simultaneous average 10 min wind speed data (see figure 2), and the annual maximum average 10 min wind speed with the simultaneous 24 h rainfall data (see figure 3) were used in this paper. A total of 32 years (1971～2002) of data sets were collected from ground-based instruments of Shenzhen. The two figures show clearly that the large rainfalls are not always occurring together with large wind speeds.

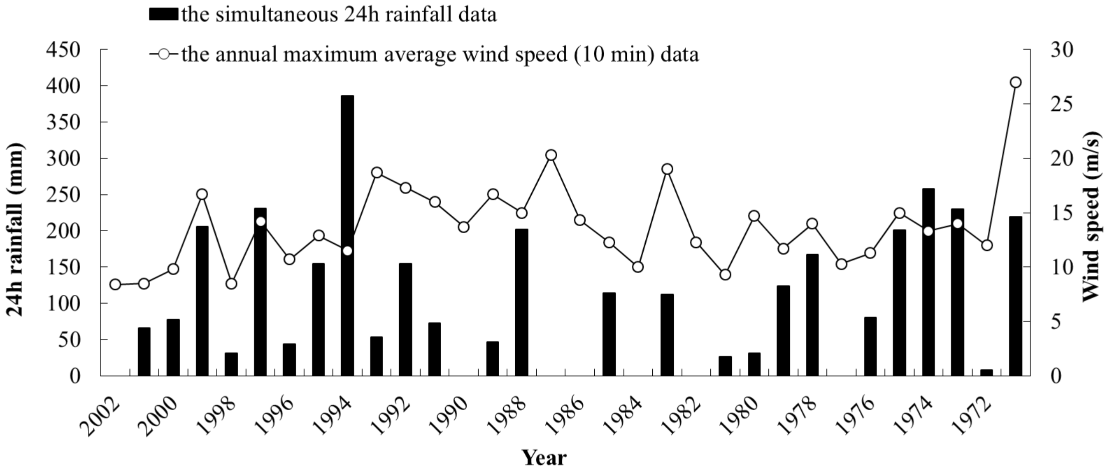
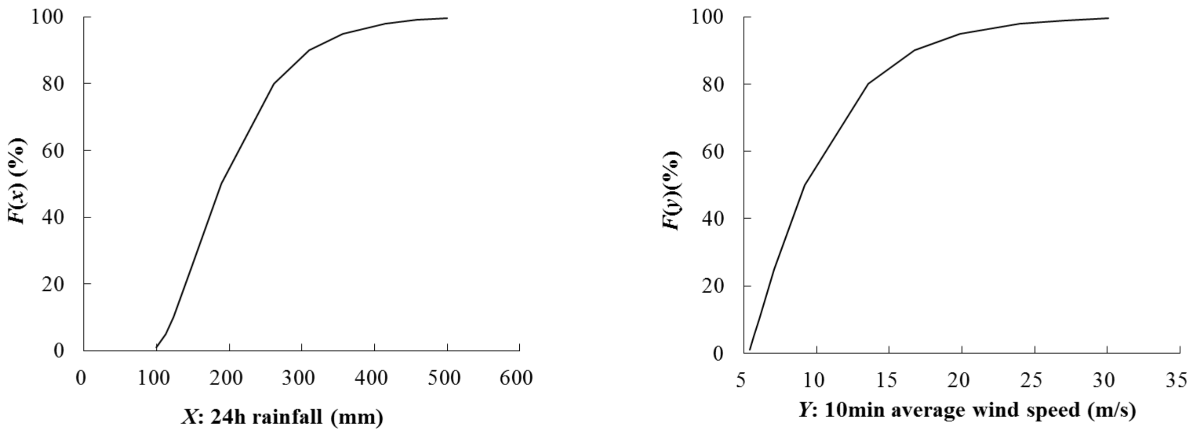


Figure 3. The annual maximum average wind speed data and its simultaneous 24h rainfall data.

3.2. Marginal distribution and joint distribution

For marginal distribution models, Pearson Type III distribution [10,11] widely used in hydrological field of China was adopted. Figure 4 shows the marginal distributions of the annual maximum 24 h rainfall and its simultaneous average wind speed. Figure 5 shows the marginal distributions of the annual maximum average 10min wind speed and its simultaneous 24 h rainfall.



(a) (b)

Figure 4. Marginal distributions: (a) *X*− 24h rainfall; (b) *Y*− simultaneous average wind speed.

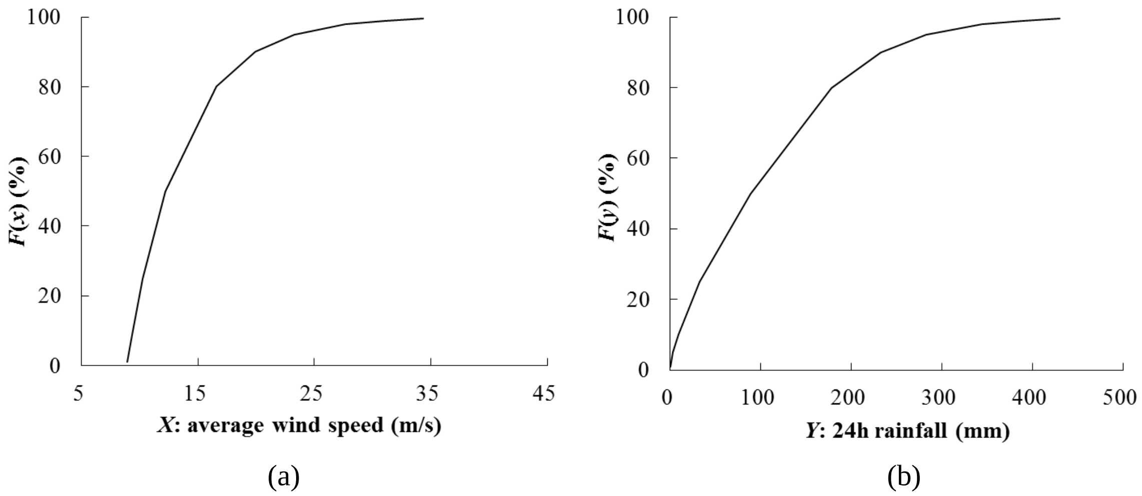


Figure 5. Marginal distributions: (a) *X*- wind speed; (b) *Y*- simultaneous 24h rainfall.

Several one-parameter archimedean copulas [12,13] can be considered as potential candidates for the joint cumulative distribution $F(x,y)$. Using the Clayton copula, the joint cumulative distribution $F(x,y)$ can be expressed as

$$F(x,y) = (F(x)^{-\theta} + F(y)^{-\theta} - 1)^{-1/\theta} \tag{12}$$

where θ is the parameter. Figure 6 shows the excellent correlation between empirical and theoretical joint distribution obtained by equation (12). Thus, Clayton copula is suitable for constructing the joint distribution of the correlated rainfall and wind speed.

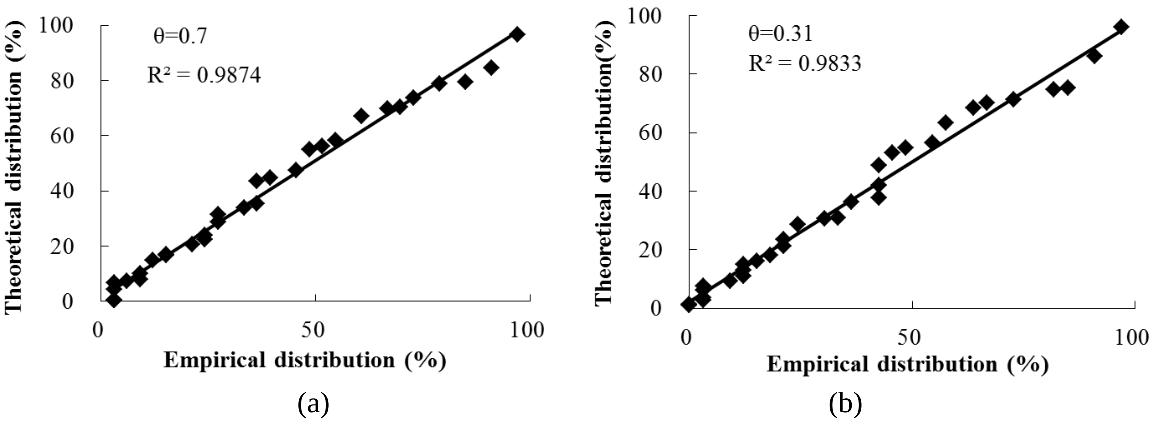


Figure 6. Correlation between the empirical and theoretical distribution: (a) the annual maximum 24 h rainfall and the simultaneous average 10 min wind speed. (b) the annual maximum average 10 min wind speed and the simultaneous 24 h rainfall.

3.3. Optimal combination values (x_T, y_T) design

The results of conditional risk probability (CRP_{II}) analysis are shown in tables 1 and 2. Using the probability table, the best combination values of rainfall and wind speed can be selected for any given design level of risk by calculating the coordinates of the critical point shown in figure 1 or by solving equation (9). For a given design risk probability of 1%, the 24 h rainfall with a design risk threshold of 457.7 mm and a wind speed of 27.4 m/s are the best combination, or the wind speed with a design risk threshold of 30.99 m/s and a 24 h rainfall of 396.5 mm are the best combination. For a 1% risk probability allowed by the design standard in related civil hydraulic engineering design, the two best combination conditions should be met simultaneously.

Table 1. The conditional risk probabilities (CRP_{II}) with design rainfall risk thresholds.

DRP (%)	*X*: 24 h rainfall (mm)	*Y*: wind speed (m/s)	CRP_{II} (%)
0.5	499.9	19.95	4.93
(*T*=200 yr)		23.27	2.50
		27.66	0.90
		30.99	0.43
1	457.7	19.95	4.92
(*T*=100 yr)		23.27	2.50
		27.66	0.89
		30.99	0.43
2	414.7	19.95	4.90
(T=50 yr)		23.27	2.49
		27.66	0.88
		30.99	0.42

Table 2. The conditional risk probabilities (CRP_{II}) with design wind speed risk thresholds.

DRP (%)	*X*: wind speed (m/s)	*Y*: 24h rainfall (mm)	CRP_{II} (%)
0.5	34.31	310.0	3.66
(T=200 yr)		356.1	1.72
		414.7	0.68
		457.8	0.35
1	30.99	310.0	3.64
(T=100 yr)		356.1	1.71
		414.7	0.68
		457.67	0.35
2	27.66	310.0	3.62
(T=50 yr)		356.1	1.70
		414.7	0.67
		457.67	0.34

4. Conclusions

In this paper, the main aim is to perform a bivariate analysis using copula approach, for finding an optimal combination design values with a given design risk probability. The work is split into four main parts: (a) the research background; (b) the establishment of the optimal combination design method of two hydrologic variables; (c) the optimal design combinations of 24 h rainfall and wind speeds at Shenzhen are analyzed; (d) a set of conclusions closes the paper. The main results show that:

- The failure probability is critical in the evaluation of the reliability of a structure. If hydrologic variables which cause the failure are dependent, the probability of failure can not be calculated only by a given extreme value of one variable.
- The large values of hydrologic variables are not always occurring together. In the traditional method on civil hydraulic engineering design, the assumption of the two extreme events occurring simultaneously is unreasonable and uneconomic.
- The proposed method takes into account joint probability distributions and conditional probability distributions of hydrologic variables. This can effectively improve safety, credibility

and economy of a structure design.

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