IOP Conf. Series: Earth and Environmental Science **189** (2018) 022084 doi:10.1088/1755-1315/189/2/022084

Development and Preliminary Application of Resonant Column-Bending Elements Combined Testing Apparatus

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Abstract Due to the influence of near-field effect and soil filterability, there are still many disputes for determining the shear wave propagation time, which hinders further development and application of the bending element test. Based on the Stokoe type resonance column instrument (RC), a pair of bending elements (BES) was implanted into the system, which achieved the combined test of RC and BES on a same soil specimen. In order to test the device reliability, combined tests of dry Fujian standard sand with different initial densities under different confining pressure were carried out to measure the dynamic shear modulus. Results show that the improved equipment can give play to both the advantages of RC and BES. For the same specimen, the BES calibration is achieved by RC system, which effectively overcomes the problem of time determination in BES test. When BES excitation frequency satisfies the condition ($2<R_d<7$), the near-field effect can be weakened effectively. When the initial arrival wave method is used to determine the shear wave propagation time in BES test, the take-off point of the previous peak of the received main wave crest should be taken as the arrival time of the shear wave.

1. Introduction

As the primary parameters of geotechnical dynamic characteristics, dynamic modulus and damping ratio are essential dynamic parameters in the seismic response analysis of soil layer and foundation. They are also indispensable contents in the seismic safety evaluation of a site. In order to determine the dynamic modulus and damping ratio of the soil, a few of scholars have carried out extensive research and obtained many valuable research results [1]. However, this is still a complex topic of study owing to the problems from the variability of soil and the reliability of test instrument and equipment [2].

There are generally two methods for measuring the geotechnical dynamic modulus, field test and laboratory test. The two methods complement and verify each other. The field test is mainly based on the wave method, which calculates the shear modulus by measuring the shear wave velocity of soil layer. The commonly used techniques include the surface wave method [3], the down-hole method [4], and the cross-hole method [5]. Laboratory tests are divided into three categories, the static method, the dynamic method, and the wave method. The static method is represented by the high-precision triaxial test [6] and the cyclic torsional shear test [7]. Such method obtains the dynamic parameters and indexes according to hysteresis curve of the relationship between dynamic shear stress and strain. However, in order to obtain the shear modulus, it is necessary to carry out the conversion based on the assumption of a constant Poisson's ratio, which is inconsistent with the actual situation. The dynamic method utilizes a resonant column (RC) [8], where the resonant (natural) frequency of system is measured by applying different excitation frequencies to a soil specimen through the drive system of RC. The RC method is internationally recognized as the most reliable method for determining the small strain shear modulus of soil. The wave method utilizes a pair of bending elements (BES), where the propagation velocity of shear wave in soil is measured. The shear modulus of soil under small strain is then calculated according to the wave theory, and the damping ratio is determined as per the attenuation curve of the shear wave. Shirley et al. [9] applied the BES technique to the geotechnical engineering field for the first time. Dyvik et al. [10] further developed the technique and compared the test methods of BES and RC. Since then, BES has been widely used as a new method to measure the modulus of soils. For example, Jocicic, et al. [11] combined the BES with the geotechnical triaxial apparatus. Zeng, et al. [12] combined the BES with the consolidation apparatus. Dyvik, et al. [10] combined the BES with the direct shear apparatus.

Up to now, the application of BES to determine the duration of wave propagation in soil is still uncertainty owing to phenomena such as the near-field effect, the overshoot effect, and the filtering effect [13]. However, there is a lot of room for development of the BES as a research tool because of its advantages including clear principle, simple operation, convenient transplantation, and nondestructive testing. Some scholars have attempted to use RC to judge the reliability of BES. Dyvik, et al. [10] tested five clay soil specimens with BES and RC apparatus, respectively. The test results of BES and RC were relatively consistent. Youn, et al. [14] conducted BES, RC, and cyclic torsional shear tests on dry sand and saturated sand specimens, respectively. It was found that test results of the different methods were consistent. Studies carried out by Sotou, et al. [15] showed that the degree of consistency between the results of BES and RC was related to soil particle size. Test results of two methods were consistent when soil particle size was small, but gradual deviation occurred with the increase of particle size (results of RC were smaller than those of BES). Yang et al. [16] used glass beads with four different particle sizes to carry out BES and RC tests. The test results were contrary to those obtained by Sotou et al., and showed that the small strain shear modulus of soil was independent of its particle size. Dong et al. [17] carried out BES and RC tests on dry sand and saturated sand under different pore ratios and confining pressures. It was found that two methods showed high consistency with a difference of less than 3% in dry sand, but the dispersion for saturated sand with a difference of 6-10%. From the above, we can see that the existing research conclusions are not completely consistent or may even be mutually contradictory. The comparison showed that judging whether the test results of BES and RC are consistent was often carried out on separate equipment, and the combined test of BES and RC for a same specimen was rarely achieved. Although RC test has been identified by domestic and international scholars in laboratory tests for determining the dynamic modulus of soil, the RC apparatus has a relatively large level of strain (10^{-5}) that causes the soil specimen to produce disturbance, which, in turn produces a vibrating compaction effect on the soil specimen. It could change the initial compactness of specimen, which is not conducive to repeated tests on a same specimen, especially for the loose sand specimen. BES test can easily withstand the repeated test of the small strain modulus of soils under the strain level of 10^{-6}, but there is still controversy about the determination of arrival time of the shear wave. How to use the coupling method of BES and RC to form a combined testing technique such that they complement each other's advantages, how to reasonably calibrate BES, and how to carry out the cross checks, are questions that need to be addressed urgently in studying the small strain dynamic characteristics of soil.

In order to solve the above-mentioned problems, based on the Stokoe RC system produced by the company of Geotechnical Digital System (GDS) in UK, a pair of BES was implanted into the equipment The test system can carry out the combined test of BES and RC for a same soil specimen and achieve the aims of calibrating BES by using the RC apparatus. As a result, the subjectivity problem of judging the arrival time of shear wave in BES test could be resolved.

2. Test principle for testing small strain shear modulus of soil

2.1. The BES method

BES is also called piezoelectric ceramic bending element, which is usually formed by laminating two piezoelectric ceramic sheets. Owing to the piezoelectric characteristics of piezoelectric ceramics (piezoelectric ceramic sheets undergoes mechanical deformation, such as elongation or shortening under the action of voltage, and, in turn, voltage is generated when piezoelectric ceramic sheets undergo mechanical deformation), the piezoelectric ceramic bending element is essentially a transducer that can achieve the inter conversion between mechanical energy and electrical energy. Piezoelectric ceramic BES include the emission bending element (EBE) and the receiving bending element (RBE). The EBE refers to a transducer that can convert electrical energy into mechanical energy and the RBE refers to a transducer that can convert mechanical energy into electrical energy. A picture of BES produced by the Geotechnical Digital Systems (GDS) is shown in Figure1.

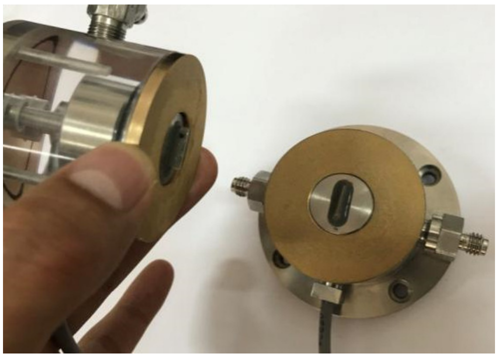


Figure 1. The pedestal and top cap of specimen with BES.

When the shear wave velocity of a specimen is tested by BES, EBE and RBE usually inserted into the ends of the soil specimen, and the voltage pulse signal with a certain frequency is generated by a function signal generator (the frequency can be adjusted according to test conditions) and applied to the EBE. The free end of EBE produces a transverse oscillation with a certain frequency at this moment, forcing the soil particles around EBE to oscillate laterally. Meanwhile, the generated shear wave is transmitted to RBE through the specimen. RBE is then forced to produce a transverse oscillation after receiving the shear wave, and the mechanical oscillation is converted to an electrical signal (the principle is shown in Figure 2). The oscilloscope can simultaneously display and store the signals generated by EBE and the signal received by RBE. The propagation time (t_s) of the wave can be calculated by comparing the emitted signal with the received signal, and the height of the soil specimen can also be measured during the test. Consequently, the propagation velocity (V_s) of wave in the specimen and the shear modulus (G_{max}) of soil can be calculated:

$$V_s = h/(t_s - t_0) \tag{1}$$

$$G_{max} = \rho V_s^2 \tag{2}$$

where h is the distance between the tips of the EBE and the RBE (Tip-to-Tip); t_0 is the system delay time of BES; ρ is the density of the soil.

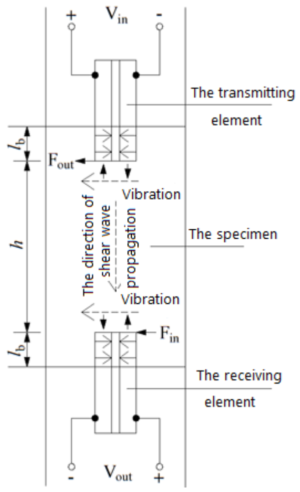


Figure 2. The shear wave testing principle of BES.

Owing to the existence of damping in soil, the amplitude of shear strain decreases gradually along the propagation direction of the shear wave. Due to the amplitude of soil particles near the free end of the EBE is the largest (estimated to be around 10^{-6}), the shear modulus measured by BES method should be the maximum shear modulus (G_{max}) of soil.

Only the wave propagation distance and propagation time need to be measured when BES method is used to measure the wave propagation velocity in soil. The propagation distance generally refers to the distance between the tips of EBE and RBE, which is easily determined in the test. However, different researchers have put forward different methods, including general time domain analysis [18], frequency domain analysis [19], and cross-correlation [20], to determine the propagation time of shear wave. However, up until now, owing to the influence of phenomena such as the near-field effect, overshoot effect, filtering effect, and other factors, there is still no unified understanding. How to accurately determine the propagation time of shear wave in the BES method remains open to debate.

2.2. The RC method

The most commonly used RC is the type of Stokoe, in which the soil specimen is fixed on the pedestal, and the hood of specimen is connected with the drive system (it can move freely after receiving the driving force). Since the principle of Stokoe RC is relatively simple, most of scholars use this type of RC. This study had adopted the Stokoe RC produced by GDS as shown in Figure 3. Figure 4 illustrates the structural diagram of drive system of the apparatus, which is the core component.

When the instrument is used to measure the shear modulus of a soil specimen, a sinusoidal voltage is first applied to the electromagnetic coil, which generates a driving force on the driving head after the electromagnetic coil is powered. A torsional force is then applied to the specimen. Different excitation frequencies and amplitude values can be applied to the specimen through adjusting the frequency and amplitude of the current applied to the electromagnetic coil. The vibration amplitude of the specimen reaches the maximum when the inherent (resonant) frequency of system is the same as the driving frequency. The maximum amplitude of the specimen can be monitored by an accelerometer at this moment. The shear modulus of soil specimen can be calculated from the resonant frequency. The basic calculation equation of the Stokoe RC is:

$$I / I_0 = \beta \tan(\beta) \tag{3}$$

where I is the rotational inertia of the specimen; I_0 is the rotational inertia of all components connected to the hood of the specimen (including the top cap, drive head, magnet, acceleration sensor, and vertical displacement sensor, etc.).

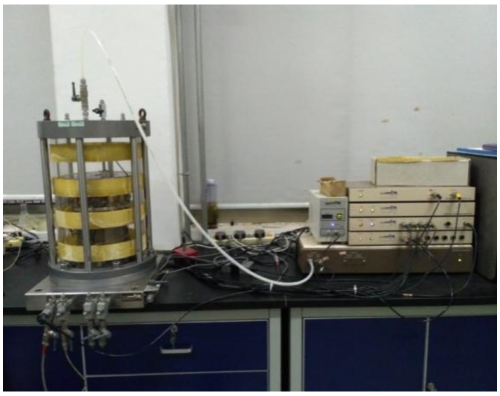


Figure 3. The Stokoe RC produced by GDS.

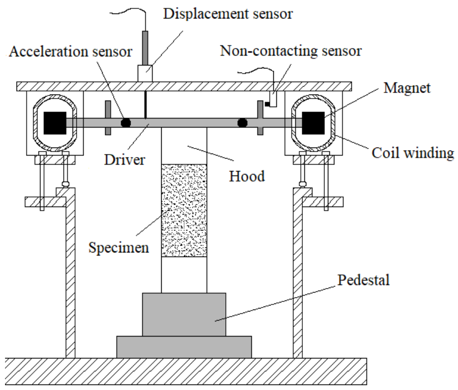


Figure 4. The structure of Stokoe RC.

The moment of inertia I for a solid cylinder specimen can be calculated according to formula (4):

$$I = \frac{md^2}{8} \tag{4}$$

where d is the diameter of a specimen, and m is the mass of specimen.

Owing to the irregular shapes of components connected to the top of the specimen, it is difficult to obtain the accurate moment of inertia I_0 through calculation. I_0 is usually obtained by calibration in experiments.

After I and I_0 are obtained, the β value can be calculated using formula (3), and it can also be obtained by checking the parameter table of the instrument as the value of I/I_0 obtained by calculation.

The shear wave velocity (V_s) of the specimen can be calculated according to formula (5) after getting the resonant frequency (f) and the corresponding β value:

$$V_s = \frac{2\pi fl}{\beta} \tag{5}$$

where l is the height of soil specimen.

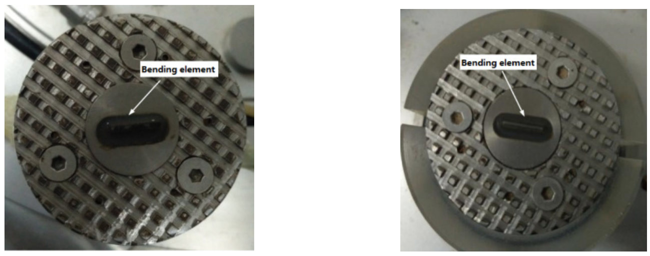
The shear modulus of specimen is calculated according to formula (6):

$$G=\rho V_s^2 \tag{6}$$

The resonant column method is currently recognized as the most reliable method for determining the small strain shear modulus of soil, and is incorporated into the geotechnical test specification as a standard method. However, owing to the limitations of the instrument itself, the RC method can only apply hydrostatic pressure and cannot apply axial pressure to the specimen. Meanwhile, the shear strain of the resonant column test should be within the range of 10^{-5} to 10^{-4}. It belongs to a small strain range but continuous excitation could cause the specimen to become dense, especially for loose sand specimens.

3. The combined testing apparatus

In this study, a pair of BES was implanted into the central areas of the pedestal and the specimen cap of the RC apparatus, respectively, which is shown in Figure 5.Since BES were installed in the specimen top cap and the pedestal of the RC apparatus, the mass and rotational inertia of the specimen cap were different from those of the initial status. Therefore, the rotational inertia I_0 was required to be recalibrated. In order to accurately calibrate, it was necessary to add a calibration gasket between the top plate and the specimen cap, which could make them tightly connect to each other, and also protect the BES. The connection of the calibration gasket is shown in Figure 6.



(a) top cap (b) pedestal

Figure 5. The specimen top cap and pedestal in RC after installation with BES.

Three copper calibration blocks (completely identical) and three aluminium calibration rods (the diameters of the rod-shaped part in the middle of the three calibration rods are different and the other parts are identical, i.e. the corresponding shear moduli are different) and calibration gaskets were required during the calibration test. The connection of each component is shown in Figure 6. Since the values I of the aluminium calibration rods are much smaller than I_0, formula (8) could be derived from formula (7) and formula (3).

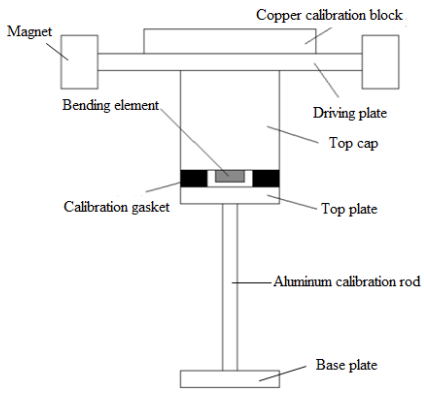


Figure 6. The connection diagram of calibration gasket.

$$G=\rho V_s^{2}=\rho\left(\frac{2\pi fl}{\beta}\right)^{2} \tag{7}$$

$$I_0=I_1+I_2=\frac{I}{\rho(2\pi l)^{2}}\cdot\frac{1}{f^{2}} \tag{8}$$

where I_1 is the mass moment of inertia of the copper calibration block, which can be calculated according to the geometrical size; I_2 is the mass moment of inertia of remaining parts (including the calibration gasket, specimen cap, drive head, magnet attached to the drive head, and acceleration sensor, etc.), which cannot be calculated owing to the irregular geometrical shapes and needs to be determined by calibration.

Three aluminium calibration rods were installed in sequence in the calibration process and four tests were implemented for each calibration rod. Different numbers of calibration blocks were attached each time, and the number of attached calibration blocks was zero, one, two and three, respectively. In this way, each calibration rod could obtain four systematic resonance frequencies, and correlation curves between I_1 and $1/f_2$ could be drawn. Three such curves, which are shown in Figure 7, could be obtained from the three calibration rods. The average value obtained from the slope of the three diagrams could be considered the I_2 value. The slope the three curves in the calibration test in this study were 3.649×10^{-3}, 3.5155×10^{-3} and 3.6588×10^{-3}. Therefore, the average value I_2 was 3.6051×10^{-3}.

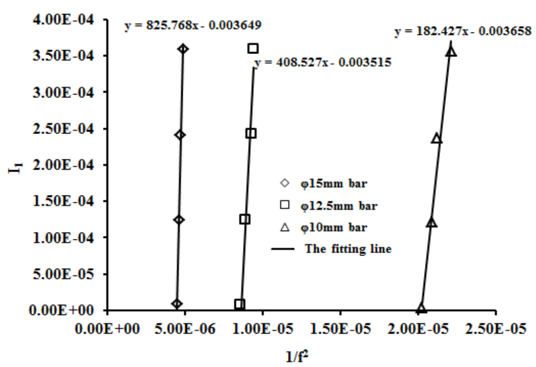


Figure 7. Calibration results of the RC installed with BES.

The calibration of BES calibrated mainly the delay time of signals propagating in the system. Direct contact was established between the emission end and the receiving end of the BES, and the shear wave was stimulated, such that the delay time of shear wave propagating in the system could be determined. The system delay time of shear wave determined at different frequencies is shown in Table 1. There is a slight difference in the system delay time under different frequencies. The system delay time was set as 5 μ_s in this study for convenience.

Table 1. System delay time of shear wave at different excitation frequencies

Excitation Frequency (kHz)	System Delay Time (μ_s)
5	6.5
10	5.75
14.28	5.25
20	4.75
25	4.5
50	4.25
100	3.75

4. Preliminary application

In order to verify the reliability of the RC-BES combined testing apparatus, Fujian standard sand was used to carry out comparative tests on the dynamic shear modulus of sand under different stress levels.

Dry sand was used as the sample in the tests in order to reduce the test time and to gauge the test equipment's efficiency.

4.1. Soil sample
The sand sample was dried and sifted with a 1mm sieve to remove particles larger than 1 mm. The particle sizes in the prepared sand ranged from 0.075 mm to 1 mm. The particle size distribution and

Table 2. Parameters of Fujian standard sand.

Type	Specific gravity (g/cm^3)	e_{min} (g/cm^3)	e_{max} (g/cm^3)
Fujian standard sand	2.68	1.489	1.836

cumulative volume percentage measured by the Mastersizer 3000 laser particle size analyzer (made in UK), are shown in Figure.8, and the basic physical parameters of the sand sample are outlined in Table 2.

4.2. Test methods
The specimen was 50 mm in diameter and 100 mm in height. Dry sand was used to prepare specimens with relative density of 30%, 50% and 70%. The specimens were formed by five layers on the RC-BES combined testing apparatus using the vibrating and tamping method. Comparisons between RC test and BES test for each sand specimen were carried out under confining pressure of 100 kPa, 200 kPa, 300 kPa, 400 kPa, and 500 kPa, receptively. The test under each level of confining pressure was implemented after one hour of consolidation. The test of BES was first carried out on the specimen and then the RC test. The test time was controlled within 10 min. The excitation frequency of 5 kHz, 10 kHz, 15 kHz, 20 kHz, 25 kHz, 33.3 kHz, 50 kHz, and 100 kHz was selected for BES test under each level of confining pressure, receptively. The received signal in the BES test was the result of undergoing 50 times of waves superposition.

4.3. Analysis of results
Figure 9 shows the typical BES test results for sand specimens. As is shown, BES could generate the compression wave when the shear wave was excited. The initial polarization direction of the compression wave was opposite to that of the shear wave excitation signal, and the compression wave arrived at the receiving end earlier than the shear wave, which generated a significant phase pull-down

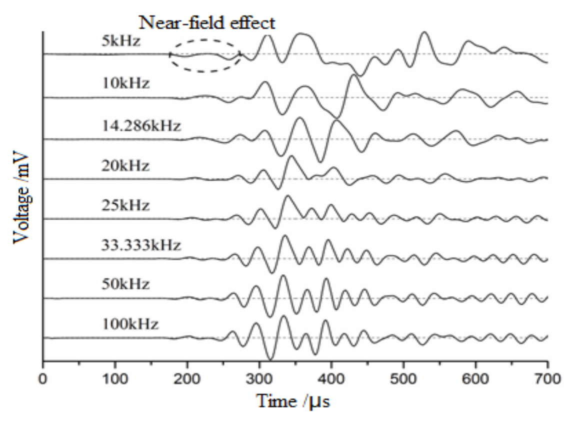


Figure 8. BES test results of sand specimens with the density ratio of thirty percent under 300kPa confining pressure.

in the initial received signal. This affected the jump point judgment of the initial wave of the shear wave, i.e. near-field effect. Near-field effect was most obvious at 5 kHz but its impact gradually weakened with the increase of frequency. However, the increase in frequency did not completely eliminate the near-field effect. When the frequency exceeded 33.3 kHz, the received signal of the shear wave began to oscillate in the initial receiving section and the signal quality decreased. Sanchez-Salinero et al. [21] suggested that the ratio of the BES's signal between the propagation distance L and the wavelength λ should be greater than 2 and less than 4, i.e., 2< R_d <4. The test results in this study showed 2< R_d <7, which were basically consistent with the conclusions of Sanchez-Salinero et al. Therefore, it is suggested that the shear wave test with BES should start at a frequency of 2< R_d, and then gradually increase until a frequency that can output a stable signal is identified. Thus, the result from this excitation frequency is selected as the final test results.

For determining the propagation time of the shear wave in BES test, the initial arrival wave method has been widely used because of its clear principle and simplicity of operation. This method is taken the time difference between the initial take-off point of the excitation signal and the received signal as the propagation time of the shear wave. It is relatively accurate in determining the propagation time of the compressed wave. However, due to the impact of the near-field effect on shear wave, the subjective factor in judging the initial take-off point of the received signal is relatively large, and the obtained propagation time is relatively random. It can be seen from Figure 8 that initial arrival wave method is significantly influenced by the excitation frequency of the BES: the higher the excitation frequency is, the shorter the measured propagation time becomes. It shows that there is frequency dispersion characteristic when BES is used for the shear wave test. This found is consistent with the previous study results. In order to eliminate human factors in determining the initial take-off point of the received wave, the RC method can be used to calibrate the arrival time of the BES. For instance, Gu et al. [22] have come to the conclusion through tests that the first zero phase point of the shear wave should be used as the take-off point of the received wave. In this study, the similar method was

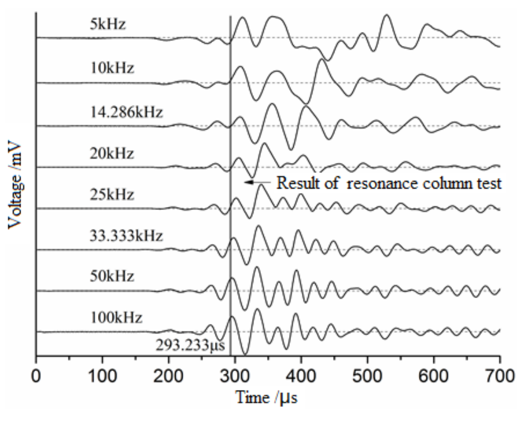


Figure 9. The test results of RC for calibrating BES.

used to calculate the maximum shear modulus (G_{max}) of sand specimens based on the Hardin model, through using RC to measure the change law of shear modulus under strains of 10^{-5}~10^{-4}. Then the shear wave velocity (V_s) corresponding to the maximum shear modulus was calculated according to formula (6). The propagation time of shear wave in specimens was determined by formula (1). The propagation time of shear wave calculated by the RC test is marked in the test results of BES, as shown in Figure 9.

The solid line perpendicular to the abscissa in the figure is the result of the RC test. When the excitation frequency of BES was between 10 kHz and 33.3 kHz, the received signals of shear wave were relatively stable (2< R_d <7). The time position of the shear wave calibrated by the RC test corresponded to the initial take-off point position of the previous peak of the received main peak of BES signal (i.e. main peak is the largest amplitude of the wave). Therefore, when the initial arrival wave method is used to determine the propagation time of the shear wave, it is more appropriate to take the take-off point of the previous peak of the main peak of the received wave as the initial arrival time of the shear wave.

5. Conclusions

In order to achieve the RC-BES combined test on a same soil specimen. This study proposed to implant a pair of BES in the Stokoe RC apparatus. Through the combined test of RC and BES on dry sand, the following conclusions could be drawn:

1) The newly improved equipment can give full play to the respective advantages of test systems of RC and BES. Based on the dynamic parameter test of the same soil specimen, the calibration of BES test system can be achieved by using the RC apparatus, which can effectively overcome the problem of subjective factors existing in the initial arrival wave method widely used in current BES tests.

2) BES test shows obvious dispersion in the test of shear wave for dry sand. The higher the excitation frequency is, the greater the measured wave velocity becomes. When the ratio between the wave propagation distance L and wavelength λ satisfies the condition of 2< R_d <4, the influence of near-field effects can be reduced effectively, and a stable received signal of shear wave can be easily obtained.

3) For the dry sand, when the initial arrival wave method is used to determine the shear wave propagation time in BES test, it is more appropriate to take the take-off point of the previous peak of the main peak of the received wave as the initial arrival time of shear wave. Whether the law is applicable to other soil remains to be verified in the future.

Acknowledgments

Authors appreciate the support of the National Natural Science Foundation of China(51579237 & 51309027).

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