

# Fracture Analysis for Torsion Problems of a Spar Platform with Cracks under Wind Load

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**Abstract.** Ocean platforms are widely used to extract submarine oil. For a long time, they are subjected to a variety of environment loads, such as those from winds, waves, currents. In this study, the torsion problems of a spar platform with cracks under wind load were investigated. The main body of spar was assumed to be a homogeneous material cylinder, then the torsion fracture problem of the cylinder was considered. The problem was reduced to solve the boundary integral equations on crack and boundary, torsion problem of the spar platform main body with a straight crack under various wind loads was calculated by used the boundary element method. The obtained results showed that with the increase of wind scale, the allowed maximum crack size in the main body of spar platform was decreased gradually, cracks had some effect on the safety of the spar's main body in marine engineering. Especially in severe ocean environments, the risks of fracture failure of main body with cracks should not be overlooked, and should be considered during the process of marine structure construction and usage.

## 1. Introduction

Spar platforms are used for deep sea environment. In the world, they can be divided into three generations: Classic Spar, Truss Spar and Cell Spar. Spar has lots of functions, such as drilling, completion, production and storage. Platform structures are mainly divided into four parts, the top is composed of two to four layers rectangular deck structure, the main body supports the upper deck, riser system mainly composed the production riser, drilling riser, output riser and line components, the bottom part is mooring system (Shen and Tang, 2011). Spar structures are strong and complicated, therefore, they are suitable for deep water operations (Li et al., 2011). Ocean platforms are subjected to a variety of environment loads such as those from wind, wave and current (Liu et al., 2012). Wave or current loads only lead to bending or shearing deformation of platform. But wind loads can cause torsional deformations of platform main body because of asymmetric wind load areas or action points. Natural disasters such as earthquakes can lead to the twisting destruction of the main body (Li et al., 2011). Especially when there are cracks in the main body, torsion fractures cannot be neglected. Considering the complexity of the spar platform's main body, we may simplify it as a cylinder. Therefore the torsion fracture problem of a cylinder was considered in this study. In the following, the derivation of new boundary integral equations for the Saint-Venant torsion problem of a cylinder containing curvilinear boundary cracks is presented, a typical torsion problem of a simply main body involving a straight crack under wind load is analyzed.



## 2. Torsion Problems of a Spar Platform's Main Body under Wind Load

Main body plays an important role in the whole spar structure. Its safety may directly affect the stability and safety of the whole platform. Consider a spar platform as shown in Figure.1. It is likely subjected to a variety of loads such as fixed loads, live loads and environmental loads. In this paper, we only consider wind load which causes torsional deformations of the main body. The spar deck usually consists of two to four layers rectangular structures, it includes drilling equipment, a powerplant with various equipment, personnel working and living facilities, helicopter lifts and so on. Because the wind load areas or action points may vary on the deck, the main body may suffer torsional deformations under winds (Pan, et al., 2010).

Wind load on the spar can be expressed as (He and Hong, 2003)

$$F_w = k \cdot k_z \cdot P_0 \cdot A \quad (1)$$

where  $k$  is the shape coefficient,  $k_z$  the lift coefficient,  $A$  the projected area perpendicular to the wind velocity,  $P_0 = \kappa \cdot V_t^2$ ,  $\kappa$  being the wind pressure coefficient,  $V_t$  the wind velocity.

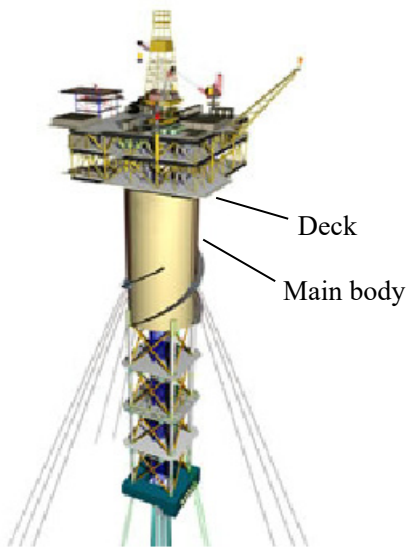


Figure. 1 Spar platform

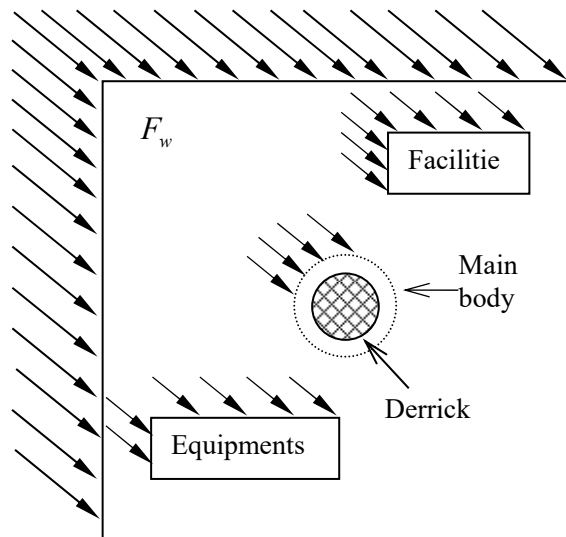


Figure. 2 Wind load acting on the deck and facilities

The deck structure of a spar under the wind load is shown in Figure.2. There are many facilities and equipment items on the deck. The wind load acting on them is related to their projected areas perpendicular to the wind velocity, this will cause a torque acting on the main body. In addition, their distances from the main body center are not the same, which would lead to torsional deformations of the body. Let  $A_i$  be the projected area perpendicular to the wind velocity of the  $i$ th facility or equipment item,  $l_i$  be the distance between the acting point and the main body center,  $k_i$  be the shape coefficient, then the wind load torque  $T$  acting on the main body can be expressed as (Dong and Kong, 2005):

$$T = \sum_i T_i = \sum_i k_i k_z P_0 A_i l_i \quad (2)$$

where  $i \in (1, m)$ ,  $m$  being the total number of facilities and equipment items on the deck.

## 3. Basic Formulation of Main Body Torsion

### 3.1. Mechanical Model of a Spar Platform's Main Body

The main body, with which the deck and risers are connected, is the supporting component of the spar. Considering the bad environment conditions and the requirement of drilling operation, the main body is

often designed as a cylinder. The material of the outermost layer has anti-seawater corrosion resistance or anti-fatigue effect. The middle layer is hard tank, it is a large diameter cylinder structure, central well through it, it could set fixed floating and variable ballast tank, provide most buoyancy for spar, and adjust spar floating state. The innermost layer is center well, it is throughout the main body from bottom to up, full of water. A schematic diagram of spar major structures is shown in Figure 3.

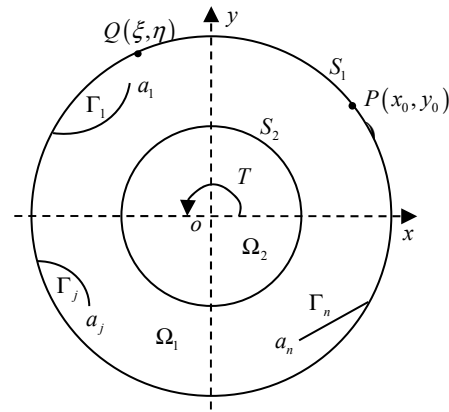
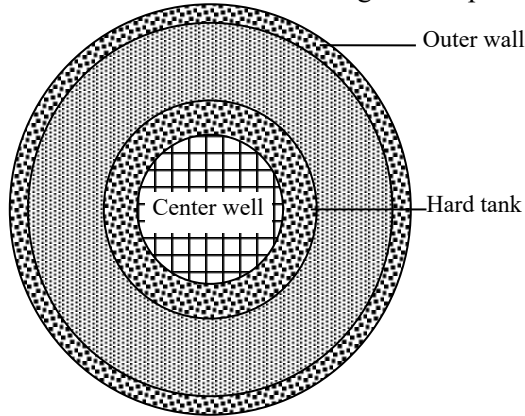


Figure. 3 Major structures of spar platform

Figure. 4 Section geometry of a cracked cylinder

The main body of a spar is not an ordinary cylinder. It is composed of various materials, there are a great number of equipment items and facilities installed in it. Therefore it is necessary to simplify it to a simple cylinder then to study the torsion problem in connection with wind load. For simplicity, suppose the main body is composed of the same kind of material as shown in Figure.4. The external section is  $\Omega_1$  with an outer boundary  $S_1$  and some boundary curvilinear cracks  $\Gamma_k$  ( $k=1, 2, \dots, n$ ), whose shear modulus and Poisson's ratio are  $G$  and  $\nu$ , respectively. The internal section is  $\Omega_2$  with interface  $S_2$ , whose corresponding material constants are  $G$  and  $\nu$  too. There are no cracks in  $\Omega_2$ . It is assumed that the spar main body is subjected to wind load torque  $T$ .

### 3.2. Torsion Rigidity $D$ and Crack Stress Intensity Factor $K_{III}$

We all know the new boundary integral equations which suitable for cylinder torsion problem can be expressed as (Tang and Wang, 1982)

$$\int_{S+\Gamma} \frac{\partial \phi^*(P, Q)}{\partial s(P)} F(Q) ds(Q) = \int_S \frac{\partial \phi^*(P, Q)}{\partial n(P)} q(Q) ds(Q) - \frac{1}{2} f(P) \quad (p(x, y) \in S + \Gamma) \quad (3)$$

Where,  $F(Q)$  denote  $\partial \phi(Q) / \partial s(Q)$  for  $Q \in S_1 \cup S_2$ , but  $F(Q)$  denote  $\partial \Delta \phi(Q) / \partial s(Q)$  for  $Q \in \Gamma$ .

The unknown function  $F(Q)$  in the boundary integral equations must satisfy the following single-valued conditions of displacement (Tang and Wang, 1982)

$$\int_{S+\Gamma} F(Q) ds(Q) = 0 \quad (4)$$

The derived Eqs. (3) together with the single-valued conditions (4) are the boundary integral equations for the present problem.

The torsion rigidity of a cylinder can be expressed as

$$D/G = \int_S [n_2(Q)\xi + n_1(Q)\eta] \xi \eta ds(Q) + \frac{1}{2} \int_{S+\Gamma} (\xi^2 + \eta^2) F(Q) ds(Q) \quad (5)$$

The stress intensity factor  $K_{III}$  is calculated by the following formulas

$$K_{III}(a_j) = -\frac{G\alpha}{2} \lim_{Q \rightarrow a_j} [\sqrt{2\pi|Q - a_j|} F(Q)] \quad (j=1,2,\dots,n) \quad (6)$$

Where  $a_j$  is the initial point of the  $j$ -th crack.

Once the unknown function  $F(Q)$  is obtained by solving the new boundary integral equations with single-valued conditions, the torsion rigidity  $D$  and stress intensity factor  $K_{III}$  can be calculated directly.

#### 4. Boundary Elements Numerical Method

As it is impractical to obtain exact solutions of the boundary integral equations presented above (Li *et al.*, 1996), a numerical technique is proposed (Zheng *et al.*, 2006). For simplicity, the boundary and crack are divided into linear elements, as shown in Figure.5. Each element is mapped onto the interval  $-1 \leq t \leq 1$  (Brebbia, 1978). The shape functions are defined as

$$N_1(t) = \frac{1}{2}(1-t), \quad N_2(t) = \frac{1}{2}(1+t)$$

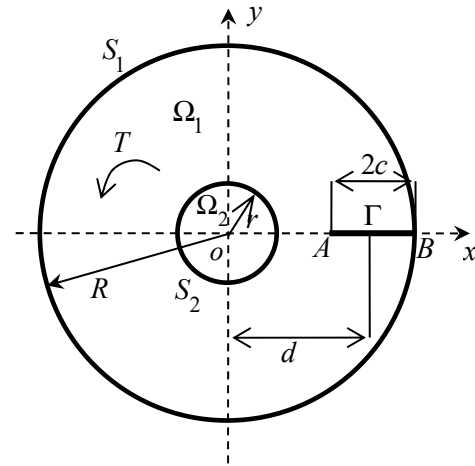
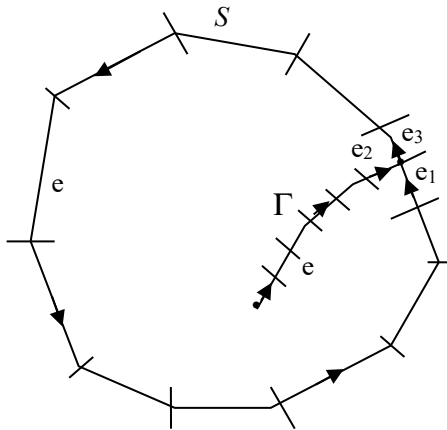


Figure. 5 Division on boundary and crack      Figure. 6 Torsion of a spar with a straight crack

The interpolation functions of boundary  $S$  are taken as

$$F(Q) = F_1^e N_1(t) + F_2^e N_2(t) \quad (|t| \leq 1, Q \in L_e \subset S) \quad (7)$$

The interpolation functions of crack  $\Gamma$  (do not contain crack tip) are taken as

$$F(Q) = F_1^e N_1(t) + F_2^e N_2(t) \quad (|t| \leq 1, Q \in L_e \subset \Gamma) \quad (8)$$

For an initial crack element

$$F(Q) = \frac{F_1^e}{\sqrt{N_2(t)}} + (F_2^e - F_1^e) N_2(t) \quad (|t| \leq 1, Q \in L_e \subset \Gamma) \quad (9)$$

For element  $e_1$  which on the boundary  $S$ , as shown in Figure. 5

$$F(Q) = (F_1^{e_1} - F_2^{e_1}) N_1(t) + \frac{F_2^{e_1}}{[N_1(t)]^{1-\lambda_1}} \quad (|t| \leq 1, Q \in L_{e_1} \subset S) \quad (10)$$

For element  $e_3$  which on the boundary  $S$ , as shown in figure. 5

$$F(Q) = \frac{F_1^{e_3}}{[N_2(t)]^{1-\lambda_2}} + (F_2^{e_3} - F_1^{e_3}) N_2(t) \quad (|t| \leq 1, Q \in L_{e_3} \subset S) \quad (11)$$

For element  $e_2$  which on crack  $\Gamma$  and intersect at boundary  $S$

$$F(Q) = \frac{F_{2c}^{e_2}}{[N_1(t)]^{1-\lambda_2}} + (F_{1c}^{e_2} - F_{2c}^{e_2} - Q_{2c}^{e_2})[N_1(t)] + \frac{Q_{2c}^{e_2}}{[N_1(t)]^{1-\lambda_4}} \quad (|t| \leq 1, Q \in L_{e_2} \subset \Gamma) \quad (12)$$

Substituting the formulas into (6) gives the following expressions for the stress intensity factor

$$K_{III}(a_j) = -\frac{G\alpha}{2} \sqrt{2\pi l_e} F_1^e \quad (13)$$

We can obtain the following two complementary equations from (28) to (30)

$$F_{2c}^{e_2} = (l_{e_2})^{\lambda_2-1} (l_{e_3})^{1-\lambda_2} F_1^{e_3} \quad (14)$$

$$Q_{2c}^{e_2} = (l_{e_1})^{1-\lambda_4} (l_{e_2})^{\lambda_4-1} F_2^{e_1} \quad (15)$$

The discretization of the boundary integral equations involves integrals on singular and non-singular elements. The integrals on the singular elements can be exactly obtained (Chien and Yhe, 1956), the integrals on the non-singular elements can be calculated by using the Gauss-Legendre integral equations.

### 5. A Numerical Example of a Spar's Main Body

To demonstrate the applicability of the present method, we present an example of the torsion problem of a spar platform main body containing a straight crack under wind load in practical situations. The facilities on the deck are of various structures. The distance from the ocean surface to the deck is 16–26 m. Let  $l=26$  m,  $A=200$  m<sup>2</sup>. The main body is simplified to a cylinder, the outermost being high strength concrete with a straight crack in it (see figure.6). When the calculation parameters are taken as

$$k = 1.5, k_z = 1.1, \kappa = 0.613 \cdot N \cdot s^2 / m^4, K_{IC} = 1.105 MN / m^{3/2}, [\tau] = 2.22 MPa; R = 16.155 m, r/R = 0.4, c/R = 0.3, d/R = 0.7$$

By using the present method and a FORTRAN program, the numerical results are obtained as

$$D^* = 0.996652, K_{III}^*(A) = -0.597362, K_{III}^*(B) = 0$$

$$\text{Where, } D^* = D / \left( \frac{\pi}{2} GR^4 \right), K_{III}^* = K_{III} / (G\alpha R \sqrt{\pi c})$$

From the strain energy density factor criterion (Li, 1998), we can obtain:

$$K_{IIIc} = \sqrt{1 - 2\nu} K_{Ic} = 0.816 K_{Ic} = 0.90168 (MN / m^{3/2}), (\nu = 0.167)$$

Use fracture criterion  $K_{III} \leq K_{IIIc}$ , we calculate the necessary maximum crack length of the main body with a straight crack under various wind loads in order to ensure that no torsional fracture occurs in the main body, as shown in figure. 7.

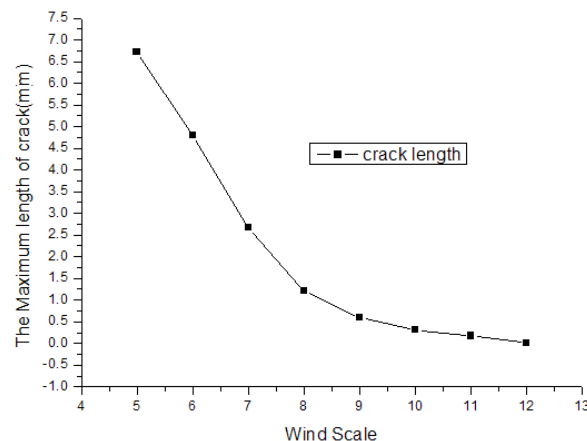


Figure. 7 The maximum crack length of main body under various wind scale

Figure.7 shows that, in order to ensure the spar's main body could not be destroyed, the allowed maximum length of straight boundary crack is more and more smaller with the increase of wind loads. When there is a fresh breeze, the allowed maximum length of straight boundary crack is 6.723mm, when the wind increased to hurricane, it is 0.01049mm. Thus, cracks have some effect on the safety of the spar main body in marine engineering. Especially in severe ocean environments, the risks of fatigue failure of the main body with cracks should not be overlooked. We should try to reduce the boundary crack length during the process of marine structure construction and usage.

## 6. Conclusions

Main body is the most important supporting component of a spar, its safety is very important. Among the factors influencing the platform, the wind load may cause torsion failure of the main body. Cracks are liable to occur during marine structure construction and usage. Therefore, it is necessary to investigate the torsion problem of a spar platform main body with boundary cracks under wind load.

We simplified the complicated structures of the main body to a simply cylinder. The displacement and stress on the boundary were expressed in terms of boundary integrals. The boundary element method was suggested. The torsion problem of a spar main body containing a straight boundary crack under wind load was calculated. The maximum crack length of main body under various wind scale was analyzed. Results show that cracks in the spar's main body should not be overlooked.

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