

# A Three-level Location-inventory Problem With Perishable Product

Zhengqiang Wu, Weili Li and Liying Song\*

School of traffic and transportation, Beijing Jiaotong University, Beijing, 100044, China

\*Liying Song's e-mail: lysong@bjtu.edu.cn

**Abstract.** This article studies a three-level multi-product location-inventory problem in which location, allocation and inventory decisions are taken in a three-level supply chain including plants, distribution centers and retailers. The direct shipment from plant to retailer is allowed and two-echelon inventory and demand correlation among retailers are considered in this supply chain network. A mixed integer nonlinear programming formulation is presented to describe the problem and trade off the costs of setup, transportation, inventory and deterioration. Then it is transformed to a conic quadratic mixed integer programming formulation and solved with a standard optimization software packages-CPLEX. In addition, we demonstrate that the distribution network allowing for direct shipment from plant to retailer is more optimal than the distribution network ignoring it in the three-level supply chain network with perishable product.

## 1. Introduction

The competition among enterprises is becoming violent in an increasingly open market environment, in which enterprises are with the pressure of shortening the delivery time, improving the quality of the products and service and reducing the cost. An effective supply chain network design can significantly reduce their cost and improve enterprises' overall competition. In general, the decisions of supply chain network design include strategic decisions involving the number and location of facilities (plants and distribution centers( DCs)) and operational decisions(assignment and inventory management decisions).

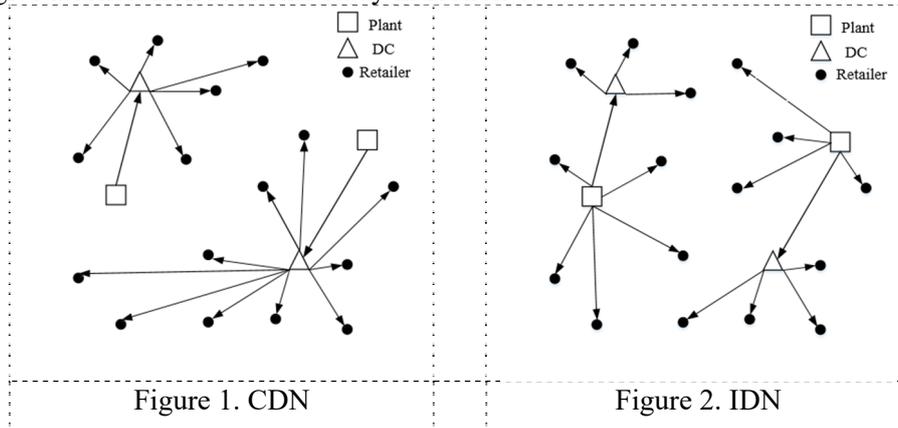
Historically, these decisions were separately studied, which would lead to suboptimal network structure. Integrated location-inventory problems were thus introduced, in which facility location, assignment and inventory management were integrated to minimize the total costs of facility setup, transportation and inventory in the whole supply chain network[1-4]. Usually, certain safety stocks are centrally maintained in DC to cope with variable demand of retailer. However, the inventory at retailer level was ignored in previous studies. In this paper, we address the more realistic issue of two-echelon inventory including retailer inventory and DC inventory.

In most studies, the products manufactured in plant, are centrally shipped to DC and then to retailer, which can effectively reduce transportation and inventory cost because of economies of scale and risk-pooling effects[5-7]. However, DC, as an intermediate link of supply chain network, increases the length of supply chain, which results in degrading the product quality and service level, especially perishable product. Therefore, in this paper, an integrated distribution network(IDN) is proposed to study location-inventory problem with perishable product. In common distribution network(CDN), DC servers connection between plant and retailer and retailer must be assigned DC(as shown in figure 1).



Unlike CDN, in IDN, retailer can be served by either single plant or single DC (as shown in figure 2). This assumption is valid in many supply chain network design problems. It may be possible to allow for direct shipment from upper layer facility to retailer [8].

The remainder of this paper is organized as follows. In section 2, the mathematical formulation of the problem is given. Section 3 presents numerical experiments on different scale network problem. Finally, we give a conclusion about the study in section 4.



## 2. Problem Description and Model

In this section, we firstly present a nonlinear integer programming model for the three-level multi-product location-inventory problem with perishable product where demand correlation between retailers and two-echelon inventory are considered and direct shipment from plant to retailer is allowed in IDN. In this network, given a set of potential plants and DCs, and a set of retailers, products are manufactured in plants and finally delivered to retailers through DCs or not. The aim of the model is to minimize the total costs of supply chain, which consists of the facility location cost, transportation cost, inventory cost and deterioration cost. Then, to tackle the nonlinear programming model, it is transformed into the mix integer conic quadratic programming formulation. To model this problem, some notations are defined below:

### Indices

$i, l$  Index for retailers (1, ...,  $I$ )

$j$  Index for DCs (1, ...,  $J$ )

$k$  Index for plants (1, ...,  $K$ )

$f$  Index for products (1, ...,  $F$ )

### Parameters

$A_{fj}$  Per order fixed ordering cost for product  $f$  at DC  $j$

$a_{fi}$  Per order fixed ordering cost for product  $f$  at retailer  $i$

$H_{fj}$  Per unit per cycle inventory holding cost for product  $f$  at DC  $j$

$h_{fi}$  Per unit per cycle inventory holding cost for product  $f$  at retailer  $i$

$Cap_{fj}$  Daily throughput capacity for product  $f$  at DC  $j$

$l_{fjk}$  Order lead time for product  $f$  in days from plant  $k$  to DC  $j$

$l_{fik}$  Order lead time for product  $f$  in days from plant  $k$  to retailer  $i$

$\lambda$  Number of working days per cycle

$p_k$  Per cycle fixed setup cost of plant  $k$

$q_{jk}$  Per cycle fixed setup cost of DC  $j$  assigned to plant  $k$

$t_{fij}, t_{fik}, t_{fjk}$  Per-unit transportation cost for product between two facilities

$\mu_{fi}$  Mean daily demand for product  $f$  at retailer  $i$

$\sigma_{fi}$  Standard deviation of daily demand for product  $f$  at retailer  $i$

$\rho_{il}$  Correlation coefficient of daily demand between retailer  $i$  and retailer  $l$

$\theta_f$  Deterioration rate of product  $f$  in transit between two facilities

$O_f$  Per-unit deterioration cost for product  $f$

$Z_\alpha$   $\alpha$ -percentile of the standard normal distribution

#### Variables

$Q_{fj}$  Optimal order quantity for product  $f$  at DC  $j$

$Q_{fi}$  Optimal order quantity for product  $f$  at retailer  $i$

#### Decision variables

$x_{ij} \in \{0,1\}$  Take value 1 if retailer  $i$  is assigned to DC  $j$ ; 0 otherwise

$y_{jk} \in \{0,1\}$  Take value 1 if DC  $j$  is assigned to plant  $k$ ; 0 otherwise

$z_{ik} \in \{0,1\}$  Take value 1 if retailer  $i$  is assigned to plant  $k$ ; 0 otherwise

$w_k \in \{0,1\}$  Take value 1 if plant  $k$  is opened; 0 otherwise

Assume that product  $f$  at retailer  $i$  has an uncertain demand that follows a normal distribution with mean  $\mu_{fi}$  and variance  $\sigma_{fi}$ . Considering the correlation of product demand between retailers and deterioration rate in transit between DC and retailer, the daily demand of product  $f$  at DC  $j$  is defined to follow a multivariate normal distribution with mean  $D_{fj} = \sum_i \mu_{fi} x_{ij} / (1 - \theta_f)$  and variance  $U_{fj} = \sum_i \sum_l \rho_{il} \sigma_{fi} \sigma_{fl} x_{ij} x_{il}$ . According to the inventory theory, the order lead times for product  $f$  at DC  $j$  and at retailer  $i$  can be expressed as  $L_{fj} = \sum_k l_{fjk} y_{jk}$  and  $L_{fi} = \sum_k l_{fik} z_{ik}$  respectively.

### 2.1. Mixed integer nonlinear programming formulation for integrated distribution network

#### Assumption

- No inventory control at plant is considered and a safety stock is held at DC or retailer.
- Each DC is assigned to a single plant and each retailer is assigned to single DC or single plant.
- Each DC and the retailer assigned to plant adopt the continuous review inventory  $(R, Q)$  policy.
- The service level is high enough to ignore the stock out cost.
- Unit cost of shipping the same kind of product is proportional to the Euclidean distance between two facilities.
- Each DC has a finite handing capacity and each plant isn't subject to any capacity restriction.
- The deterioration cost of product in transit is considered. The deterioration rate of product is a constant in transit between any two facilities.

#### Formulation

The inventory costs of DC include holding working inventory cost, ordering cost and safety stock cost, given as:

$$\sum_f \sum_j \frac{Q_{fj} H_{fj}}{2} + \sum_f \sum_j \frac{\lambda A_{fj} D_{fj}}{Q_{fj}} + \sum_f \sum_j Z_\alpha H_{fj} (U_{fj} L_{fj})^{1/2} \quad (1)$$

Where  $\alpha$  is the probability for stock-out during the lead times. At the level of retailer, there are two following situations. The first is when retailer is assigned to DC, which can place a group order and centrally hold the safety stock. Only holding working inventory cost is thus considered at retailer, given as:

$$\sum_f \sum_i \sum_j \frac{h_{fi} \mu_{fi} l_{fij} x_{ij}}{2} \quad (2)$$

Second, when assigned to plant, retailer maintains safety stock and places order individually. The inventory costs at the retailer assigned to plant include holding working inventory cost, ordering cost and safety stock cost, obtained as:

$$\sum_f \sum_i \sum_k \frac{Q_{fi} h_{fi} z_{ik}}{2} + \sum_f \sum_i \sum_k \frac{\lambda a_{fi} \mu_{fi} z_{ik}}{Q_{fi} (1 - \theta_f)} + \sum_f \sum_i Z_\alpha h_{fi} \sigma_{fi} L_{fi}^{1/2} \quad (3)$$

The optimal value  $Q_{fj}^*$  and  $Q_{fi}^*$  can be obtained by differentiating the equation (1) and equation (3) with respect to  $Q_{fj}$  and  $Q_{fi}$  respectively:

$$Q_{fj}^* = \left( \frac{2\lambda A_{fj} \sum_i \mu_{fi} x_{ij}}{H_{fj}(1 - \theta_f)} \right)^{1/2} \tag{4}$$

$$Q_{fi}^* = \left( \frac{2\lambda a_{fi} \mu_{fi}}{h_{fi}(1 - \theta_f)} \right)^{1/2} \tag{5}$$

Therefore, substituting  $Q_{fj}^*$  and  $Q_{fi}^*$ , the total inventory costs of supply chain can be written as:

$$\sum_f \sum_j \left( \frac{2\lambda A_{fj} H_{fj} \sum_i \mu_{fi} x_{ij}}{1 - \theta_f} \right)^{1/2} + \sum_f \sum_j Z_\alpha H_{fj} \left( \sum_i \sum_l \sum_k \rho_{il} \sigma_{fi} \sigma_{fl} l_{fjk} x_{ij} x_{lj} y_{jk} \right)^{1/2} \tag{6}$$

$$+ \sum_f \sum_i \sum_j \frac{h_{fi} \mu_{fi} l_{fij} x_{ij}}{2} + \sum_f \sum_i \sum_k \left( \frac{2\lambda a_{fi} h_{fi} \mu_{fi}}{1 - \theta_f} \right)^{1/2} z_{ik} + \sum_f \sum_i Z_\alpha h_{fi} \sigma_{fi} \left( \sum_k l_{fik} z_{ik} \right)^{1/2}$$

The three-level multi-product location-inventory model (P1) with perishable product for IDN is as follows:

P1

$$\text{Min} \sum_k p_k w_k + \sum_j \sum_k q_{jk} y_{jk} \tag{7}$$

$$+ \sum_f \sum_i \sum_j \sum_k \frac{\lambda \mu_{fi} t_{fjk} x_{ij} y_{jk}}{(1 - \theta_f)^2} + \sum_f \sum_i \sum_j \frac{\lambda \mu_{fi} t_{fij} x_{ij}}{1 - \theta_f} + \sum_f \sum_i \sum_k \frac{\lambda \mu_{fi} t_{fik} z_{ik}}{1 - \theta_f} \tag{8}$$

$$+ \sum_f \sum_j \left( \frac{2\lambda A_{fj} H_{fj} \sum_i \mu_{fi} x_{ij}}{1 - \theta_f} \right)^{1/2} + \sum_f \sum_j Z_\alpha H_{fj} \left( \sum_i \sum_l \sum_k \rho_{il} \sigma_{fi} \sigma_{fl} l_{fjk} x_{ij} x_{lj} y_{jk} \right)^{1/2} \tag{9}$$

$$+ \sum_f \sum_i \sum_j \frac{h_{fi} \mu_{fi} l_{fij} x_{ij}}{2} + \sum_f \sum_i \sum_k \left( \frac{2\lambda a_{fi} h_{fi} \mu_{fi}}{1 - \theta_f} \right)^{1/2} z_{ik} + \sum_f \sum_i Z_\alpha h_{fi} \sigma_{fi} \left( \sum_k l_{fik} z_{ik} \right)^{1/2} \tag{10}$$

$$+ \sum_f \sum_i \sum_j \sum_k \frac{\lambda O_f \theta_f \mu_{fi} x_{ij} y_{jk}}{(1 - \theta_f)^2} + \sum_f \sum_i \sum_j \frac{\lambda O_f \theta_f \mu_{fi} x_{ij}}{1 - \theta_f} + \sum_f \sum_i \sum_k \frac{\lambda O_f \theta_f \mu_{fi} z_{ik}}{1 - \theta_f} \tag{11}$$

Subject to:

$$y_{jk} \leq w_k, \quad \forall j, k \tag{12}$$

$$z_{ik} \leq w_k, \quad \forall i, k \tag{13}$$

$$x_{ij} \leq \sum_k y_{jk}, \quad \forall i, j \tag{14}$$

$$\sum_i \mu_{fi} x_{ij} \leq Cap_{fj}, \quad \forall f, j \tag{15}$$

$$\sum_j x_{ij} + \sum_k z_{ik} = 1, \quad \forall i \tag{16}$$

$$\sum_k y_{jk} \leq 1, \quad \forall j \tag{17}$$

$$x_{ij}, y_{jk}, z_{ik}, w_k \in \{0, 1\} \quad \forall i, j, k \tag{18}$$

The above model is to minimize the total cost in three-level supply chain which is comprised of fixed setup costs(7), transportation costs(8), inventory costs(9-10) and deterioration costs(11). Equations (12)-(14) represent that each retailer and each DC can only be assigned to one of their opened providers. Equation (15) states the daily throughput capacity for each DC. Equations (16) and (17) assure the single-sourcing strategy for each retailer and each DC. Equation (18) enforces the binary variables restrictions.

## 2.2. Mixed integer conic quadratic programming formulation.

In this section, we introduce some mathematical methods to transform the mixed integer nonlinear programming model P1 into a conic quadratic mixed integer program (CQMIP). The CQMIP about integrated location-inventory optimization can be efficiently solved directly using standard optimization software packages such as CPLEX[9-10]. To formulate the P1 as COMIP, the objective function needs to be linearized, which denotes that the quadratic terms and square-root terms including decision variable should be substituted in linearized component.

We can introduce a new binary variable to replace the quadratic term and add two new constraints.

$$\sum_k M_{ijk} = x_{ij} \quad \forall i, j \quad (19)$$

$$M_{ijk} \leq y_{jk} \quad \forall i, j, k \quad (20)$$

The quadratic term  $x_{ij}y_{jk}$  takes value 1 if and only if  $x_{ij} = y_{jk} = 1$  and the above equations effectively make  $M_{ijk}$  to be 1 if and only if  $x_{ij} = y_{jk} = 1$ , so  $M_{ijk}$  can replace the  $x_{ij}y_{jk}$ . Particularly, because  $x_{ij}, y_{jk}$  and  $M_{ijk}$  are binary variables, it can be expressed in their quadratic form (as  $x_{ij}^2 = x_{ij}, y_{jk}^2 = y_{jk}, M_{ijk}^2 = M_{ijk}$ ). In addition, three positive auxiliary variables ( $v_{1fj}, v_{2fj}, v_{3fi}$ ) are introduced to remove the two complicated square-root terms in the objective function. The model P1 can be transformed into mixed integer conic quadratic programming formulation(P2) as below:

P2

$$\begin{aligned} \text{Min} \quad & \sum_k p_k w_k + \sum_j \sum_k q_{jk} y_{jk} + \sum_f \sum_i \sum_j \sum_k \frac{\lambda \mu_{fi} t_{fjk} M_{ijk}}{(1 - \theta_f)^2} + \sum_f \sum_i \sum_j \frac{\lambda \mu_{fi} t_{fij} x_{ij}}{1 - \theta_f} \\ & + \sum_f \sum_i \sum_k \frac{\lambda \mu_{fi} t_{fik} z_{ik}}{1 - \theta_f} + \sum_f \sum_j v_{1fj} + \sum_f \sum_j Z_\alpha H_{fj} v_{2fj} + \sum_f \sum_i \sum_j \frac{h_{fi} \mu_{fi} l_{fij} x_{ij}}{2} \\ & + \sum_f \sum_i \sum_k \left( \frac{2\lambda a_{fi} h_{fi} \mu_{fi}}{1 - \theta_f} \right)^{1/2} z_{ik} + \sum_f \sum_i Z_\alpha h_{fi} \sigma_{fi} v_{3fi} + \sum_f \sum_i \sum_j \sum_k \frac{\lambda O_f \theta_f \mu_{fi} M_{ijk}}{(1 - \theta_f)^2} \\ & + \sum_f \sum_i \sum_j \frac{\lambda O_f \theta_f \mu_{fi} x_{ij}}{1 - \theta_f} + \sum_f \sum_i \sum_k \frac{\lambda O_f \theta_f \mu_{fi} z_{ik}}{1 - \theta_f} \end{aligned} \quad (21)$$

Subject to:

Equations:(12)-(20)

$$\frac{2\lambda A_{fj} H_{fj} \sum_i \mu_{fi} x_{ij}}{1 - \theta_f} \leq v_{1fj}^2, \quad \forall f, j \quad (22)$$

$$\sum_i \sum_l \sum_k \rho_{il} \sigma_{fi} \sigma_{fl} l_{fjk} M_{ijk} M_{ljk} \leq v_{2fj}^2, \quad \forall f, j \quad (23)$$

$$\sum_k l_{fik} z_{ik} \leq v_{3fi}^2, \quad \forall f, i \quad (24)$$

$$v_{1fj}, v_{2fj}, v_{3fi} \geq 0, \quad \forall f, i, j \quad (25)$$

$$M_{ijk} \in \{0,1\}, \quad \forall i, j, k \quad (26)$$

The model of CDN can be obtained by neglecting the terms about decision variable  $z_{ik}$  in the model P2 and the details of CDN's model are no longer described in this paper.

### 3. Numerical experiment

In this section, the transformed model P2 is setup in GAMS 23.8.2 and solved using a standard software packages-CPLEX. Since P2 is second order conic quadratic programs, CPLEX can guarantee global optimality for the model.

#### 3.1. Data generation

In this paper, the required data has been randomly generated based on the strategy proposed by Park et al.[11]. The locations of retailers, potential plants and DCs are uniformly distributed in the square of (0, 10]. The delivery costs are calculated to be proportional to Euclidean distance in the plane.  $V_1$ ,

$V_2, V_3$  is the value of per unit per distance delivery costs from plant to DC, from plant to retailer and from DC to retailer respectively. The demand correlation coefficient ( $\rho_{il}$ ) belongs to (0, 1]. Also,  $\alpha$  is 95%, and so  $Z_\alpha = 1.65$ . The other required data is generated in the way as shown in table 1. In this table,  $\bar{\mu}_f$  denotes the average value of the mean retailer demands for product  $f$ ,  $\tau$  refers to the percentage of variation,  $\bar{c}_0$  refers to the average value of the per unit delivery cost between plant and DC,  $\bar{c}_1$  denotes that between plant and retailer,  $f_0$  means the number of product types,  $\bar{q}$  refers to the average value of  $q_{jk}$  for all  $j$  and  $k$ . We set the base case parameters  $V_1 = 0.5, V_2 = V_3 = 1, \mu_0 = 10, \sigma_0^2 = 6, C_0 = 8, h_0 = 3, h_1 = 4, A_0 = 10, \tau = 0.5, q_0 = 8, \lambda = 120$ . In addition, the test problems are given in table 2, which consists of different combination of products, plants, DCs and retailers.

Table 1. Data generation

$\mu_{fi} = \mu_0 \cdot U[1,5]$	$\sigma_{fi} = (\sigma_0^2 \cdot U[1,5])^{1/2}$
$Cap_{fj} = 3 \cdot C_0 \cdot \bar{\mu}_f \cdot (1 + U[-\tau, \tau])$	$\theta_f = U[0.05, 0.1]$
$O_f = U[5, 10]$	$H_{fj} = h_0 \cdot (1 + U[-\tau, \tau])$
$h_{fi} = h_1 \cdot (1 + U[-\tau, \tau])$	$l_{fjk} = 3 \cdot U[1, 5]$
$l_{fik} = 3 \cdot U[1, 5]$	$l_{fij} = U[1, 5]$
$A_{fj} = A_0 \cdot \bar{c}_0 \cdot \bar{\mu}_f \cdot (1 + U[-\tau, \tau])$	$A_{fi} = (A_0 + 2) \cdot \bar{c}_1 \cdot \bar{\mu}_f \cdot (1 + U[-\tau, \tau])$
$q_{jk} = q_0 \cdot f_0 \cdot \bar{c}_1 \cdot \sum_f Cap_{fj} \cdot (1 + U[-\tau, \tau])$	$p_k = 3 \cdot \bar{q} \cdot (1 + U[-\tau, \tau])$

Table 2. Test networks

Problem instance	PB1	PB2	PB3	PB4	PB5	PB6	PB7	PB8	PB9	PB10	PB11
$F$	3	3	3	3	3	5	5	5	5	5	5
$K$	3	3	3	3	3	3	5	5	5	5	5
$J$	5	5	5	10	10	5	5	10	10	10	10
$I$	15	20	30	30	40	20	20	20	40	60	80

### 3.2. Numerical experiments

The goal of this part of the numerical experiments is to examine the benefits of IDN considering direct shipment from plant to retailer for a location-inventory problem with perishable product. The effect of IDN has been demonstrated by calculating the savings achieved in the total cost of the network considering and ignoring direct shipment from plant to retailer. Savings in the total cost can be achieved by calculating the ratio of difference for objective function in CDN and IDN to objective function in CDN.

**3.2.1. Effect of IDN.** This part of numerical experiment proves that new IDN is more competitive than CDN because of lower cost across different scale supply chain network with perishable product. The results presented in table 3 where the total cost in IDN, the total cost in CDN and savings for all test network are reported. It is clear that the total cost in IDN is generally lower than CDN's. The savings of PB7 is 0.54% that is minimum for all test instance, which indicates that IDN and CDN have almost same rational network structure leading to nearly same cost in this network. However, the savings in several test networks is relatively high. In PB1, PB2, PB5 and PB6, for example, the savings is beyond 5% and the highest is up to 10.79%, which states that it is of great significance for optimization of network structure to allow direct shipment from plant to retailer in three-level supply chain design with perishable product.

Table 3. Savings

Problem instance	PB1	PB2	PB3	PB4	PB5	PB6	PB7	PB8	PB9	PB10	PB11
Total cost of IDN(million ¥)	1.10	1.47	1.98	1.97	2.69	2.75	2.55	2.53	5.35	6.87	9.65
Total cost of CDN(million ¥)	1.18	1.59	2.03	2.06	2.83	3.08	2.57	2.59	5.60	7.19	9.82
Savings(%)	6.38	7.43	2.62	4.38	5.05	10.79	0.54	2.39	4.51	4.39	1.80

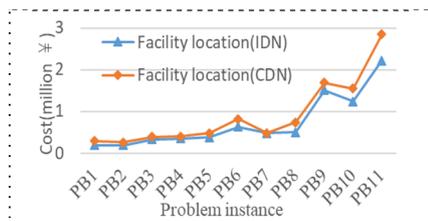


Figure 3. Total facility location costs of IDN vs. CDN

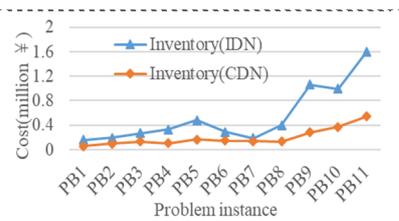


Figure 4. Total inventory costs of IDN vs. CDN

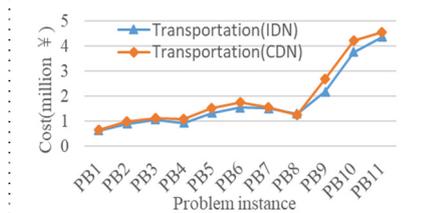


Figure 5. Total transportation costs of IDN vs. CDN

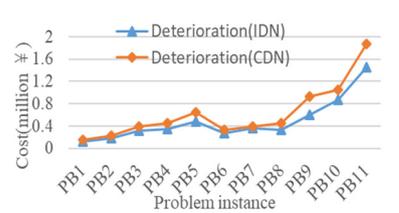


Figure 6. Total deterioration costs of IDN vs. CDN

Table 4. Number of DC location

Problem instance	PB1	PB2	PB3	PB4	PB5	PB6	PB7	PB8	PB9	PB10	PB11
Number of DC location in IDN	2	1	2	2	3	1	2	1	1	3	3
Number of DC location in CDN	2	2	3	3	4	2	2	2	4	4	5

Furthermore, we analyze the difference on concrete costs including the costs of facility location, transportation, inventory and deterioration between IDN and CDN. From figure 3, figure 5 and figure 6, we can see that the costs of facility location, transportation and deterioration in IDN are generally lower than that in CDN, while IDN leads to higher inventory cost compared to CDN, which presented in figure 4. The costs saved in facility location, transportation and deterioration are almost higher than extra inventory cost. That is why IDN can lead to lower total cost of system compared to CDN.

The trade-offs between various costs in this problem are multiple and complex. We make a specific analysis from facility location, transportation, inventory and deterioration costs respectively as follows:

First, it is clear that the costs of facility location in IDN tend to be lower than that in CDN for all test problem in figure 3. This is because the number of DCs in CDN is usually more than that in IDN (see table 4). In CDN, retailer must be served by DC and decision-maker locates more DCs in order to achieve economies of scale and centralize safety stocks, which can reduce total transportation and inventory cost. In contrast, in IDN, retailer can be served either plant or DC, which leads to locate less DCs.

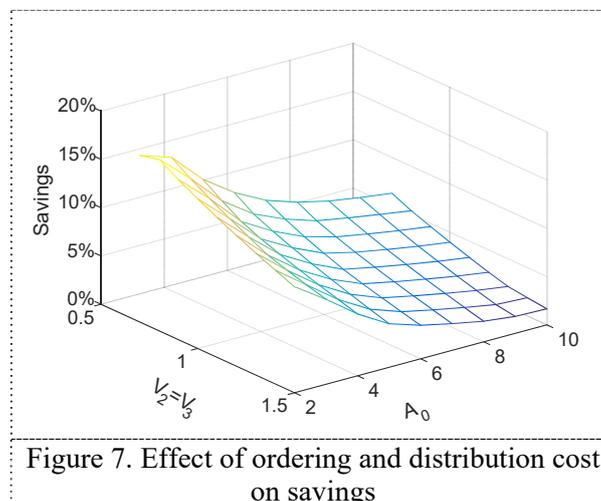
Second, the distance between retailer and plant is usually larger than that between retailer and DC, but smaller than total transportation distance (the sum of distance between plant and DC and distance

between DC and retailer)[12]. However, because the shipment size between retailer and plant is relatively smaller, per-unit transportation cost is higher. In figure 5, the transportation cost in IDN is generally lower than that in CDN, which indicates that, for the retailers assigned to plant, the distance between retailer and plant in IDN is small enough to ignore the relative higher unit transportation cost.

Third, the total safety stock cost of centralized mode, that safety stock is maintained at DC, is lower than that of decentralized mode that every retailer maintains its own safety stock. Therefore, the total safety stock cost in IDN are higher than that in CDN. In addition, in IDN, because retailer orders product and holds safety stock individually, the number of orders in IDN is far more than that in CDN, which will lead to higher order cost that is proportional to the number of orders. The total inventory cost in IDN are thus higher than that in CDN, which can be clearly seen in figure 4.

Fourth, it is clear that, for all test network, the deterioration cost can be reduced in IDN compared to that in CDN, which is shown in figure 6. In IDN, we allow that retailer can order perishable product from plant directly, thus reducing the traditional intermediate circulation. Naturally, the total deterioration cost of system in IDN is smaller.

*3.2.2. Effect of ordering and distribution cost on savings.* This part of the numerical experiment is designed to examine the effect of ordering and distribution cost on savings. We set up PB9 as a test network. From figure 7, a conclusion can be drawn that the savings increases with the decrease of DCs and retailers' ordering cost and per unit per distance distribution costs from plant/DC to retailer. The retailers assigned to plant need to place orders individually, which will increase the total number of orders. Therefore, compared to CDN, a decreasing of fixed order cost can reduce more cost in IDN. Today, the fixed cost of placing orders sharply reduces because of ecommerce, which make IDN meaningful in practice. In addition, the total distribution mileage from plant to retailer and from DC to retailer is larger in IDN. Thus, the savings with smaller unit distribution cost presents larger.



#### 4. Conclusion

This paper studies a location-inventory problem in an integrated distribution network, in which direct shipment from plant to retailer and multi-level inventory are considered. The proposed problem is formulated as a nonlinear integer programming model, which can simultaneously make three types of decisions: (i) the number and location of plants and DCs, (ii) the assignment of retailers to either located plants or located DCs and located DCs to located plants, (iii) the inventory control decisions at each located DCs and retailers assigned to located plants. The nonlinear integer programming model is transformed to a mixed integer conic quadratic program, which can be solved directly using commercial optimization solver-CPLEX. The numerical experiment is firstly designed to prove that integrated DC network allowing direct shipment from plant to retailer is superior to common

distribution network ignoring direct shipment from plant to retailer. Second, we analyze the difference of the facility location, transportation, inventory and deterioration costs between two modes of distribution network. Through analysis of numerical result, we demonstrate that the structure of IDN is more optimal than CDN's in the three-level supply chain network with perishable product.

## Appendices

### Acknowledgments

This work was supported by the National Natural Science Foundation of China (project T14A100051). The authors want to express their gratitude to the anonymous referees for their helpful comments.

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