

# Intercity Multi-modal Traffic Assignment Model and Algorithm for Urban Agglomeration Considering the Whole Travel Process

Pu Xu<sup>1\*</sup>

<sup>1</sup> School of Traffic and Transportation, Beijing Jiaotong University, Beijing, 100044, China

\*Corresponding author's e-mail: 16120755@bjtu.edu.cn

**Abstract.** Based on the full analysis of the whole process of intercity travel of urban agglomerations, a super network model of urban agglomeration intercity multi-mode traffic system is constructed. The super network link is divided into travel link, transfer link, boarding link and alighting link, in which the travel link includes the travel link on the urban transportation network and the travel link on the intercity transportation network. Considering the factors of travel time, cost and degree of comfort, the generalized cost function and generalized hyper-path cost function are established for all links. On the basis of this, the stochastic equilibrium assignment model of urban agglomeration intercity multi-modal transportation system is proposed and the corresponding algorithm is designed based on MSA algorithm. Finally, an example is presented to illustrate the proposed method.

## 1. Introduction

The development of Chinese urban agglomerations has entered the fast lane. The “13th Five-Year Plan” proposes to build 19 urban agglomerations such as Beijing-Tianjin-Hebei, Yangtze River Delta and Pearl River Delta. In the future, the urban agglomeration intercity passenger traffic volume will increase greatly, so the comprehensive transportation network structure is urgently needed as support to meet the increasing demand, and thus it is of great practical significance to forecast the sharing of multiple traffic modes according to the characteristics of intercity passenger trips.

For urban transportation systems, the transportation network structure is the premise and basis for traffic system analysis. At present, the description of the network topology of single mode has been relatively mature at home and abroad. Most of these studies are based on graph theory by transforming the single-mode road network into a network structure consisting of nodes and edges. However, with the diversified development of urban transport modes, the transportation network structure has become more and more complex and simple network topology has been unable to describe complex multi-modal transportation systems. Chen and Si et al. describe a multi-modal urban transportation system using a hierarchical network structure [1, 2]. In this structure, each layer represents a running network of one mode. This structure can be well described the operational characteristics of each mode but cannot describe the relationship between different modes. Lozano and Storchi have established a super network for urban multi-modal transportation systems based on super network theory [3,4], which is a network of multi-layer independent networks. Each layer of independent network is a running network of a certain transport mode, which can not only express the characteristics of the independent operation of each mode, but also describe the connection between



different modes. Based on the full analysis of the intercity passenger travel process of urban agglomerations, this paper constructs a super network model of urban agglomeration intercity multi-modal transportation system by using topological structure and establishes a stochastic user equilibrium model to determine the share amount of various intercity transport modes.

## 2. Intercity passenger travel process and traffic network model for urban agglomerations

Urban agglomeration intercity trip is the whole process of traveling in a single direction through urban roads and intercity transportation lines by using one or several modes from one place in a city to another place in the other city in the urban agglomeration in order to achieve a certain living or production purpose. Passengers' travel between cities, as the middle stage of intercity travel, cannot exist alone. It must be linked to the urban travel of two ends of the city to complete the trip. Therefore, the study of urban agglomeration intercity travel should be based on the whole process of passenger travel so as to grasp the law of travel as a whole. The whole process of urban intercity passenger travel is divided into three stages: the first stage is the urban travel in the starting city: passengers reach a certain external transport hub through the urban traffic network from the starting point of a traffic district in city A. The second stage is the intercity trip between city A and city B: passengers reach the external transport hub of city B via the intercity transportation line from the external transport hub in city A. The third stage is the urban travel in the destination city: passengers reach the destination of a traffic district in city B through the urban traffic network from the external transport hub of city B.

The intercity traffic system is a multi-level comprehensive traffic network which consists of the main traffic network of the starting and destination cities and the intercity traffic network. The traveller's trip from the origin to the destination generally includes the boarding process, the travel process, the transfer process and the alighting process. According to the trip characteristics of travellers, this paper constructs a super network structure to describe the intercity transportation system of urban agglomeration. In the super network structure, the urban trunk traffic network and the intercity traffic network are connected by transfer nodes of external traffic hubs. Meanwhile, the urban trunk traffic network and the intercity traffic network are composed of multi-modal subnets. Therefore, the super network is a system composed of multilevel and multi-modal sub networks, which can be expressed as:

$$ST = (N, A, M) \quad (1)$$

$$N = N_1 \cup N_2 \quad (2)$$

$$A = A_1 \cup A_2 \cup A_3 \cup A_4 \quad (3)$$

$$A_2 = A_u \cup A_i \quad (4)$$

$$M = M_1 \cup M_2 \quad (5)$$

Where:  $N$  is the set of all nodes in the super network;  $A$  is the set of all links;  $M$  is the set of traffic modals;  $N_1$  is the set of starting and ending nodes,  $N_2$  is the set of nodes except starting and ending points;  $A_1, A_2, A_3, A_4$  represents the set of boarding link, travel link, transfer link and alighting link respectively, and the travel link  $A_2$  includes urban links  $A_u$  and intercity links  $A_i$ ;  $M_1$  is the set of the urban transport modes and  $M_2$  is the set of intercity transport modes.

The following is an example to illustrate the structure of the urban agglomeration intercity transportation system. Figure 2 shows an example of an intercity transportation network structure from city A to city B within a certain urban agglomeration. The origin is a residential area in city A and the destination is a business area in city B. From the starting point of city A to the destination of city B, the available urban transportation modes are cars, buses and subways, which are represented by numbers 1, 2, and 3, respectively. Intercity transportation modes options include cars, trains and intercity buses, which are represented by *Car*, *Train* and *Intercity bus* respectively. Travellers can transfer from urban traffic network to intercity traffic network or from intercity traffic network to urban traffic network via external transport hubs, among which the node 3 and 10 are the cities' car

hubs, the node 5 and 14 are the cities' railway hubs, and the node 9 and 16 are the cities' intercity bus hubs, respectively.

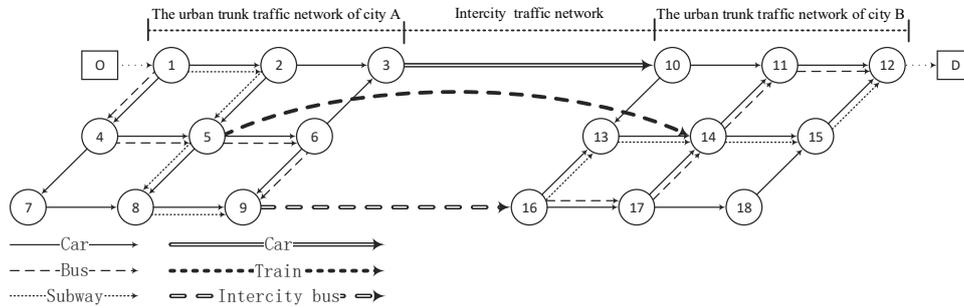


Figure 1. Physical structure map of urban agglomeration intercity multi-modal traffic network

According to the urban agglomeration intercity traffic network in Figure 1, the super network is constructed as shown in Figure 2. The letters *C*, *S*, *B*, *R*, and *I* represent car, subway, bus, train, and intercity bus respectively. In this paper, the path from the city's starting point to the city's destination in Figure 3 is called hyper-path. The transfer times of every hyper-path is two which includes the transfer from urban transportation network to intercity transportation network in the origin city and the transfer from intercity transportation network to urban transportation network in the destination city.

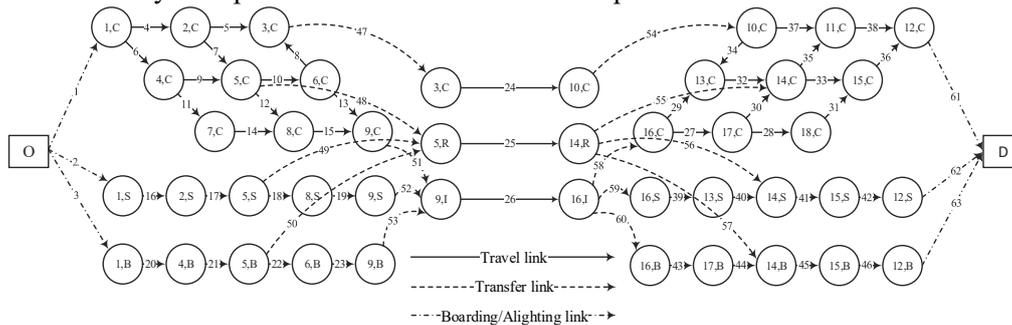


Figure 2. Urban agglomeration intercity multi-modal travel super network

### 3. Generalized travel cost

Considering the whole process of intercity passenger trips in urban agglomerations, the links of above super network models are divided into boarding link, travel link, transfer link and alighting link. Travellers' different travel activities on various links lead to different definitions of generalized link costs. The calculation methods for the generalized cost of each types of links are given as follows.

#### 3.1 Boarding link

The cost of boarding link is the time spent for travelers to arrive at the urban transport network site from the starting point. For car drivers, the time for boarding is mainly the travel time of boarding link. For subway and bus travelers, the time for boarding is mainly the time to reach the site and the waiting time at the site. The cost of boarding link can be expressed as follows:

$$G_{a_1}^m = vot \cdot T_{a_1}^m \quad \forall m \in M_1, a_1 \in A_1 \quad (6)$$

where  $G_{a_1}^m$  is generalized cost of boarding link,  $vot$  is the time value and  $T_{a_1}^m$  is boarding time which can be formulated as follows

$$T_{a_1}^m = \begin{cases} T_{a_1} & m = 1 \\ T_{a_1}^{walk} + T_{a_1}^{wait} & m = 2 / 3 \end{cases} \quad (7)$$

where  $T_{a_1}$  is the driving time of cars on the boarding link,  $T_{a_1}^{walk}$  and  $T_{a_1}^{wait}$  are travelers' walking time and waiting time on boarding link respectively. In this paper we assume that  $T_{a_1}$ ,  $T_{a_1}^{walk}$  and  $T_{a_1}^{wait}$  are all fixed values.

### 3.2 Travel link

Travel links include travel links on urban traffic networks and travel links on intercity traffic networks. The generalized cost of travel links on urban traffic network highlights the time cost and traffic congestion cost. The generalized cost of travel links on intercity traffic network highlights the cost of time, money and comfort.

#### (1) Travel links on urban traffic network

The time cost of a car on a link can be determined by BPR function and its monetary cost is mainly fuel price. Here we assume that the link travel time of bus and subway is fixed, but the time cost is determined by the BPR function considering the time cost of the road is prolonged by the traffic congestion, and the corresponding monetary cost is mainly the line fare. This paper assumes that subway and bus are all single fares. The cost of travel links on urban traffic network can be expressed as follows:

$$G_{a_2}^m = \text{vot} \cdot T_{a_2}^m + P_{a_2}^m \quad \forall m \in M_1, a_2 \in A_u \quad (8)$$

where  $G_{a_2}^m$  is the generalized cost of travel links on urban traffic network,  $T_{a_2}^m$  and  $P_{a_2}^m$  respectively indicate the time and monetary cost of urban traffic modes on travel links, which can be formulated as follows:

$$T_{a_2}^m = \begin{cases} t_{a_2}^m \left\{ 1 + \alpha_m \left[ (x_{a_2}^m + x_{a_2}^{m0}) / C_{a_2}^m \right]^{\beta_m} \right\} & m = 1 \\ t_{a_2}^m \left\{ 1 + \alpha_m \left[ (x_{a_2}^m + x_{a_2}^{m0}) / C_{a_2}^m \right]^{\beta_m} \right\} & m = 2 / 3 \end{cases} \quad (9)$$

where  $t_{a_2}^m$  ( $m=1$ ) is the free flow travel time of car on link  $a_2$ ,  $t_{a_2}^m$  ( $m=2/3$ ) is the fixed travel time for mode  $m$  on link  $a_2$ . Due to the existence of a certain number of internal traffic flows on the urban transport network,  $x_{a_2}^{m0}$  is the existing traffic flow on the link and  $x_{a_2}^m$  is the flow of intercity trip on the link. Here we assume that the average number of car passenger is one.  $C_{a_2}^m$  is the capacity or design passenger volume of link  $a_2$ .  $\alpha_m$  and  $\beta_m$  are retardation factors.

$$P_{a_2}^m = \begin{cases} p_s \cdot l_{a_2} & m = 1 \\ p^m & m = 2 / 3 \end{cases} \quad (10)$$

Where  $p_s$  is the fuel price per unit mileage of a car,  $l_{a_2}$  is the distance of link  $a_2$ ,  $p^m$  is the fare of mode  $m$ .

#### (2) Travel links on intercity traffic network

For cars, the time cost can also be determined by BPR function, and the corresponding monetary costs are mainly fuel and tolls. For trains and intercity buses, it is assumed that they run strictly according to the timetable, so the running time is determined. However, as the number of passengers increases, the sense of comfort is reduced and the travel time is prolonged, so it is necessary to amplify the travel time to indicate the loss of comfort, and the monetary costs of trains and intercity buses are link fares. The cost of travel link on intercity traffic network can be expressed as follows:

$$G_{a_2}^m = \text{vot} \cdot T_{a_2}^m + P_{a_2}^m \quad \forall m \in M_2, a_2 \in A_i \quad (11)$$

where  $G_{a_2}^m$  is the generalized cost of travel links on intercity traffic network,  $T_{a_2}^m$  and  $P_{a_2}^m$  respectively indicate the time and monetary cost of intercity traffic modes on travel links, which can be formulated as follows:

$$T_{a_2}^m = \begin{cases} t_{a_2}^0 \left\{ 1 + \alpha \left[ (x_{a_2}^m + x_{a_2}^{m0}) / C_{a_2}^m \right]^{\beta} \right\} & m = \text{car} \\ T_m \cdot (1 + \rho) & m = \text{train / bus} \end{cases} \quad (12)$$

where  $t_{a_2}^0$  is the free flow travel time of car on link  $a_2$ . In order to simplify the model, the traffic flow of other types of vehicles is given and converted to the number of equivalent cars, as there are a considerable number of other types of vehicles on the intercity car network.  $x_{a_2}^{m0}$  is the flow of other types of vehicles on link  $a_2$ .  $x_{a_2}^m$  is the car flow of intercity trip on the link.  $C_{a_2}^m$  is the capacity of

link  $a_2$ .  $\alpha$  and  $\beta$  are retardation factors.  $T_m$  is the fixed running time of mode  $m$  on link  $a_2$ , and  $\rho$  is the penalty parameter which is formulated as follows:

$$\rho = \begin{cases} 0 & x_{a_2}^m \leq \eta C_{a_2}^m \\ b \cdot (x_{a_2}^m - \eta C_{a_2}^m) / \eta C_{a_2}^m & \eta C_{a_2}^m < x_{a_2}^m \leq C_{a_2}^m \\ b \cdot (x_{a_2}^m - \eta C_{a_2}^m) / \eta C_{a_2}^m + c \cdot (x_{a_2}^m - C_{a_2}^m) / C_{a_2}^m & x_{a_2}^m > C_{a_2}^m \end{cases} \quad (13)$$

where  $C_{a_2}^m$  is the design passenger carrying capacity for intercity traffic mode on link  $a_2$ . It is assumed that there is a proportional relationship between the design passenger carrying capacity and the number of seats for intercity traffic mode  $m$ .  $\eta$  is the scale factor and  $\eta C_{a_2}^m$  represents the number of seats.  $b$  and  $c$  are parameters.

$$P_{a_2}^m = \begin{cases} p_s \cdot l_{a_2} + r_{a_2} & m = car \\ p^m & m = train / bus \end{cases} \quad (14)$$

where  $p_s$  is the fuel price per unit mileage of a car,  $l_{a_2}$  is the distance of link  $a_2$ ,  $r_{a_2}$  is the toll on link  $a_2$ ,  $p^m$  is the fare of mode  $m$ .

### 3.3 Transfer link

The cost of the transfer link is the cost incurred by traveller transferring between urban transportation network and intercity transportation network, including the transfer time and the loss of comfort caused by the transfer. The transfer time is composed of the walking time of the travellers on transfer link and the transfer waiting time between different modes. The loss of comfort is related to the transfer time. The longer the time is, the higher the loss of comfort is. The cost of transfer link is defined as:

$$G_{a_3}^{transfer} = vot \cdot (T_{a_3}^{transfer} + S_{a_3}^{transfer}) \quad a_3 \in A_3 \quad (15)$$

where  $G_{a_3}^{transfer}$  is the cost of transfer link,  $T_{a_3}^{transfer}$  is transfer time and  $S_{a_3}^{transfer}$  is comfort loss, which are defined as follows:

$$T_{a_3}^{transfer} = T_{a_3}^{walk} + T_{a_3}^{wait} \quad (16)$$

where  $T_{a_3}^{walk}$  and  $T_{a_3}^{wait}$  are travelers' walking time and waiting time on transfer link respectively, in this paper we assume that  $T_{a_3}^{walk}$  and  $T_{a_3}^{wait}$  are both fixed values.

$$S_{a_3}^{transfer} = \gamma \cdot T_{a_3}^{transfer} \quad (17)$$

where  $\gamma$  is the transfer penalty parameter.

### 3.4 Alighting link

The cost of alighting link is the time cost of travellers travelling from urban traffic network site to destination. For car travellers, the alighting time is mainly the driving time on alighting link. For subway and bus travellers, the alighting time is mainly for walking down the network to reach the destination. The cost of alighting link can be expressed as:

$$G_{a_4}^m = vot \cdot T_{a_4}^m \quad \forall m \in M_1, a_4 \in A_4 \quad (18)$$

where  $G_{a_4}^m$  is the generalised cost of alighting link,  $T_{a_4}^m$  is the alighting time which is defined as follows:

$$T_{a_4}^m = \begin{cases} T_{a_4} & m = 1 \\ T_{a_4}^{walk} & m = 2 / 3 \end{cases} \quad (19)$$

where  $T_{a_4}$  is the driving time of the car on alighting link,  $T_{a_4}^{walk}$  is travelers' walking time on alighting link. Here we assume that  $T_{a_4}$  and  $T_{a_4}^{walk}$  are both fixed values.

In this paper, the cost of the hyper-path is defined as the superposition of the cost of all links made up of the path, which can be expressed as:

$$G_k^w = \sum_a G_a \cdot \delta_{a,k}^w \quad \forall k \in K_w, w \in W \quad (20)$$

where  $G_k^w$  represents the cost of hyper-path between OD pair  $w$ ,  $G_a$  is the cost of all types of links which can be obtained from equation 16 to 19.  $K_w$  is the set of all hyper-paths between OD pair  $w$ .  $W$  is the set of all OD pairs.  $\delta_{a,k}^w$  is a link-path incidence indicator, which equals one if link  $a$  is on route  $k$ , zero otherwise.

#### 4. Stochastic user equilibrium assignment model

A stochastic equilibrium assignment model is proposed to describe the traffic assignment problem in multimodal traffic networks.

$$\min Z(f) = \frac{1}{\theta} \sum_w \sum_k f_k^w \ln f_k^w + \sum_a \int_0^{x_a} G_a(x) dx \quad (21)$$

$$s.t. \begin{cases} \sum_k f_k^w = q^w & \forall w \in W \\ f_k^w \geq 0 & \forall w \in W, k \in K_w \end{cases} \quad (22)$$

$$x_a = \sum_w \sum_{k \in K_w} f_k^w \cdot \delta_{a,k}^w \quad \forall a \in A \quad (23)$$

where  $f_k^w$  is the flow of hyper-path  $k$  between OD pair  $w$ ,  $q^w$  is the demand for OD pair  $w$ ,  $\theta$  indicates the random characteristic of the model, and the greater the value, the more accurate the travelers' perception of the network is.  $G_a(\cdot)$  is the generalized cost of link  $a$ .

It can be proved that the optimal solution of model (21) ~ (24) satisfies the logit traffic assignment condition of equation (25). The proof can be found in [5].

$$f_k^w = \frac{\exp(-\theta G_k^w)}{\sum_l \exp(-\theta G_l^w)} \cdot q^w \quad \forall k, w, l \quad (24)$$

#### 5. Solution method

Method of Successive Average can be used to solve the model. The specific steps are as follows:

Step1 Initialize. Set  $x_a^0 = 0, \forall a$ . The cost of each link can be obtained according to the cost formula.

Step2 The traversal graph method is used to determine all paths between OD pairs and their costs  $G_k^w$ . The shortest path and its cost  $G_{k,min}^w$  can be obtained at the same time. The effective paths are searched from the traversal paths according to the criteria:  $G_k^w \leq (1 + \sigma)G_{k,min}^w, \sigma > 0$ . According to the logit model, a random assignment is carried out to get the flow of each link on the effective path and set the number of iteration  $n = 1$ .

Step3 Calculate the cost of each link according to the current traffic flow  $x_a^n$ , then calculate  $G_k^w$  of all paths between OD pairs and get  $G_{k,min}^w$ .

Step4 The effective paths are searched from all paths according to the criteria:

$$G_k^w \leq (1 + \sigma)G_{k,min}^w, \sigma > 0$$

Step5 According to the logit model, a random assignment is carried out to load the flow on effective paths and get auxiliary link flow  $y_a^n$ .

Step6 Update the current flow of each link:  $x_a^{n+1} = x_a^n + 1/n(y_a^n - x_a^n)$

Step7 If  $\sqrt{\sum (x_a^{n+1} - x_a^n)^2} \cdot (\sum x_a^n)^{-1} \leq \varepsilon$ , where  $\varepsilon$  is an acceptable error, then stop and output best solutions  $x_a^{n+1}$ . Otherwise, return to step 3.

#### 6. Numerical studies

Taking the urban agglomeration intercity traffic network structure shown in Figure 1 as an example, the model and algorithm are checked. This paper mainly simulates the calculation process, and the accuracy of the parameters is not high, so the parameters of this paper are set as follows:

(1)  $p_s = 0.6 \text{ yuan/km}$ ,  $p_2 = 1 \text{ yuan}$ ,  $p_3 = 2 \text{ yuan}$ ,  $r_{a_2} = 10 \text{ yuan}$ ,  $q = 5000$ ,  $vot = 10 \text{ yuan/h}$

(2)  $\sigma = 0.5, \gamma = 0.2, b = 0.1, c = 0.2, \eta = 0.8, \theta = 1, \varepsilon = 0.001$

(3)  $\alpha = 0.15, \beta = 4, \alpha_1 = 0.15, \beta_1 = 4, \alpha_2 = \alpha_3 = 0.1, \beta_2 = \beta_3 = 2$

Travel information of various modes is shown in Table 1 to Table 5.

Table 1. The travel information of subway and bus

link	$t_{a_2}^m$	$C_{a_2}^m$	$x_{a_2}^{m0}$												
16	0.067	1000	500	20	0.083	1000	300	39	0.067	1000	500	43	0.125	1000	300
17	0.05	1000	600	21	0.067	1000	400	40	0.1	1000	500	44	0.125	1000	300
18	0.05	1000	500	22	0.167	1000	300	41	0.125	1000	600	45	0.125	1000	400
19	0.067	1000	700	23	0.2	1000	300	42	0.125	1000	600	46	0.167	1000	300

Table 2. The travel information of car

link	$t_{a_2}^m$	$C_{a_2}^m$	$l_{a_2}$	$x_{a_2}^{m0}$	link	$t_{a_2}^m$	$C_{a_2}^m$	$l_{a_2}$	$x_{a_2}^{m0}$	link	$t_{a_2}^m$	$C_{a_2}^m$	$l_{a_2}$	$x_{a_2}^{m0}$
4	0.125	1000	5	500	12	0.067	1000	3	500	31	0.167	700	5	700
5	0.125	800	5	800	13	0.167	1000	4	600	32	0.125	800	5	800
6	0.1	700	4	700	14	0.083	700	5	400	33	0.167	1000	5	1000
7	0.1	1000	4	1000	15	0.167	800	5	500	34	0.125	1000	4	1000
8	0.2	800	4	800	27	0.1	800	3	800	35	0.067	800	3	800
9	0.083	1000	4	1000	28	0.125	1000	4	1000	36	0.167	800	5	800
10	0.125	800	4	800	29	0.1	800	3	400	37	0.167	800	5	800
11	0.083	700	4	500	30	0.067	1000	3	1000	38	0.125	1000	4	1000

Table 3. The travel information of intercity traffic modes

link	link information			link	link information			link	link information		
24	$t_{a_2}^0=1h$	$C_{a_2}^{car}=1500$	$x_{a_2}^{car0}=1000$	25	$T_{train}=0.5h$	$C_{a_2}^{train}=1000$		26	$T_{bus}=1.5h$	$C_{a_2}^{train}=1000$	

Table 4. The travel information of transfer link

link	$T_{a_3}^{walk}$	$T_{a_3}^{wait}$									
47	0	0	51	0.083	0.33	55	0.167	0	59	0.167	0.05
48	0.083	0.5	52	0.167	0.33	56	0.167	0.05	60	0.25	0.167
49	0.167	0.5	53	0.25	0.33	57	0.25	0.167			
50	0.25	0.5	54	0	0	58	0.083	0			

Table 5. The travel information of boarding or alighting link

link	$T_{a_1}^{walk}$	$T_{a_1}^{wait}$	link	$T_{a_1}^{walk}$	$T_{a_1}^{wait}$	link	$T_{a_1}^{walk}$	$T_{a_1}^{wait}$
1	0.083	0	3	0.167	0.167	62	0.25	0
2	0.25	0.05	61	0.083	0	63	0.167	0

First, the convergence of the MSA algorithm is analyzed. The convergence of the algorithm is expressed by the flow change of boarding link 1 and alighting link 62 in each iteration, and Figure 3 shows the convergence of the MSA algorithm with a fixed total demand. It can be seen that the algorithm has obvious convergence properties. After 59 iterations, the solution meets the convergence requirements.

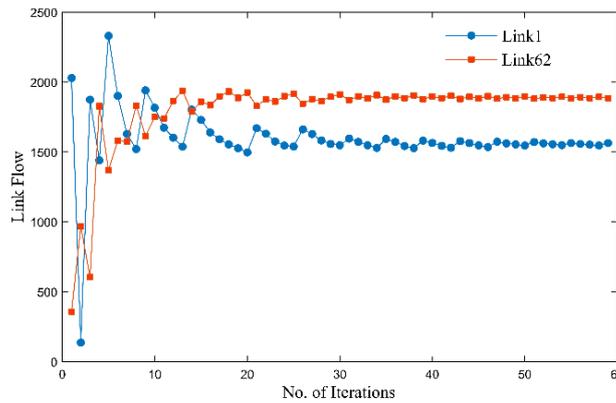


Figure 3. The convergence of algorithm

For the super network in the numerical studies, when the program achieves equilibrium, there are 74 effective hyper-paths in the super network. Figure 4 shows the traffic flow of intercity trips in various links when the super network reaches equilibrium. It can be seen that among various intercity traffic modes, 18.9% of travellers choose cars, 57.1% of travellers choose inter-city trains, and 24% of travellers choose intercity buses.

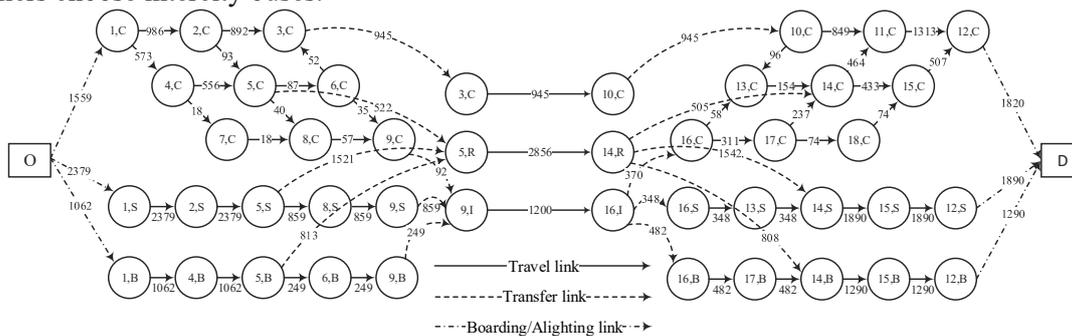


Figure 4. The link flow of super network at equilibrium

### 7. Conclusion

Based on the analysis of the urban agglomeration intercity travel process, this paper constructs an intercity multimode traffic super network model which includes the main traffic network of both origin and destination cities and the intercity traffic network. Through the super network model, the traffic assignment problem of the intercity multimode traffic system can be easily transformed into an ordinary network traffic assignment problem. According to the characteristics of various traffic modes and the super network model, the generalized cost functions of different links and the multi-modal traffic assignment model are established. The corresponding algorithm is proposed based on the MSA algorithm. Finally, the effective of the model and algorithm is verified with an example.

### References

- [1] Chen S, Peng H, Shuang L, et al. A Multimodal Hierarchical-based Assignment Model for Integrated Transportation Networks[J]. Journal of Transportation Systems Engineering & Information Technology, 2009, 9(6):130-135.
- [2] Bingfeng Si, Ming Zhong, Liang Gao, et al. Hierarchical network-based equilibrium model and algorithm for a mixed-traffic urban transport system[J]. Transportation Planning & Technology, 2012, 35(5):545-559.
- [3] Lozano A, Storchi G. Shortest viable path algorithm in multimodal networks[J]. Transportation Research Part A, 2001, 35(3):225-241.

- [4] Lozano A, Storchi G. Shortest viable hyperpath in multimodal networks[J]. *Transportation Research Part B Methodological*, 2002, 36(10):853-874.
- [5] SI B F, LING X Q. Stochastic user equilibrium model for parking and route choice under advanced travel information system[J]. *Journal of Management Science in China*, 2007, 10(2): 19-25.