

Car-following Model Considering Multiple Distances and Numerical Simulation of Mixed Traffic Flow

Wang Wei, Wu Zhong

College of Civil and Transportation Engineering, Hohai University, Nanjing 210098, China

wangwei_952@163.com

Abstract. A new car-following model with consideration of multiple distances (MDOV) was proposed based on the OV model, and the linear stability was analyzed theoretically, showing the MDOV model is more stable than OV model and the stability may enhance if considering more distances. Numerical simulation was demonstrated to study the evolution between high-velocity steady state and low-velocity steady state under different car proportions in the mixed traffic flow. The results indicate that the steady state evolution is usually influenced by the car number and the distances considered in MDOV model, the traffic wave spreads faster and the steady state evolution is shortened in the mixed flow than the single flow. In addition, the congestion is inhibited in the mixed flow and the effect is more obvious as the mixture ratio increases.

1. Introduction

Numerical simulation is widely used to study traffic flow. The car-following model, describing the interaction between the adjacent vehicles in the non-free traffic flow, is a main method of microscopic traffic flow survey, as well it is very benefit on comprehending the evolution of the traffic flow. Generally, the model is composed of several parameters, such as the velocity difference and distance between adjacent vehicles.

Just like the velocity gradient model ^[1], the car-following model holds that the rear vehicle's acceleration is caused by the velocity difference between adjacent vehicles, which is formulated as a mathematical expression according to the kinetic method ^[2]. Since then, a series of models have been developed, which take the front's acceleration and distance into consideration. However, these models believe that drivers' perception of velocity difference and driving behavior decisions are simultaneous. In fact, driving behavior change is slower than the perception and there is a time difference between them. On this basis, the time-delayed car-following model is proposed, which adds the reaction delay term into the kinetic formula so that it can reflect the acceleration variation of the following car more accurately ^[3]. Because of the subjectivity and randomness of the driver's decision, a model based on the fuzzy logic, considering the uncertainty in the car-following behavior, is developed ^[4]. It can be applied more widely and more efficient to control the driving behavior.

In most above models, there is no limit on the acceleration and velocity and it can be controlled flexibly. To solve this problem, Bando et.al conduct an OV model which adopts the optimal velocity function to simulate that drivers adjust the velocity in accordance with the adjacent distance ^[5]. The model synthesizes many factors, such as the reaction-stimulation, acceleration change, safe distance and upper limit of velocity. Owing to its superiority on explaining the complex car-following behavior, it draws scholars' attention and a series of improved OV models have been raised.



In addition, many scholars conduct studies on the linear and nonlinear stability of these models. In terms of the linear stability analysis, the stability and adaptability could be obtained [6]. Meanwhile, these characteristics could be significantly improved while considering more forward information., and it is also effective to relieve the congestion [7].

The aforementioned researches explore inherent characteristics of the car-following behavior from different angles and improve the accuracy of the model. However, most existing models are built on the single flow and ignore the otherness between oversize vehicles (cart) and mini vehicles (car) in the mixed traffic flow. Actually, the cart driver can perceive more distances ahead because of the wider vision, and it may have an effect on the driving behavior. Therefore, it is necessary to take the cart proportion and their relative positions in the mixed traffic flow into account.

In this paper, a new car-following model considering multiple distances (MDOV) is put forward based on the OV model, which brings the different perception of ahead traffic conditions into a general formula. Then, the linear stability analysis is conducted and a comparison is made among different cases. Finally, a numerical simulation of mixed traffic flow is carried out to study the steady evolution process under different cart proportions.

2. Car-following model

2.1. OV car-following model

Bando et al. apply the optimal velocity function to describe the acceleration change of the following vehicle in the OV model. It is expressed as a differential equation:

$$\ddot{x}_n(t) = k[V(\Delta x_n(t)) - v_n(t)] \quad (2.1)$$

Where $k = \frac{1}{\tau}$ is a sensitivity coefficient and τ is the reaction time; $x_n(t)$, $v_n(t)$ and $\ddot{x}_n(t)$ is the head position, velocity and acceleration of the nth vehicle at time t; $\Delta x_n(t)$ is the distance between the nth and (n+1)th vehicle satisfying $\Delta x_n(t) = x_{n+1}(t) - x_n(t) - l_{n+1}$, l_{n+1} is the length of the leading vehicle; $V(\bullet)$ is the optimal velocity achieved by the following car within reaction time τ and can be expressed as

$$V(\Delta x_n(t)) = \frac{v_{\max}}{2} [\tanh(\Delta x_n(t) - h_c) + \tanh(h_c)] \quad (2.2)$$

Where v_{\max} is the expected velocity and h_c is the safety distance.

2.2. Car-following model considering multiple distances

The traditional OV model only considers the distance between adjacent vehicles but ignores the more forward traffic condition. This makes it impossible to depict the phenomenon that the rear car has been accelerating in spite of short distance while the platoon starts. In order to make up for the deficiency, a new model is established considering multiple distances ahead [8-10] and it is expressed as

$$\ddot{x}_n(t) = k \left\{ V \left[\sum_{j=1}^m \alpha_j \Delta x_{n+j-1}(t) \right] - v_n(t) \right\} \quad (2.3)$$

Where n is the serial number of the following vehicle, m is the number of distances considered in the model. α_j is the influence coefficient representing the influence of distances between different positions on the following vehicle's acceleration. Obviously, the impact is stronger while the vehicle is closer and it can be formulated as $\alpha_j < \alpha_{j-1}$. In addition, it satisfies the normalization shown

as $\sum_{j=1}^m \alpha_j = 1$ and $\alpha_j > 0$. With regard to the condition the further information cannot be gained, m is chosen to be 1 and the model may degenerate to OV model.

The influence coefficient α_j is defined as

$$\alpha_j = \frac{1}{\sum_{j=1}^m \frac{x_{n+j}(t) - x_n(t)}{1}} \quad (2.4)$$

Where $x_{n+j}(t)$ is the head position of the $(n+j)$ th vehicle at time t . As the expression shows, the smaller is the impact on the acceleration while the vehicle is farther.

2.3. Stability analysis

It is assumed that in the platoon each adjacent distance is b , safety distance is h_c , length of the vehicle is l , reaction coefficient is k and the steady velocity is $V(b)$. Accordingly, the head positions in the steady traffic flow can be expressed as

$$x_n^0(t) = (b + l)n + V(b)t \quad (2.5)$$

To perturb the steady state with $y_n(t)$, and it can be expressed as

$$x_n(t) = x_n^0(t) + y_n(t) \quad (2.6)$$

Which is equal to

$$y_n(t) = x_n(t) - x_n^0(t) \quad (2.7)$$

To get the first and second derivations of equation (2.7) and substitute them into equation (2.3), which can be simplified as

$$\begin{aligned} \ddot{y}_n(t) = \ddot{x}_n(t) &= k[V(\sum_{j=1}^m \alpha_j \Delta x_{n+j-1}) - v_n(t)] \\ &= k[V(b + \sum_{j=1}^m \alpha_j \Delta x_{n+j-1}) - V(b) - \dot{y}_n(t)] \end{aligned} \quad (2.8)$$

According to the Taylor's formula, it is expanded as

$$\ddot{y}_n(t) = \ddot{x}_n(t) = k[V'(b) \sum_{j=1}^m \alpha_j \Delta y_{n+j-1} - \dot{y}_n(t)] \quad (2.9)$$

$$\text{Here, } V'(b) = \left. \frac{dV(\Delta x_n)}{d\Delta x_n} \right|_{\Delta x_n=b}.$$

Let $y_n = \exp(i\alpha_k n + zt)$, and substitute it into equation (2.9)

$$z^2 + kz + kV'(b) \sum_{j=1}^m \alpha_j [\exp(ia_k j) - \exp(ia_k (j-1))] = 0 \quad (2.10)$$

And then substitute $z = \lambda + \omega i$ into equation (2.11)

$$\lambda^2 - \omega^2 + k\lambda + kV'(b)\sigma_c + i[2\lambda\omega + k\omega - kV'(b)\sigma_s] = 0 \quad (2.11)$$

$$\text{Where } \sigma_c = \sum_{j=1}^m \alpha_j [\cos a_k j - \cos a_k (j-1)], \quad \sigma_s = \sum_{j=1}^m \alpha_j [\sin a_k j - \sin a_k (j-1)].$$

In the analysis, only take the perturbation in the one-dimensional into consideration instead of the lateral offset, and let $\lambda = 0$.

$$-\omega^2 + kV'(b)\sigma_c + i[k\omega - kV'(b)\sigma_s] = 0 \quad (2.12)$$

While the real and imaginary part equal to 0, it can be expressed as

$$V'(b) = \frac{k\sigma_c}{\sigma_s^2} = \frac{k \sum_{j=1}^m \alpha_j [\cos a_k j - \cos a_k (j-1)]}{\left\{ \sum_{j=1}^m \alpha_j [\sin a_k j - \sin a_k (j-1)] \right\}^2} \quad (2.13)$$

While a_k is close to 0, critical stability curve can be gained and expressed as

$$V'(b) = k \sum_{j=1}^m \alpha_j (2j-1) / 2 \quad (2.14)$$

Stability condition is

$$V'(b) < k \sum_{j=1}^m \alpha_j (2j-1) / 2 \quad (2.15)$$

While m equals to 1, the model may degenerate to OV model and its stability condition is

$$V'(b) < k / 2 \quad (2.16)$$

To analyze the stability condition (2.16), as the fig.1 shows, with the enlargement of m , the stability region is larger where vehicle can run steadily. This is mainly because the driver of following vehicle considers not only the position of leading vehicle but also more leading vehicles, which cuts down the sensitivity to the velocity fluctuation and enhances the stability. In addition, it is foreseeable that the stability region changes less and tends to a fixed area in the case that m increases to a certain value., indicating that considering too many distances is of little significance of the improvement of the model.

For a single curve, the stability region is of negative correlation to the sensitivity coefficient k and positive correlation to the reaction time τ , which reflects that traffic flow is closer to stability if the reaction time is smaller and too large reaction time leads to its instability and sensitivity to the velocity fluctuation ahead.

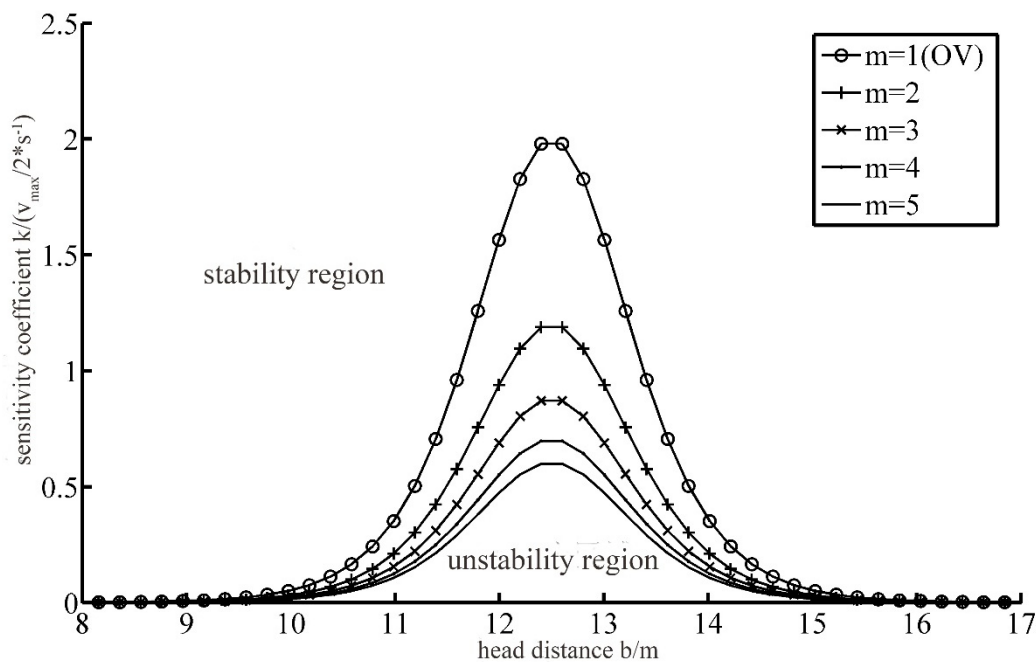


Fig. 1 The critical stability curves of the model considering different numbers of vehicles

3. Numerical simulation

In order to analyze the stability of the traffic flow consisting of two different kinds of vehicles (cart and car) intuitively and comprehensively, an experiment is carried out to study the evolution of mixed traffic flow under different cart ratios with an open boundary [7,8,11,13].

The parameters are set as follows: the total of vehicles is N equal to 100, the sensitivity coefficient k equals to $1.4s^{-1}$ and the expected velocity v_{max} is 20m/s. In addition, different vehicles distribute uniformly in the platoon and cars follow the OV model as carts follow the MDOV model. Only the traffic behavior of mixed traffic flow with different car-following characteristics are surveyed, ignoring different lengths of vehicles. During the simulation, accelerating and decelerating the leading vehicle leads the velocity change of the platoon, and then the numerical simulation is conducted with different cart ratios p ranging in 0, 0.1, 0.25, 0.5, 0.75, 0.9, 1.

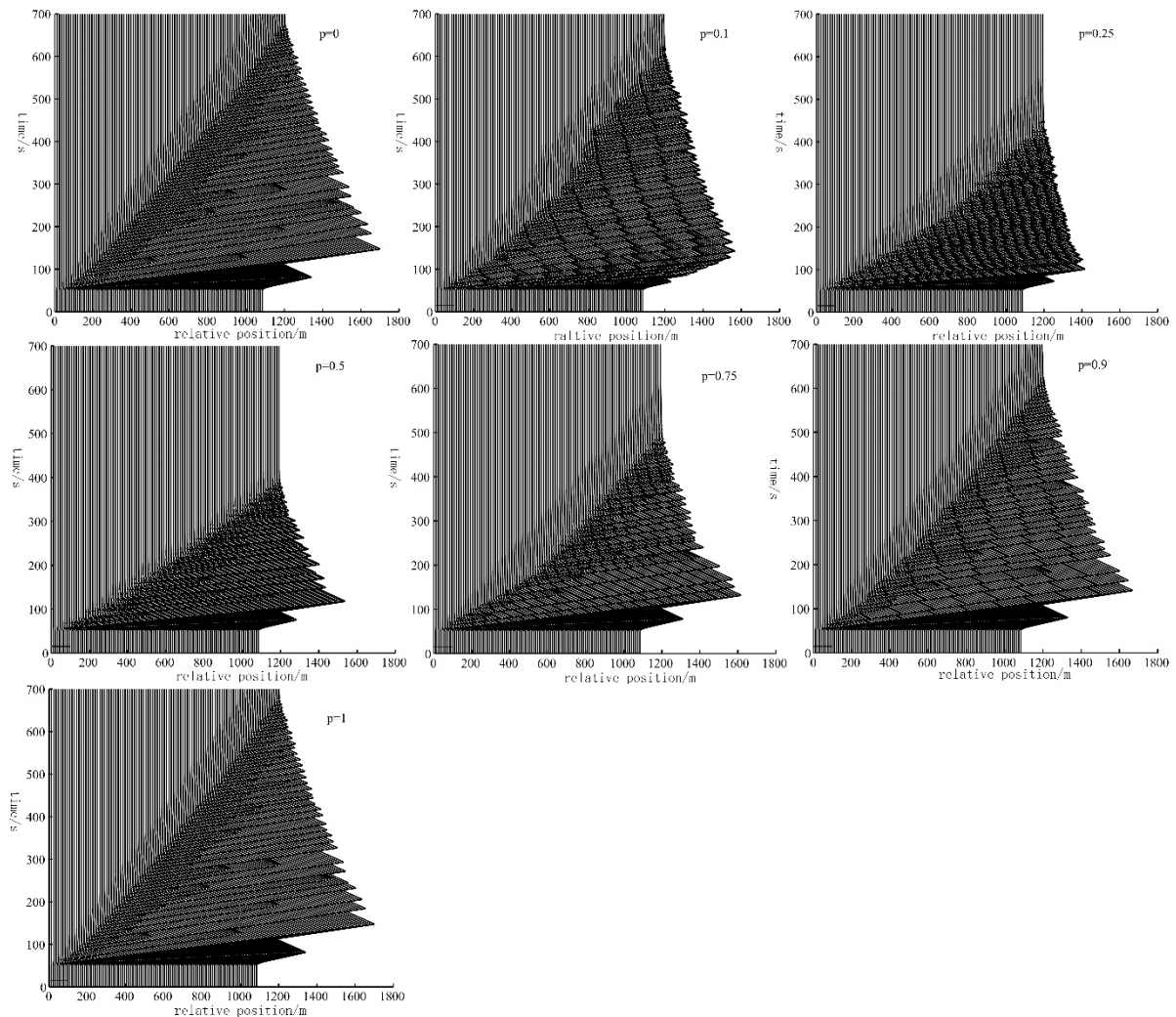


Fig.2 Temporal and spatial distribution pattern of steady state evolution from $5m/s$ to $15m/s$ under different chart proportion

Fig.2 is the temporal and spatial distribution pattern of steady state evolution from 5m/s to 15m/s under different cart proportions. In the evolution from low-velocity to high-velocity steady state, consisting of accelerating and stabilizing process, the rear vehicle accelerates slower than leading vehicles and the platoon length becomes longer while leading vehicles are accelerating because the transport of acceleration wave needs time; and then length becomes shorter while acceleration of rear vehicles. After acceleration, traffic flow would reach steady state in the course of time.

Obviously, the process of acceleration and stabilization are shortest while cart ratio equals to 0.5 and in the single traffic flow is the process is the longest. With the enlargement of the mixture of traffic flow, the steady evolution process is shortened, especially the stabilization, indicating stronger stability in higher mixed traffic flow.

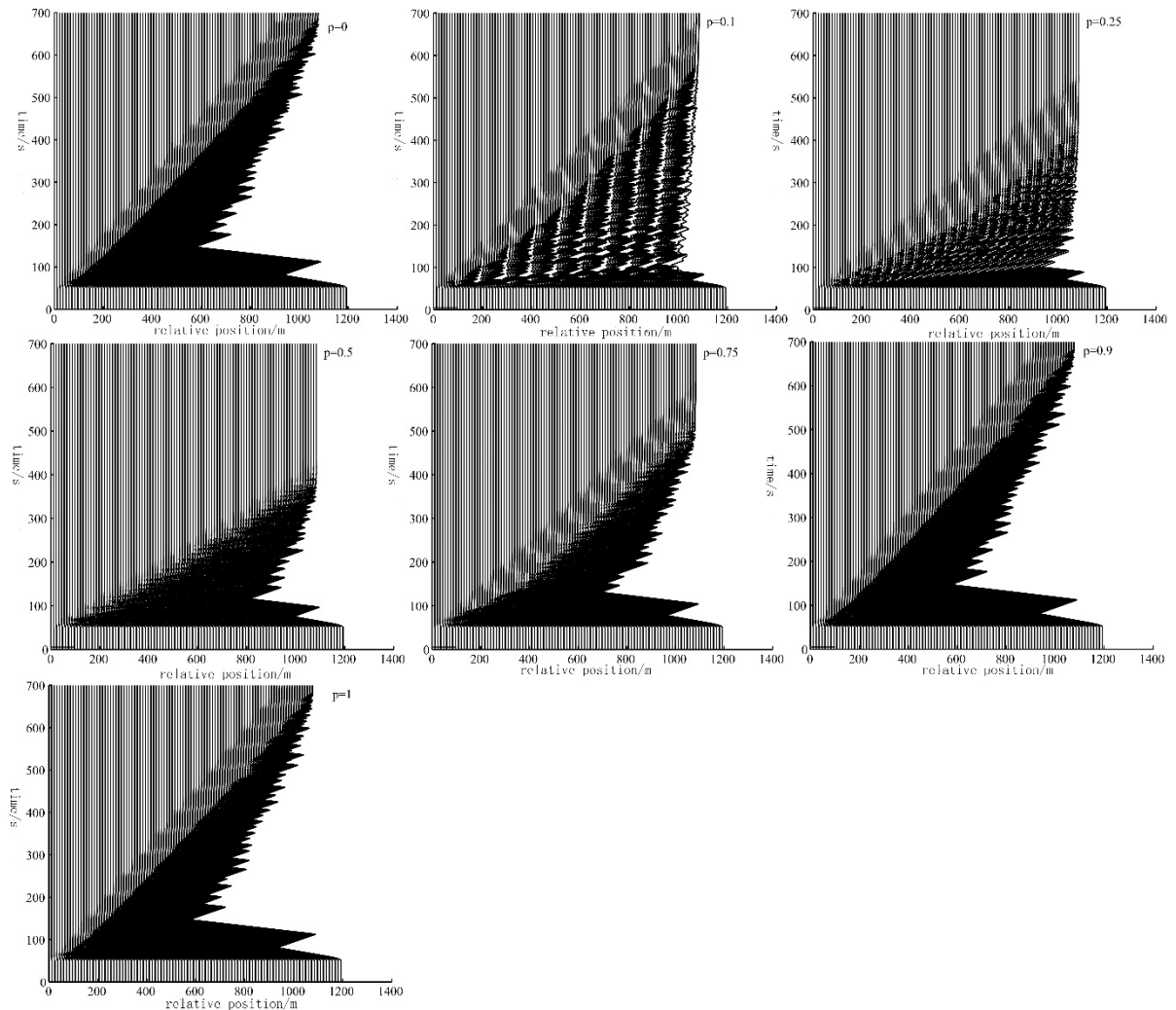


Fig. 3 Temporal and spatial distribution pattern of steady state evolution from 15m/s to 5m/s under different cart proportions

Fig.3 is the temporal and spatial distribution pattern of steady state evolution from 15m/s to 5m/s under different cart proportions. In the evolution from high-velocity to low-velocity steady state, consisting of decelerating and stabilizing process, the rear vehicle decelerates slower than leading vehicles because the deceleration wave spreads from front to backwards. As a result of time delay, rear vehicles are decelerating and distances become short while leading vehicles have finished the deceleration, easily leading to a traffic jam., and then traffic jam clears off as rear vehicles finish accelerating. Conversely in the mixed traffic flow, cart drivers can get the ahead information more quickly to fast the wave and decelerating process of the whole platoon. With the enlargement of the mixture of traffic flow, traffic flow decelerates more gently and the threshold of traffic congestion is reduced. The above results show that during the steady evolution process, carts in the mixed traffic flow may speed up the transmission of deceleration wave, shorten the stabilization and restrain traffic jam, more obvious in the platoon of higher mixture ratio.

Hence, with the increase of cart proportion, the evolution process is shortened first and increased later and the process is quickest while proportion is 0.5. It is mainly caused by two reasons: first, when

there are fewer carts in the platoon, cart drivers could get comprehensive information ahead and rear vehicles may accelerate earlier and the number of carts has a greater impact on the stability of traffic flow than considering more distances in the model. Second, while the cart proportion increases to a larger value (>0.5), the mutual interference between adjacent vehicles amplifies and cart drivers can get less information ahead, their car-following behavior may consider fewer distances, which has a great impact on the stability than cart numbers in the platoon.

4. Conclusion

In conclusion, MDOV model considers more distances ahead, which is more stable than OV model and the stability is enhanced while considering more distances. In the steady state evolution process, MODV model could make the traffic wave spreads quicker and keep down traffic congestion, the effect is obvious in the higher mixed traffic flow.

The aforementioned conclusion has some reference value on the study of car-following model and stability of mixed traffic flow. In this paper, numerical simulation is only conducted on the mixed traffic flow of uniform distribution. Further study could be made on other distributions.

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