The Maximization of Air Tickets Income Based on Logit Competition Model

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Abstract: In the competition with high-speed railway, the Logit Competition Model was built according to the theory of utility maximization to maximize the airline ticket revenue. Taking the six routes centre of Zhengzhou city as an example, the probability of choosing plane or the high-speed railway under different discounts was predicted. Based on the relationship between probability and ticket price, the optimal discount rate and ticket price when selecting different routes was calculated. The results show that the Zhengzhou to Dalian route has the most optimal revenue and the minimum discount pressure, while the Zhengzhou to Beijing route has the least optimal revenue and the maximum discount pressure.

1.Preface

Before the advent of high-speed rail, the division between railway and aviation is clear and there is little competition between them. With more and more high-speed railway operation, it has produced serious competition with civil aviation. High speed railway has the characteristics of high speed, comfort and low price, which inevitably brings the diversion effect to civil aviation. Today, the "four verticals and four horizontals" high-speed railway network will be completed, and the "eight verticals and eight horizontals" plan has been presented. The mileage of the high speed rail will increase rapidly. Therefore, it is particularly important to study how to make a price for maximize revenue under the impact of high-speed railway.

2.Model construction

If the stochastic utility theory is used to explain the way of people chose traveling. Then the stochastic utility function is:

$$U = V + \varepsilon \tag{1}$$

In the formula, the V is a fixed item, the$\varepsilon$is an unpredictable random item.

If$\varepsilon$is subject to exponential distribution:

$$F(\varepsilon) = 1/\left(1 + e^{\varepsilon}\right) \tag{2}$$

Today, Chinese high-speed transportation network has two ways: high-speed railway and air transportation.

The respective probability of air transportation and high-speed rail is:

$$P_A = P_{(U_A \geq U_G)} = \frac{1}{1 + e^{V_A - V_G}} = \frac{e^{V_A}}{e^{V_A} + e^{V_G}} \tag{3}$$

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$$P_G = 1 - P_A \tag{4}$$

In the formula, the A represents the aircraft and the G represents the high-speed rail.

It can be obtained form (3) and (4):

$$V_A - V_G = \ln\left(P_A/1 - P_A\right) \tag{5}$$

It can be seen that the probability P_A of taking plane is related to the difference of the fixed term V in the utility function. The people consider various factors to select travel mode. V represents the combined results of these factors. Assuming this is a Linear function relation between V and the factors, then its expression is:

$$V_A - V_G = \sum_{i=1}^{n} \theta_i x_i \tag{6}$$

In the formula, i represents the number of influencing factors, X_i represents the difference of influencing factors and θ_i is the corresponding parameter of the influencing factors.

Thus the expression (7) can be obtained.

$$\ln\left(P_A/1 - P_A\right) = \sum_{i=1}^{n} \theta_i x_i \tag{7}$$

After linear regression, we can figure out the parameters θ_i , get the probability of people choosing to travel and the relationship between the various factors.

3.Example analysis and solution

3.1Range and route selection

Due to the different distances, the impact of high-speed rail on civil aviation is also different. After a comprehensive consideration, we selected some cities, centre in Zhengzhou, in the area around 600km-1500km as survey routes. These cities are Beijing, Shanghai, Guangzhou, Dalian, Lanzhou and Chongqing. The distance from Zhengzhou to these cities is shown in Table 1

Table 1. The distance of Zhengzhou to 6 cities /km.

	Shanghai	Guangzhou	Dalian	Beijing	Lanzhou	Chongqing
High-speed rail	998	1614	1689	695	1180	1598
Airplane	887	1389	988	646	890	1042

3.2Selection of influential factors

Regarding the selection of influencing factors, I consider the characteristics of the two travel modes:

- The aircraft is fast but the fare is expensive. High-speed rail fare is relatively cheap but slow, so "Fare" and "Time" are two influencing factors.
- The aircraft possibly has one flight in one hour, but the high-speed rail only two trains in one day because of limit by the geographical environment. On the other hand ,the aircraft is often delayed because of affection by the weather, but the high-speed rail can basically start on time. So "Shift" and "Punctuality rate" are also two influencing factors.
- The airport is far away from the centre of the city. The high-speed rail station is closer than the airport to the city centre even if it is located in the suburbs. The distance is directly related to the traveler's convenience and travel expenses. In addition, the waiting time for taking a plane is generally longer than waiting time for a high-speed train. So "Extra cost" and " Extra time" are also two influencing factors.
- As mentioned above, the different distances, the impact of high-speed rail on civil aviation is also different. Therefore, "Distance" is also an influencing factor.

"Fare" "Time" and "Shift" of high-speed rail and flights can be obtained from the booking website. Ticket prices consider discounts."Extra cost" and "Extra time" can be obtained from the map navigation software. The influencing factor data from Zhengzhou to the cities is shown in Table 2.

Table 2. Impact Factor Data Sheet.

		C/￥	T/h	L/km	W_T/h	W_C/￥	Z	B
Shanghai	G	500	5	998	2.5	8.5	0.97	18
4.6discount	A	546	2	887	4	10	0.78	26
Guangzhou	G	720	6.5	1614	3.3	11	0.96	20
7.3discount	A	1050	2.3	1389	4.2	11	0.75	27
Dalian	G	705	9	1689	2.3	6	0.96	2
7.8discount	A	773	1.9	988	3.8	10	0.7	8
Beijing	G	416.5	3	695	2	7	0.98	50
7.5discount	A	952	1.5	646	3.7	32	0.8	4
Lanzhou	G	446	5.9	1180	2.4	4	0.95	11
3discount	A	376	2.1	890	4.8	33	0.78	13
Chongqing	G	522	8.7	1598	2.1	7	0.94	2
5.1discount	A	546	2	1042	4.2	10	0.73	12

[a] "G" means high-speed rail, "A" means aircraft.

[b] "C" "T" "L" "W_T" "W_C" "Z" "B" respectively indicate "Fare" "Time" "Distance" " Extra time" " Extra cost" "Punctuality rate" and "Shift".

[c] "5.1discount" means the number of discounts about airplane.

After the analyzing of the correlation, we eliminate "Distance", "Punctuality rate" and "Extra time" according to the data in Table 2. Finally, the effect factors are chosen: "Fare", "Time", "Extra cost" and "Shift".

3.3Parameter determination

x_1, x_2, x_3 and x_4 respectively indicate the "Fare diffierence", "Time diffierence", "Extra cost difference" and "Shift diffierence". Thus the expression (8) can be obtained.

$$\ln\left(P_A/1 - P_A\right) = \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4 \tag{8}$$

We get the probability $P_A$of people choosing plane and the data table for regression calculation according to the survey data in Table 2.They are shown in Table 3.

Table 3. Regression Data Sheet.

	X_1/￥	X_2/h	X_3/￥	X_4	P_A	$\ln(P_A/1-P_A)$
Shanghai	46.0	-3.0	1.5	8	0.67	0.708
Guangzhou	330.0	-4.2	0.0	7	0.45	-0.201
Dalian	68.0	-7.1	4.0	6	0.87	1.901
Beijing	535.5	-1.5	25.0	-46	0.17	-1.586
Lanzhou	-70.0	-3.8	29.0	2	0.89	2.091
Chongqing	24.0	-6.7	3.0	10	0.85	1.735

Calculation of parameters by linear regression:

θ_1=-0.004; θ_2=-0.263; θ_3=0.028; θ_4=0.010

The parameters put into the expression (8), then:

$$\ln\left(P_A/1 - P_A\right) = -0.004x_1 - 0.263x_2 + 0.028x_3 + 0.010x_4$$

$$P_A = \frac{\exp(-0.004x_1 - 0.263x_2 + 0.028x_3 + 0.010x_4)}{1 + \exp(-0.004x_1 - 0.263x_2 + 0.028x_3 + 0.010x_4)} \quad (9)$$

3.4Model verification

The data in Table 3 put into the expression (9). We get the model calculation value of the probability of people taking the airplane, then compare the survey values in Table 3 and the calculated values of the models. The validation results are shown in Figure 1.

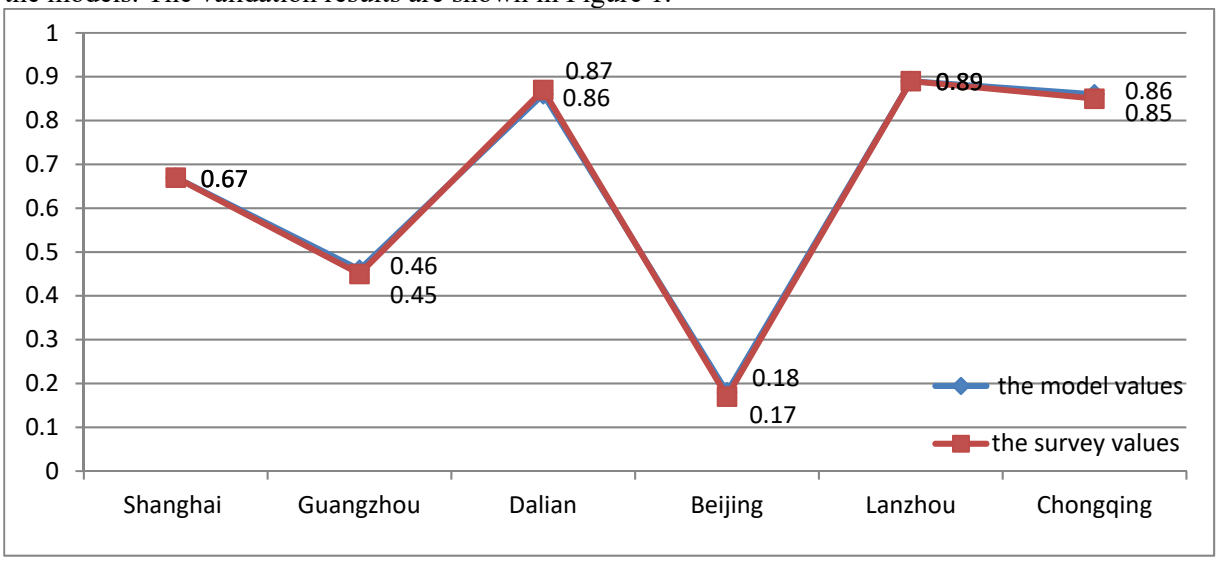


Figure 1. Choose the probability of the plane traveling

From Figure 1, we can see the probability of people choosing airplane to travel. The model value is basically consistent with the survey value.

3.5Optimal pricing and discount rate

Adding one passenger is almost negligible for the flight costs when this means passenger changes have nothing to do with variable costs. Moreover, the fixed costs of most of airlines are sunk costs which managers do not consider when pursuing profits. So airlines are often pursuing the revenue maximization in operations, and a large part of the airline is the ticket. Therefore, in order to get maximization ticket revenue, the airline must study the relationship between probability of people choosing travel mode and ticket price. According to expressions (9) and table 2, the expected revenue function of civil aviation is R="Ticket price" $\times P_A$. Then:

$$R = (x_1 + C_G) \times \frac{\exp(-0.004x_1 - 0.263x_2 + 0.028x_3 + 0.010x_4)}{1 + \exp(-0.004x_1 - 0.263x_2 + 0.028x_3 + 0.010x_4)} \quad (10)$$

In the formula, the R represents the expects return, the C_G represents the high-speed rail ticket price.

Putting x_2, x_3, x_4 in the table 3 and C_G in the table 2 into expressions(10), we get the relationship between the expected return R and the fare difference x_1 in the six routes. As shown in Figure 2:

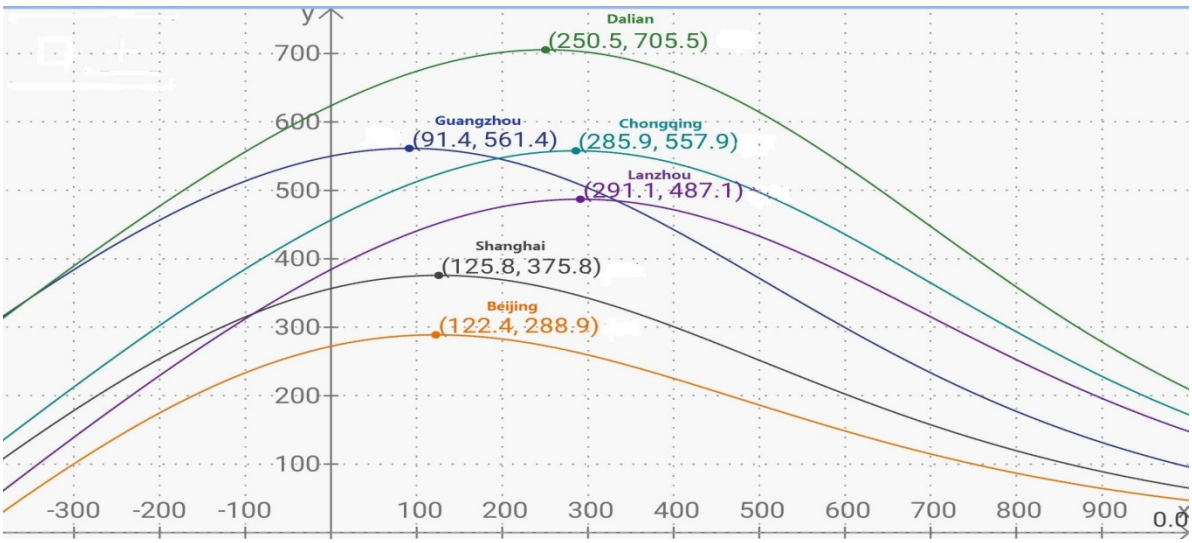


Figure 2. "Yield — Price Difference" Curve Chart.

R, X_1, the ticket price, P_A and the discount rate on six routes are shown in Table 4.

Table 4 . Best Price and Discount Statistics.

Zhengzhou——	Shanghai	Guangzhou	Dalian	Beijing	Lanzhou	Chongqing
X1/￥	125.8	91.4	250.5	122.4	291.1	285.9
Optimal expects return /￥	375.8	561.4	705.5	288.9	487.1	557.9
Optimal ticket price/￥	625.8	811.4	955.5	538.9	737.1	870.9
P_A	0.601	0.691	0.738	0.536	0.662	0.640
Optimal discount rate	0.527	0.564	0.960	0.424	0.588	0.489

Seen from Table 4, on the Zhengzhou-Dalian route, even if the ticket is 9.6 discount, still a 73% probability of choice, the optimal expects return is highest, but the ticket revenue pressure is minimal. In contrast, on the Zhengzhou-Beijing route, even if the ticket is 4.2 discount, only a 53.6% probability of choice, the unit expects return is lowest, and the ticket revenue pressure is maximal.

4.Conclution

The logit competition model was selected to analyse the probability of people choosing high speed rail or aircraft. According to the theory of maximization of income, the expected income of civil aviation on each line was calculated, and the optimal ticket price and discount rate were figured out when the income is the maximum.

Shortcomings: Adopting intention survey may be different from the actual choice; the location and survey data are few. The method of this paper only provides a reference for the pricing. More detailed price is bases on differentiated pricing strategy, which will continue to improve in the future.

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