

Prediction of bearing capacity of pile foundation in karst area based on model of metabolic GM (1,1)

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Abstract: There are many methods to predict the bearing capacity of bridge pile foundation in karst area, and the accuracy of each method is different. In order to find out that the bearing capacity of pile foundation in karst area is affected by the change of cave size and predict the bearing capacity of pile foundation in karst area better, this paper is based on the engineering characteristics of pile foundation in karst area and the gray system theory and the finite element numerical simulation. The numerical simulation models of different cave sizes were used to analyze the changing rules of the bearing capacity of piles when the cave size was changed. At the same time, a metabolic GM (1,1) model was established, and the time response of pile foundation bearing capacity was deduced. Compared with the original GM (1,1) model, the P-S curve obtained from the FEM simulation was predicted. Finally, the engineering formula is used to validate the predicting formula of the pile bearing capacity in karst area. The results show that the bearing capacity of pile foundation in karst area is significantly affected by the size of caverns, of which the influence by cave caverns is the most obvious, and the bearing capacity of pile foundations decreases with the increase of cave spans. When the hole span increases from 6m to 9m, The bearing capacity decreased by 33.44%. Compared with the bearing capacity obtained by numerical simulation of the finite element method, the prediction error of the pile foundation by the metabolic GM (1,1) model can reach 1% and the precision is high. The obtained bearing capacity and precision of the obtained pile prediction formula can be used to predict the bearing capacity of pile foundation in karst area, which has some theoretical and engineering practical value.

1. Introduction

There is a large area of karst development in the southwest of China. With the increase in the demand for highways and bridges, more and more piles of bridge directly built in karst areas^[1]. In the karst development area, the upper rock layer of the cavern is severely eroded, and the size of the cavern, the thickness of the roof, etc. have a certain influence on the bearing capacity of the bridge pile foundation^[2-5]. In the actual engineering, due to the limitation of actual engineering conditions, it is impossible to accurately discuss and analyze all the factors. Besides, the static load test cannot be performed to the pile foundation damage in the actual engineering, so that it is impossible to accurately predict bearing capacity of pile foundation during design^[6-11]. Chunlin He et al^[12] used the finite element numerical simulation method to discuss the influence of the size of karst caves on the bearing capacity of the bridge pile foundation, but they were only in the qualitative discussion stage.



The quantitative calculation of the bearing capacity of the pile foundation was not analyzed when the size of the karst cave was different. Yong Lei et al.^[13] used the indoor model test to study the influence of different roof thickness of karst caves on the ultimate bearing capacity of the pile foundation, but he didn't consider the influence of the size of the cavern on the ultimate bearing capacity of the pile foundation. Zhongju Feng et al.^[14] used numerical simulation to simulate and analyze the variation law of bearing capacity of pile foundation with the change of soil material properties. Shenggen Huang et al.^[15] combined with the static load test to study the bearing characteristics of pile foundations in karst areas, but didn't explore the influence of different sizes of karst caves on the variation of the bearing behavior of pile foundations. Guojun Cai et al.^[16] used the CPTU prediction method to predict the ultimate bearing capacity of friction piles, and the prediction accuracy was high. Yiwen Yao et al.^[17] studied the grey prediction method for the ultimate bearing capacity of a single pile using the grey system theory, but the accuracy of the model needed to be improved. Weidong Yang^[18] proposed the influence of the height and span of the cavern on the ultimate bearing capacity of the bridge pile foundation, and gave the formula for the bearing capacity of the pile foundation under the joint action of the cavern span and the cavern height combined with the gray system theory, but the grey system model is lack of precision.. The grey system theory, proposed by Prof. Ju long Deng in 1982, and could be used to solve the problem of prediction of ultimate bearing capacity of piles containing both known and unknown information^[19-21]. At present, the research on the ultimate bearing capacity of bridge pile foundation in karst areas is less affected by the size of the karst caves and the prediction of the bearing capacity of the pile foundation by using the metabolic GM(1,1) model^[22-25].

The bearing capacity of the pile foundation in karst area is affected by various factors of the karst cave. In this paper, the influence law of the bearing capacity of the pile foundation and the prediction method of the bearing capacity of the pile foundation are studied. The finite element numerical simulation was used to analyze the trend of the ultimate bearing capacity of the bridge pile foundation in the karst area under the influence of the cave height and cave span. Based on the analysis results, the prediction formula of bearing capacity of pile foundation with high precision in karst area was deduced by using the metabolic GM(1,1) model. The comparison the prediction results accuracy of the common GM (1,1) and metabolic GM (1,1) is used to verify the scientificity and rationality of the prediction formula of this article. The research results can provide technical reference for the design of bridge pile foundation in the karst development area.

2. Influence analysis of the size of the underlying cave on the bearing capacity of the pile foundation

In the numerical simulation, the established constitutive model and the selected material parameters determine the accuracy of the results. In the numerical simulation analysis in this paper, the bridge pile foundation is a solid pile model. The structure adopts concrete materials and the ideal elastic constitutive model is selected. Due to the non-linear stress-strain relationship of rock and soil, the elastic-plastic constitutive model and the Mohr-Coulomb yield criterion are selected for the analysis.

2.1. Model Establishment, Parameter Value Selection and Working Condition Selection

2.1.1. The establishment of numerical simulation model

In this paper, the simulation model of the bearing capacity of the pile foundation under different working conditions is established by using the nonlinear finite element program to simulate the distribution of the soil layer and the layout of the pile position.

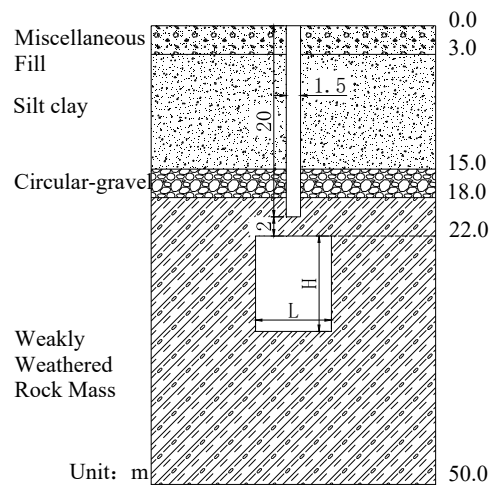


Figure 1. Soil distribution and position of pile.

During the modeling process, in order to ensure the convergence and the accuracy of the calculation results, mesh encryption processing was performed on the pile foundation, soil around the pile foundation and the soil around the cavern. The distribution of the soil layers and the position of the piles are shown in Figure 1. The cross-section of the model based on Figure 1. is shown in Figure 2.

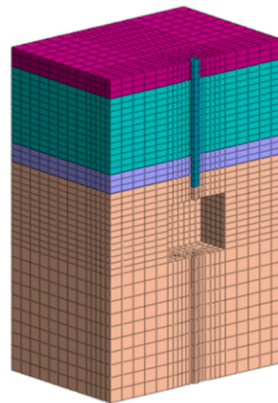


Figure 2. Model.

2.1.2 Determination of Calculation Parameters

In the numerical simulation, the values of the material parameters of each soil layer and pile foundation are shown in Table 1.

Table 1. Model material parameters in numerical simulation.

Material	Elastic Modulus E / Pa	Poisson ratio μ	Cohesion c / Pa	Internal friction angle $\varphi / (^{\circ})$	Bulk density $\gamma / (\text{kN} \cdot \text{m}^{-3})$
Pile	3×10^{10}	0.2	—	—	25
Miscellaneous Fill	1.3×10^7	0.4	1.0×10^4	8	18.5
Silt clay	4×10^7	0.3	1.8×10^4	10	19.5
Circular-gravel	8×10^8	0.3	0	28	24.5

Weakly Weathered Rock Mass	2×10^{10}	0.23	2×10^5	25	25
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2.1.3. Selection of model conditions

Different models are established in this paper. According to the size of large caverns, the influence of different cavern heights and cave span variations on the bearing capacity of the pile foundation is simulated. The analysis scheme is shown in Table 2 and Table 3.

Table 2. Calculation condition of variation of cavern height.

Cavern span l/m	Cavern high H/m	Pile diameter D/m	Pile length L/m
9	4、6、9、12、15	1.5	20

Table 3. Calculation condition of variation of cavern span.

Cavern high h/m	Cavern span H/m	Pile diameter D/m	Pile length L/m
4、9	4、6、9、12、15	1.5	20

2.2 Analysis of Change Law of Bearing Capacity of Pile Foundation

According to the method specified in the specification, the P-S curve of the pile foundation affected by the size of the karst cave is analyzed to determine the bearing capacity of the pile foundation in different working conditions, and the variation trend with the size of the cavern is analyzed.

2.2.1 Influence of the Height of Karst Cave on Bearing Capacity of Pile Foundation

Where the cave height is 9m, the P-S curve at a hole height of 4m, 6m, 9m, 12m, and 15m is taken as an example, as shown in Figure.3.

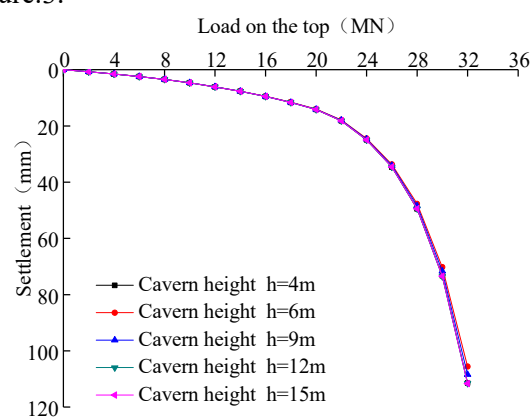


Figure.3. Influence of variation of cavern height on bearing capacity of pile foundation.

As can be seen from the above figure, only for the size of the cave under the pile foundation, the hole span is the same, the hole height is different, the P-S curve is almost completely coincident, the height of the cave has little effect on the ultimate bearing capacity of the pile and the influence of cave height on bearing capacity of pile foundation can be ignored. This curve is a "steeply descending" curve with obvious damage characteristic points. The load corresponding to the starting point of the steep descending process should be taken as the ultimate bearing capacity of the pile foundation according to "Code for Design of Foundations and Foundations of Highway Bridges and Culverts"^[26]. The ultimate bearing capacity of the pile is 26.82MN.

2.2.2. Analysis of influence of karst cave span bearing capacity of pile foundation

Here, the P-S curves with hole heights of 4m and 9m and hole spans of 4m, 6m, 9m, 12m, and 15m are used as examples, as shown in Fig. 4(a) and (b). The P-S curve across the 9m, 12m, and 15m holes is a 'steeply descending' curve. The P-S curve of 4m and 6m in the hole belongs to the "gradually changing" curve. There is no obvious damage feature point. The bearing capacity corresponding to the settlement value specified in the code is the bearing capacity of the pile foundation, and the corresponding load when the settlement value is 40mm is often used. Pile foundation bearing capacity. Pile bearing capacity-cave span is shown in Figure.4.

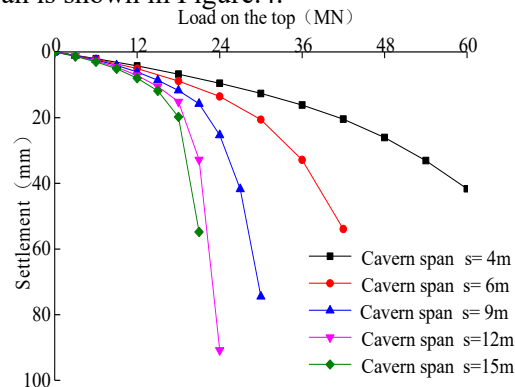


Figure.4 Influence of variation of cavern span on bearing capacity of pile foundation when cavern high is 4m.

As can be seen from Figure 4, when the height of the cave is the same, the bearing capacity of the pile significantly decreases with the increase of the span. When the span increases from 4m to 6m, the bearing capacity of the pile decreases by 33.44%. The specific reduction is shown in Figure.5.

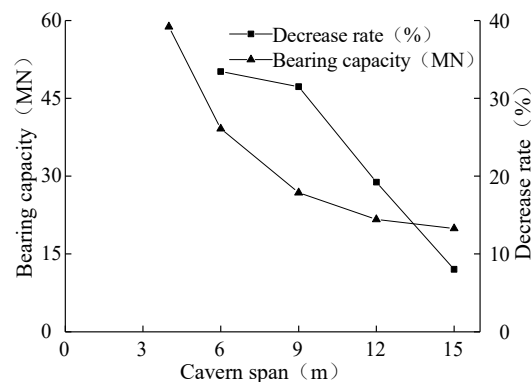


Figure.5. The variation of bearing capacity of pile foundation with the change of span and the influence degree of span.

3. Grey System Theory

The main body of the grey system theory is system analysis, evaluation, modeling, forecasting, decision making, control, optimization, and the core is the grey model (GM).

3.1. Basic Principles of Grey System Theory

The grey system theory^[20] takes the "small sample" with known information and partial information unknown and "poor information" uncertain systems as the research object. The gray model is based on discrete sequences and is established by fitting differential equations. The grey theory follows the following principles: (1) the axiom of differential information principle; (2) the axiom of non-uniqueness of the solution; (3) the principle of minimum information; (4) the principle of

cognition; (5) the principle of new information priority; (6) the principle of grey inextinction.

3.2. Establishment of Grey GM(1,1) Model

The grey GM (1,1) model is a model consisting of dynamic differential equations of one variable of one order.

The grey GM (1,1) model was used to predict the bearing capacity of the pile foundation with a height of 9m and a span of 4m. In the study of the effect of karst cave spanning 4m on the ultimate bearing capacity of pile foundation, the settlement S of pile foundation is regarded as a generalized time, and the first-order dynamic differential equation GM (1,1) model of P_j sequence of ultimate bearing capacity is established.

The initial ultimate bearing force sequence and cavern height sequence are:

$$\begin{aligned} P^{(1)} &= [P^{(1)}(1), P^{(1)}(2), \dots, P^{(1)}(n)] \\ S^{(1)} &= [S^{(1)}(1), S^{(1)}(2), \dots, S^{(1)}(n)] \end{aligned} \quad (1)$$

The above sequence is reduced by one generation (1AGO) and the new sequence is

$$\begin{aligned} P^{(0)} &= [P^{(0)}(1), P^{(0)}(2), \dots, P^{(0)}(n)] \\ S^{(0)} &= [S^{(0)}(1), S^{(0)}(2), \dots, S^{(0)}(n)] \end{aligned} \quad (2)$$

among them:

$$\begin{aligned} P^{(0)}(k) &= [P^{(1)}(k) - P^{(1)}(k-1)] \\ S^{(0)}(k) &= [S^{(1)}(k) - S^{(1)}(k-1)] \end{aligned} \quad (3)$$

In the formula: $k = 2, 3, \dots, n$.

According to the grey system theory, the establishment of first-order one-variable dynamic differential equations is:

$$\frac{dP^{(1)}}{dS} + aP^{(1)} = b \quad (4)$$

In the formula: a —Development coefficient (1/mm) , b —Grey effect amount (kN/mm) .

If the parameter column $\hat{a} = [a, b]^T$

The \hat{a} can be obtained by the least square method, which is

$$\hat{a} = (B^T B)^{-1} B^T y_n \quad (5)$$

In the formula:

$$B = \begin{bmatrix} S^{(0)}(2) & & & \\ & S^{(0)}(3) & & \\ & & \dots & \\ & & & S^{(0)}(n) \end{bmatrix} \begin{bmatrix} -z_0(2) & 1 \\ -z_0(3) & 1 \\ \dots & \dots \\ -z_0(n) & 1 \end{bmatrix} \quad (6)$$

$$z^{(1)}(k) = -\frac{1}{2} [P^{(1)}(k) + P^{(1)}(k-1)] \quad (7)$$

$$z^{(1)} = [z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)] \quad (8)$$

$$y_n = [P^{(0)}(2), P^{(0)}(3), \dots, P^{(0)}(n)]^T \quad (9)$$

The solution of the differential equation (4) is

$$\hat{P}^{(1)}(k+1) = \left[P^{(1)}(1) - \frac{b}{a} \right] e^{-a[S^{(1)}(k+1) - S^{(1)}(1)]} + \frac{b}{a} \quad (10)$$

In the formula: $\hat{P}^{(1)}(k+1)$ is the predicted value of pile top load in step $k+1$, $\hat{S}^{(1)}(k+1)$ is the settlement of the pile foundation in step $k+1$, the ultimate bearing capacity of the bridge pile foundation can be predicted by formula (10).

3.3. Accuracy test

3.3.1. Residual Test

$$\text{Residual: } \varepsilon(k) = \varepsilon(k+1) - \varepsilon(k) \quad (11)$$

$$\text{Relative error: } \Delta_k = \frac{|\varepsilon(k)|}{P_j^{(0)}(k)} \quad (12)$$

3.3.2. Posterior difference test

$$\text{Posterior error ratio: } C = \frac{R_1}{R_2} \quad (13)$$

$$\text{Small error probability: } P = P\{|\varepsilon(k) - \bar{\varepsilon}| < 0.6745 R_2\} \quad (14)$$

$$\text{Mean of residual errors: } \bar{\varepsilon} = \frac{1}{n} \sum_{i=1}^n \varepsilon(i) \quad (15)$$

$$\text{Residual variance: } R_1^2 = \frac{1}{n} \sum_{i=1}^n (\varepsilon(i) - \bar{\varepsilon})^2 \quad (16)$$

$$\text{Original data mean: } \bar{x}_1^{(0)} = \frac{1}{n} \sum_{i=1}^n x_1^{(0)}(i) \quad (17)$$

$$\text{Original data variance: } R_2^2 = \frac{1}{n} \sum_{i=1}^n (x_1^{(0)}(i) - \bar{x}_1^{(0)})^2 \quad (18)$$

Table 4. Prediction accuracy.

Prediction accuracy	Good	Qualified	Reluctant	Unqualified
P	>0.95	>0.80	>0.70	≤0.70
C	<0.35	<0.50	<0.65	≥0.65

4. Grey prediction model for the bearing capacity of pile foundation in karst area and its accuracy test

4.1. GM(1,1) Model Calculation Analysis

The initial analysis of the cave height is unchanged, and the simulation results at the time of the span of 4m cannot determine the trend of the P-S curve. Therefore, in order to ensure that the accuracy meets the requirements, the GM (1,1) model of "metabolism" and the prediction of bearing capacity are carried out using a load-settlement sequence of different step sizes.. According to the calculation, the 5-10 level simulation data series is selected to predict the ultimate bearing capacity. The relative error between the predicted value and the simulated value is small and meets the requirements. The corresponding data is shown in Table 5 and Table 6.

Table 5. Table of original date.

$P^{(1)}$	24	30	36	42	48	54
$S^{(1)}$	9.5	12.6	16.2	20.6	26.3	33.4

Table 6. Table of date after a cumulative decrease.

$P^{(0)}$	6	6	6	6	6	6
$S^{(0)}$	2.8	3.1	3.6	4.4	5.7	7.1

$$z^{(1)}(k) = -\frac{1}{2} [P^{(1)}(k) + P^{(1)}(k-1)]$$

$$z^{(1)} = [z^{(1)}(6), z^{(1)}(7), \dots, z^{(1)}(10)] = [27, 33, 39, 45, 51]$$

$$B = \begin{bmatrix} S^{(0)}(6) & & & & \\ & S^{(0)}(7) & & & \\ & & \dots & & \\ & & & S^{(0)}(10) & \\ & & & & \end{bmatrix} \begin{bmatrix} -z_0(6) & 1 \\ -z_0(7) & 1 \\ \dots & \dots \\ -z_0(10) & 1 \end{bmatrix} = \begin{bmatrix} -83.7 & 3.1 \\ -118.8 & 3.6 \\ -171.6 & 4.4 \\ -256.5 & 5.7 \\ -362.1 & 7.1 \end{bmatrix}$$

$$y_n = [P^{(0)}(6), P^{(0)}(7), \dots, P^{(0)}(10)]^T = [6, 6, 6, 6, 6]^T$$

$$\hat{a} = (B^T B)^{-1} B^T y_n = (a, b)^T = (0.04549, 3.14412)^T$$

That is, the development coefficient : $a = 0.04549$, Gray effect amount: $b = 3.14412$.

$$\hat{P}^{(1)}(k) = \left[P^{(1)}(5) - \frac{b}{a} \right] e^{-a[S^{(1)}(k) - S^{(1)}(5)]} + \frac{b}{a} = (-45.11673)e^{-0.04549[S^{(1)}(k) - 9.5]} + 69.11673$$

$$S^{(1)}(11) = 40\text{mm} , \text{Pile bearing capacity: } P = 57.8505\text{KN}.$$

In the numerical simulation, loading is continued to 60 MN. The load when the settlement is 40 mm is the bearing capacity of the pile foundation, and the bearing capacity of the pile foundation with a span of 4 m is 58.5167 MN.

To improve accuracy, take settlement $S^{(1)}(11) = 38\text{mm}$,The predicted load is

$$\hat{P}^{(1)}(11) = \left[P^{(1)}(1) - \frac{b}{a} \right] e^{-a[S^{(1)}(11) - S^{(1)}(5)]} + \frac{b}{a} = 56.7777\text{MN}$$

Add the new data (56.7777, 38) to the load-settling sequence, remove the data from the 5th level, and do the “metabolism,” repeat the above steps:

$$\hat{a} = (a, b)^T = (0.03964, 2.89781)^T$$

Get metabolic GM(1,1) prediction formula:

$$\hat{P}^{(1)}(k+1) = \left[P^{(1)}(1) - \frac{b}{a} \right] e^{-a[S^{(1)}(k+1) - S^{(1)}(1)]} + \frac{b}{a} \quad (20)$$

$$\hat{P}^{(1)}(12) = \left[P^{(1)}(1) - \frac{b}{a} \right] e^{-a[S^{(1)}(12) - S^{(1)}(6)]} + \frac{b}{a} = 58.5524\text{MN}$$

From Table 7 and Table 8, we can see that the cave height is constant and the hole span is 4m, the general GM(1,1) model is used to predict the bearing capacity of the bridge pile foundation, the relative error is 1.14%, and the metabolic GM(1,1) model's relative error of pile foundation bearing prediction is 0.06% with a high accuracy. Formulas (19)~(20) can be used to predict the ultimate bearing capacity of bridge pile foundations, and can be used to predict the settlement prediction of pile foundations under various loads. It can provide reference value for the design and calculation of pile foundations and engineering practice, saving the cost of a static load test.

Table 7. The results of 4m prediction model analysis.

Model category	a	b	Bearing capacity prediction /MN	Bearing capacity /MN	Relative error /%
Normal GM (1,1)	0.04549	3.14412	57.8505	58.5167	1.14
MetabolicGM (1,1)	0.03964	2.89781	58.5524	58.5167	0.06

Table 8. Table of analog values and error tests.

Num ber	Origin al data/ $P_j^{(0)}$	Simulatio n value/ $\hat{P}_j^{(0)}$	Residual / $\varepsilon(k) = P_j^{(0)}(k+1) - P_j^{(0)}(k)$	Relative error / $\Delta_k = \frac{ \varepsilon(k) }{P_j^{(0)}(k)}$	Posterior error ratio/ C	Small error probability/ P
1	30	30	0	0		
2	36	35.73190	0.26810	0.74%		
3	42	41.71275	0.28725	0.868%	0.11536	1
4	48	48.06040	0.06040	0.13%		
5	54	54.20252	0.20252	0.38%		
6	58.51 67	58.5524	0.03570	0.06%		

5. Conclusion

(1) When the pile length, pile diameter and slab thickness are constant, the karst cavern spans unchanged, the impact of the cavern height on the ultimate bearing capacity of the bridge pile foundation can be neglected, the cave height remains constant, the cave span increases, and the ultimate bearing capacity of the bridge pile foundation will be significantly reduced, and the bearing capacity of the pile decreases by 33.44% when the span increases from 4m to 6m. Therefore, in the actual geological exploration before the construction of the bridge pile foundation, the exploration of the cave size is important.

(2) The general GM (1,1) model predicts the bearing capacity of the bridge pile foundation, the relative error is 1.14%, and the metabolic GM (1,1) model's relative error of the prediction of the bearing capacity of the pile foundation is 0.06% with a high precision. Equation (20) can be used to predict the ultimate bearing capacity of the pile foundation of the bridge and the settlement caused by the pile foundation under various loads. It is suitable for the prediction of the ultimate bearing capacity of the bridge pile foundation when the field test conditions are insufficient in the karst development area, and the on-site static load test cannot be loaded into the pile foundation damage and the data are too few. It provides a reference value for the design of the pile foundation and saves the cost of on-site static tests.

(3) The method of GM (1,1) model for predicting the ultimate bearing capacity of pile foundation can be calculated according to actual project, and a targeted prediction formula is obtained. There are many factors influencing the bearing capacity of bridge pile foundation in karst area. The influence of slab thickness, karst corrosion and other factors of the corresponding pile foundation bearing capacity prediction need to be further studied.

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