IOP Conf. Series: Earth and Environmental Science **189** (2018) 052070 doi:10.1088/1755-1315/189/5/052070

Identification of nonlinear dynamical characteristics of the single degree of freedom system using Hilbert transform

Qiang Liu [1*], Miaofang Wang [1] and Rongjie Liu [2]

[1] College of Engineering Technology, Jimei University, Xiamen, Fujian, 361021, China

[2] College of Information and Electrical Engineering, Feng Chia University, Tai Zhong, Tai Wang, 400, China

[*]Corresponding author's e-mail: liutanq007@aliyun.com

Abstract: The method of Hilbert transform is used in applications to signal decomposition and nonlinear vibration analysis. For a single degree of nonlinear system vibration, the vibration response signal can be regarded as a single signal component, and the instantaneous frequency and instantaneous amplitude of the signal are extracted directly using the Hilbert transform method. Then, the nonlinear vibration equations is derived which can be transformed to the standard equation with the instantaneous parameters. The amplitude-frequency curve of response of system is obtained. Finally, through the numerical simulation of a typical *Duffing* system, the effectiveness of the Hilbert transform is verified for the application to nonlinear analysis.

1. Introduction

Vibrational signal analysis is one of the most important topics in nonlinear vibration. In recent years, the Hilbert transform method has been widely used in extracting instantaneous characteristics from vibration signal. The instantaneous amplitude, frequency and phase of the original signal, which have usually been utilized to analyze the nonlinearity of structures and to estimate major parameters of vibration system, can be directly obtained by applying the Hilbert transform method[1]. Wang et al has researched a two degrees of freedom nonlinear stiffness of shear structure, instantaneous amplitude and instantaneous frequency is obtained by HHT transform and identify the type of nonlinear structure[2]. Yi et al used the Hilbert transform method to identify the dynamic response of the reinforced concrete beam with crack, so as to identify the nonlinear dynamic characteristics of the beam[3]. Cheng et al measured response signal and excitation signal by experiment and used Hilbert transform to damage the cantilever beam of the time-varying stiffness model and obtain a reasonable time-varying stiffness model[4]. Wang et al used wavelet transform to identify the nonlinear vibration system, so as to research the relationship between parameter and identification error[5]. Huang et al analyzed the vibration signal through the Hilbert transform, and put forward the fault diagnosis method of rolling bearing based on HTT[6].

The aforementioned researchers have evaluated the parameters of nonlinear vibration system basing either on the wavelet transform method or on the Hilbert transform method. Although satisfying results had been obtained, there were still large errors at the end points of the signal.

In this paper, the Hilbert transform is applied to the single-degree-of-freedom nonlinear vibration analysis, which is a method of parameter identification of a single-degree-of-freedom nonlinear system. For the non-linear parameters of the unknown system, the frequency response curve of a single degree-of-freedom system can be solved based on the HHT transform method, and the nonlinear characteristics of the system are analyzed. Finally, through a single-degree-of-freedom vibration system and a single-degree-free nonlinear vibration system with time-varying damping, the nonlinear characteristic of the system is analyzed, and the effectiveness of this method is verified.

2. Calculation methods

2.1. The theoretical basis of Hilbert-Huang transformation

The core contents of Hilbert-Huang transform are divided into two parts: Empirical Mode Decomposition (EMD) and Hilbert Spectrum Analysis (HSA). Empirical Mode Decomposition (EMD) is an intrinsic Mode function that decomposes complex signals into a set of functions with better Hilbert transform performance[7-8]. That is to select the optimal and interesting IMF components, which should be suitable in nature and ensure that the Hilbert transform can be obtained with a clear instantaneous amplitude and instantaneous frequency.

For any of the eigenfunctions $c(t)$， the Hilbert transform $\hat{c}(t)$ is defined as the convolution of signal and $1/\pi t$:

$$\hat{c}(t) = \frac{1}{\pi} P \int \frac{c(\tau)}{t-\tau} \mathrm{d}\tau \tag{1}$$

Where P is cauchy principal value, in this way, the local property of the signal is emphasized, and the actual nonexistent frequency components of the Fourier transform are avoided. The analytical signal $z(t)$ of $c(t)$ is defined as:

$$z(t) = c(t) + i\hat{c}(t) = a(t)e^{i\theta(t)} \tag{2}$$

Where $a(t)$ and $\theta(t)$ are respectively the instantaneous amplitude and instantaneous phase of the signal:

$$a(t) = \sqrt{c^2(t) + \hat{c}^2(t)} \tag{3}$$

$$\theta(t) = \arctan(\ \hat{c}(t) / c(t)) \tag{4}$$

Instantaneous frequency that can be obtained by instantaneous phase:

$$\omega(t) = \mathrm{d}\,\theta(t) / \mathrm{d}\,t \tag{5}$$

As the amplitude is shown in the frequency - time plane, Hilbert amplitude spectrum $H(\omega,t)$ can be obtained：

$$H(\omega,t) = \mathrm{Re}\sum_{j=1}^{n} a_j(t) e^{i\int \omega_j(t)dt} \tag{6}$$

Where, Re is real component，r_n is omitted in the derivation, because it is a monotone function or a constant. The residual term can be used to see the fluctuations of the growth cycle in the time frequency conversion of Hilbert, since the information of interest is generally not in the residual item, so the residual term is generally omitted when making the transformation.

2.2. Nonlinear vibration response analysis based on Hilbert-Huang transform

2.2.1. Single degree-of-freedom nonlinear system free vibration

Differential equation of single degree of freedom nonlinear system:

$$\dot{x} + h(x,\dot{x}) + g(x) = 0 \tag{7}$$

Where, $g(x)$ is the resilience of the system， $h(x,\dot{x})$ is the damping force of the system：

$$\begin{cases} g(x) = \omega_x^2(t)x(t) \\ h(x,\dot{x}) = 2h_x(t)x(t) \end{cases} \tag{8}$$

Where, $h_x(t)$ and $\omega^2(t)$ are transient parameters related to the inherent characteristics of the system itself. Formula (8) is substituted into formula (7), and the complex equation of the analytic signal is obtained by an HTT transformation：

$$\ddot{X} + 2h_x(t)\dot{X} + \omega_x^2(t)X = 0 \tag{9}$$

For equation(9), the system is solved, and the two transient parameters can be obtained by the fact that the part and the imaginary part are zero[9]：

$$\begin{cases} h_x(t) = -\dfrac{\dot{A}}{A} - \dfrac{\dot{\omega}}{2\omega} \\ \omega_x^2(t) = \omega^2 - \dfrac{\ddot{A}}{A} + \dfrac{2\ddot{A}}{A^2} + \dfrac{\dot{A}\dot{\omega}}{A\omega} \end{cases} \tag{10}$$

According to equation (10), $h_x(t)$ and $\omega^2(t)$ are expressed by instantaneous amplitude (A) and instantaneous frequency (ω). For a nonlinear system of free vibration, these two transient parameters can be solved.

2.2.2. Nonlinear system response analysis based on HHT transform

The time frequency analysis of the displacement response signals obtained by Hilbert transform is adopted. In essence, the characteristic of the nonlinear vibration of the system is obtained by obtaining the frequency response curve. In this paper, the time domain response of the system is analyzed by HTT transformation, and the frequency response curve associated with the amplitude and instantaneous frequency of the vibration response is obtained, and the nonlinear characteristics of the system are further analyzed.

Assume that the formula (9) system is stimulated by a Sine excited $F(t) = f_0\exp(i\omega t)$, the complex frequency response function of the system is obtained：

$$H(\omega) = \frac{1}{\omega_x^2 - \omega_x + j2h_x\omega} \tag{11}$$

The amplitude of the steady-state response of the system：

$$A = \frac{f_0}{\sqrt{\left(1 - \dfrac{\omega^2}{\omega_x^2}\right)^2 + \dfrac{4h_x^2\omega^2}{\omega_x^2}}} \tag{12}$$

Furthermore, the formula (12) is transformed into the equation of the frequency response function [10]:

$$\omega^2 = \left(1 - 2h_x^2\right)\omega_x^2 \pm \omega_x^2\sqrt{4h_x^2\left(h_x^2 - 1\right) + \frac{f_0^2}{A^2}} \tag{13}$$

From the Equation (12) and (13), the instantaneous amplitude (A) and instantaneous frequency (ω) of the system are functions related to the transient parameter $h_x(t)$ and $\omega^2(t)$. Therefore, for vibration signal $x(t)$, after the HTT transformation, the analytical signal $X(t)$ can be easily analyzed, thus obtaining instantaneous amplitude $A(t)$ and instantaneous frequency $\omega(t)$ of the system.

3. Results and discussion

Numerical simulation example analysis: vibration characteristics of *Duffing* nonlinear single free vibration system.

Duffing nonlinear vibration system equation：

$$\ddot{x}(t)+2\xi\omega_0\dot{x}(t)+\omega_0^2 x(t)+\varepsilon x^3(t)=0$$

Where, $\omega_0=120$, $\xi=0.03$, $\varepsilon=2$, Sampling frequency is 1000HZ, Time is $0\sim 2s$, The initial amplitude is 200mm.

The fourth order Runge-Kutta method is used in MATLAB. Furthermore, in order to investigate the time-varying characteristics of the response, the EMD decomposition was applied. Thus, the EMD decomposition diagram of the response was obtained as shown in figure 1. As you can see from figure 1, due to only one single freedom of free vibration modal and vibration response signal of the EMD only first-order IMF, the response can be regarded as a single component signal, So directly the instantaneous amplitude and instantaneous frequency of the signal (the mean of the curve is smoothed) are obtained by the Hilbert transform， As shown in figure 2 and 3, The amplitude-frequency curve is shown in figure 4.

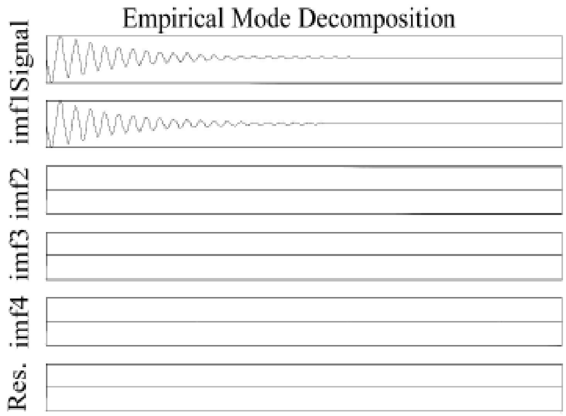

Figure 1. Empirical Mode Decomposition of system

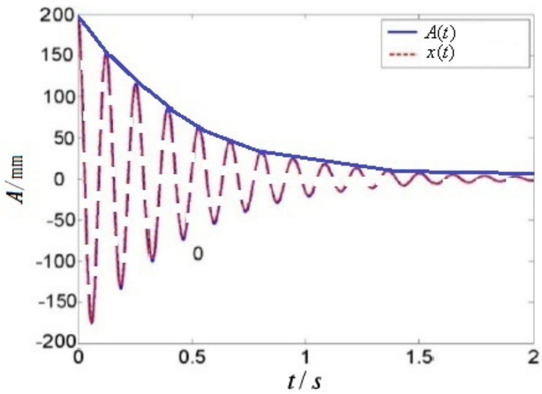

Figure 2. The instantaneous amplitude of system

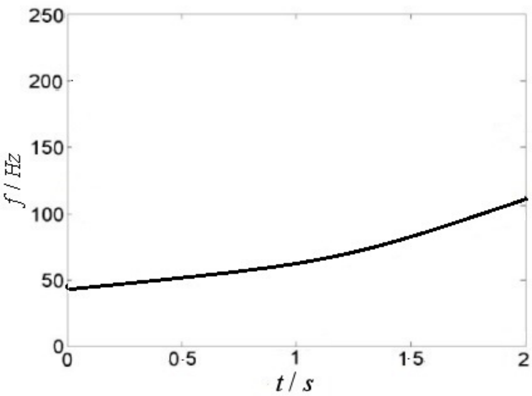

Figure 3. The instantaneous frequency of system

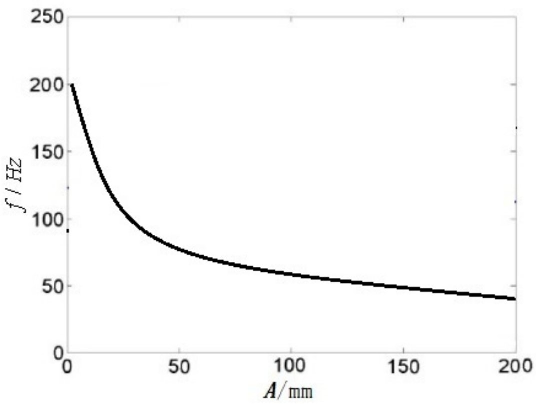

Figure 4. The amplitude-frequency curve of system

The figure 2 and figure 3 shows that the amplitude of the response of the system attenuates with time, and the frequency of vibration increases with time. As shown in figure 4, the frequency decreases with the amplitude of the amplitude. Figure 3 shows the instantaneous frequency obtained by the HTT transformation mentioned, the lowest frequency of the system is 47 Hz. Figure 4 is the first frequency response curve obtained by the HTT transformation method, and the corresponding lowest frequency is 47 Hz, which is consistent with the result of figure 3.

To sum up, the example is the application frequency response relationship equations in section 2.2. In terms of the vibration response, the dynamic characteristics of the system can be determined. In this case, the intrinsic parameters of the system are no longer important, the method can avoid or ignore the recognition errors caused by frequency side effects and frequency fluctuations. The validity of the method is verified.

4. Conclusion

In this paper, the instantaneous frequency and instantaneous amplitude envelope of the vibration signal are obtained based on the Hilbert transform. And the method is applied to the analysis of the single-degree-of-freedom nonlinear system, the vibration equation is expressed as the standard equation relating to the transient amplitude, and the nonlinear characteristics of the system are analyzed based on the HHT transform. Research shows:

(1) Since there is only one modal in the free vibration with single-degree-of-freedom, the response can be treated as one component of signal. Moreover, instantaneous frequency and instantaneous amplitude can be obtained by applying the Hilbert transform method. As a result, the vibration parameter identification value of the single-degree-of-freedom system is more efficient.

(2) The standard equation of the system frequency response with related transient parameters is derived, In the case of unknown system parameters, the frequency curve is used to determine the characteristic of the nonlinear vibration of the system. This method effectively avoids the recognition error caused by the prominent endpoint effect and frequency fluctuation. Numerical examples demonstrate the validity of the method.

Acknowledgments

This research was financially supported by the Natural Scientific Research Foundation of Fujian, China (Grant No. 2016J01242).

References

[1] Qin Y., Qin S., Mao Y.F. (2008) Research on iterated Hilbert transform and its application in mechanical fault diagnosis. Mechanical Systems and Signal Processing, 22: 967-980.
[2] Huang T., Lou M.L. (2006) The study of nonlinear structural system identification based on Hilbert transform. Earthquake Engineering and Engineering Vibration, 26: 80-85.
[3] Yi W.J., Duan S.P. (2008) Identification of nonlinear dynamical characteristics of cracked reinforced concrete beam. Journal of Vibration and Shock, 27: 27-31.
[4] Chen L., Lei M.F . (2014) Research on damage detection of the cantilever beam based on time-varying stiffness model. Journal of Railway Science and Engineering, 11: 32-38.
[5] Huang D.H., Zhou S., Ren W.X. (2014) Identification of the time-varying and typical nonlinear vibration system based on wavelet transform. Journal of Vibration and Shock, 33: 123-129.
[6] Huang Z.H.，Xie Y. (2011) Fault diagnosis of roller bearing with inner and external fault based on Hilbert transform. Journal of Central South University, 42:1992-1996.
[7] Peng Zhike, Tse P W, Chu Fulai. (2005) A comparison study of improved Hilbert-Huang transform and wavelet transform: Application to fault diagnosis for rolling bearing. Mechanical Systems and Signal Processing, 19: 974-988.
[8] Peng Zhike, Tse P W, Chu Fulai. (2005) An improved Hilbert-Huang Transform and its application in vibration signal analysis. Journal of Sound and Vibration, 286:187-205.
[9] Liu Y.Z., Chen L.Q. (2001) Nonlinear Vibration. Higher Education Press of China, Beijing.
[10] Kong X.R., Xiong H., Li H.Q. (2016) The modified method of Hilbert Transform and application in nonlinear vibration analysis. Journal of Mechanical Engineering, 52: 95-101.