

An improved optimal control strategy for hybrid AC/DC power system

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Abstract. The additional modulation function of high voltage direct current (HVDC) links can improve the system stability while the unavoidable random external disturbance and sudden changes seriously affect the controller performance and the system stability. In this paper, an improved control scheme based on inverse optimal adaptive control is proposed for the hybrid AC/DC system which considers the system parameter error, external disturbance and other random perturbation. The proposed control scheme combines the inverse optimal control approach and the adaptive control method to account for the modelling errors and uncertainty, which aims at the disturbance attenuation and the optimal operating of the hybrid AC/DC power system. Besides, the improved PI (Proportional-Integral) controller based on the lead-lag link for the current control of HVDC system is proposed to further improve system dynamic performance. Simulations conducted on a typical hybrid AC/DC system show the correctness and validity of the proposed control method.

1. Introduction

With the rapid and worldwide development of HVDC technology, scholars home and abroad have more and more researches for HVDC and hybrid AC/DC power system [1-3]. Since the advent of modern control theory with the representative of linear systems and optimal control, fruitful achievements of theoretical innovation and applications have been proposed, especially the wide applications in power system [4-6]. However, the traditional linear optimal control uses the approximate linear model based on rated conditions and is only suitable for small signal stability problem in principle. Although the nonlinear control based on differential geometry and feedback linearization method [7] can be superior to linear method in improving large disturbance stability, its modeling is still based on a fixed structure and the modeling errors and disturbance attenuation issue are not involved in the design of these controllers, which cannot ensure the robustness of the controllers. Due to the inherent robustness with respect to modeling errors and external disturbances, the nonlinear H_∞ optimal control [8] is a potential method to solve the optimal and robust problem. However, the applications of H_∞ optimal control remain open since solving the Hamilton-Jacobi-Isaacs (HJI) partial differential equation is relatively



difficult to realize. Scholars have proposed various methods [9-12] to study the H_∞ optimal control but these methods are mostly based on solving the HJI partial differential equation. To avoid solving the HJI equation, Freeman and Kokotovic proposed the inverse optimal control [13] in 1990s. In the control, a control law and a Lyapunov function are designed first, and then it is shown to be optimal with respect to a meaningful objective functional, whose minimal value can be any difference with that of the objective functional given before the design. The inverse optimal control can not only ensure the robust stability of the uncertain nonlinear system with external disturbance, but also solve the global optimization problem and some actual application achievements have been made [14-16]. In this paper, the inverse optimal control approach and the adaptive control method are combined to account for the modeling errors and uncertainty, the disturbance attenuation, and the optimality of the DC modulation controller for the hybrid AC/DC power system to enhance the safe and stable operation of the power system.

In this paper, an improved scheme based on inverse optimal adaptive control and the improved PI controller is proposed to improve the dynamic performance of the hybrid AC/DC power system. This paper proceeds as follows. Section 2 describes the system modelling of hybrid AC/DC power system. Section 3 introduces the inverse optimal adaptive control and the improved PI controller based on the lead-lag link. In Section 4, a typical hybrid AC/DC system is taken to verify the validity of the proposed control strategies. Finally, Section 5 highlights the conclusions of the whole paper.

2. System modelling of hybrid AC/DC power system

The typical hybrid AC/DC power system is shown in figure 1. In the figure, the G_A and G_B is the equivalent generators of area A and area B. There are two power transmission channel. The P_{ac} is the AC power transmission and The P_{dc} is the DC power transmission. P_{LA} and P_{LB} are the local loads.

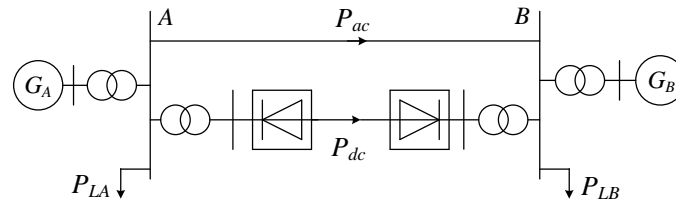


Figure 1. Hybrid AC/DC transmission system.

The dynamic equations of the system in figure 1 can be written as

$$\begin{cases} \dot{\delta}_{AB} = \omega_n \omega_{AB} \\ \dot{\omega}_{AB} = \frac{1}{M_{TA}} (P_{mA} - P_{LA} - P_{ac} - P_{dc}) - \frac{1}{M_{TB}} (P_{mB} - P_{LB} + P_{ac} + P_{dc}) - \theta \omega_{AB} + \varepsilon_D w_d \\ \dot{P}_{dc} = (P_{dcn} - P_{dc} + P_u) / T_{dc} + \varepsilon_P w_d \end{cases} \quad (1)$$

where $\delta_{AB} = \delta_A - \delta_B$, $\omega_{AB} = \omega_A - \omega_B$ and $\omega_n = 2\pi f$ is the rated angular speed; P_{mA} and P_{mB} are the mechanical inputs of the generators; P_{LA} and P_{LB} are the local loads; P_{ac} is the AC transmission power and P_{dc} is the DC transmission power; P_{dcn} is reference DC power, ΔP_0 is DC modulation signal and T_{dc} is time constant of DC system; θ is the equivalent damping coefficient, which is unknown usually; ε_D is the error coefficient of equivalent damping and external disturbance; ε_P is the error coefficient of DC system equivalent and external disturbance and w_d is the external random disturbance.

Define a new coordinate λ as

$$\lambda = \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} \frac{\delta_{AB} - \delta_0}{\omega_n} \\ \omega_{AB} \\ \frac{P_{mA} - P_{LA}}{M_{TA}} - \frac{P_{mB} - P_{LB}}{M_{TB}} - \frac{P_{ac} - P_{dc}}{M} \end{bmatrix} \quad (2)$$

where $1/M = (1/M_{TA} + 1/M_{TB})$; δ_0 is the steady value of δ_{AB} .

The mechanical power of the generator and the load power are assumed to be slowly varying with time. Thus, it can be derived by

$$\dot{\lambda}_3 = -\frac{\dot{P}_{ac}}{M} + \frac{(-P_{dc} + P_{dcn} + P_u)}{T_{dc}M} + \frac{\varepsilon_P w_d}{M} \quad (3)$$

Take virtual control v as

$$v = -\frac{\dot{P}_{ac}}{M} + \frac{-P_{dc} + P_{dcn} + P_u}{T_{dc}M} \quad (4)$$

and it results that

$$\dot{\xi}_3 = v + \frac{\varepsilon_P w_d}{M} \quad (5)$$

Then the dynamic model of the system in figure 1 can be written as

$$\begin{cases} \dot{\lambda}_1 = \lambda_2 \\ \dot{\lambda}_2 = \lambda_3 + \varphi_2 \theta + \eta_2 w_d \\ \dot{\lambda}_3 = v + \eta_3 w_d \end{cases} \quad (6)$$

where $\varphi_2 = \xi_2$, $\eta_2 = \varepsilon_D$ and $\eta_3 = \frac{\varepsilon_P}{M}$.

3. The improved optimal control for the hybrid AC/DC system

3.1. Inverse optimal adaptive control

Inverse optimal adaptive control can solve the global optimization problem of nonlinear systems and ensure the robustness of the system meanwhile. The inverse optimal control approach and the adaptive control method are combined into the hybrid AC/DC grid for the optimal operation of the system. In this section, the application of inverse optimal adaptive control is extended to hybrid AC/DC system. Besides, a sufficient condition for the solvability of this problem is given based on *Definition 1* in the *Appendix* section

Consider the following general nonlinear system

$$\dot{\lambda}(t) = f(\lambda, \theta) + g_1(\lambda)w_d + g_2(\lambda)v(t) \quad (7)$$

where $\lambda(t)$ is the state variable, w_d is the external uncertainty disturbance and $v(t)$ is the control input. The solution of the inverse optimal adaptive control for system (7) is given by solving the stability problem of the auxiliary system. Considering the auxiliary system of (8)

$$\dot{\lambda} = f(\lambda, \theta) + g_1 \ell \gamma(2|L_{g_1} V|) \frac{(L_{g_1} V)^T}{|L_{g_1} V|^2} + g_2 v \quad (8)$$

where $V(\lambda, \hat{\theta})$ is a defined Lyapunov function and γ is a class \mathcal{K}_∞ function while the derivative γ' is also a class \mathcal{K}_∞ function. $\ell \gamma$ is the Legendre-Fenchel transformation and satisfies $\ell \gamma(r) = r(\gamma')^{-1}(r) - \gamma((\gamma')^{-1}(r))$ where $(\gamma')^{-1}(r)$ is the inverse function of $d\gamma(y)/dy$. If there exists a control law v

$$v = \alpha(\lambda, \hat{\theta}) = -R(\lambda, \hat{\theta})^{-1} (L_{g_2} V)^T \quad (9)$$

which can globally asymptotically stabilize (8), then the control law v^*

$$\begin{aligned} v^* &= \alpha^*(\lambda, \hat{\theta}) \\ &= \beta \alpha(\lambda, \hat{\theta}) \\ &= -\beta R(\lambda, \hat{\theta})^{-1} (L_{g_2} V)^T, \beta \geq 2 \end{aligned} \quad (10)$$

minimizes the cost functional (11) and solves the inverse optimal adaptive control for system (7)

$$J(v) = \sup_{w_d \in D} \left\{ \lim_{t \rightarrow \infty} \left[2\beta V(\lambda, \hat{\theta}) + \int_0^t (l(\lambda, \hat{\theta}) + v^T R(\lambda, \hat{\theta}) v - \beta \mathcal{G} \gamma(|w_d|/\mathcal{G}) d\tau) \right] \right\} \quad (11)$$

where $0 < \mathcal{G} \leq 2$ and

$$\begin{aligned} l(\xi, \hat{\theta}) &= -2\beta \left[L_f V + \ell \gamma(2|L_{g_1} V|) - L_{g_2} V R^{-1} (L_{g_2} V)^T \right] \\ &+ \beta(2 - \mathcal{G}) \ell \gamma(2|L_{g_1} V|) + \beta(\beta - 2) L_{g_2} V R^{-1} (L_{g_2} V)^T \end{aligned} \quad (12)$$

3.2. Inverse optimal adaptive control of hybrid AC/DC power system

The standard form of system (5) is

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = \begin{bmatrix} \lambda_2 + \varphi_1 \theta \\ \lambda_3 + \varphi_2 \theta \\ 0 + \varphi_3 \theta \end{bmatrix} + g_1 w_d + g_2 v \quad (13)$$

where $g_1 = [\eta_1, \eta_2, \eta_3]^T$, $g_2 = [0, 0, 1]^T$, $\varphi_1 = \varphi_3 = 0$ and $\eta_1 = 0$. Let $\varphi_0 = \eta_0 = 0$ for notational convenience.

Taking $\gamma(y) = y^2/\mu$ where μ is a positive constant yet to be determined, we have $\ell \gamma(2y) = \mu y^2$. Then the auxiliary system of system (13) is

$$\begin{bmatrix} \dot{\lambda}_1 \\ \dot{\lambda}_2 \\ \dot{\lambda}_3 \end{bmatrix} = \begin{bmatrix} \lambda_2 + \varphi_1 \theta \\ \lambda_3 + \varphi_2 \theta \\ 0 + \varphi_3 \theta \end{bmatrix} + \mu g_1 (L_{g_1} V)^T + g_2 v \quad (14)$$

Taking the following coordinate change

$$\begin{cases} z_1 = \lambda_1 \\ z_2 = \lambda_2 - \alpha_1 \\ z_3 = \lambda_3 - \alpha_2 \end{cases} \quad (15)$$

where α_1 and α_2 are smooth functions yet to be determined (For notational convenience we define $\alpha_0 = 0$) and choosing the following Lyapunov function for system (15)

$$V(\lambda, \hat{\theta}) = \sum_{i=1}^3 \frac{1}{2} z_i^2 + \frac{1}{2} P \tilde{\theta}^2 \quad (16)$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\hat{\theta}$ is the estimate of the unknown parameters θ and $\tilde{\theta}$ is the estimation error, the derivative of Lyapunov function (16) can be given as

$$\begin{aligned} \dot{\mathcal{L}}V(\lambda) &= \frac{\partial V}{\partial \lambda} \dot{\lambda} + \frac{\partial V}{\partial \hat{\theta}} \dot{\hat{\theta}} \\ &= z_n v + \sum_{i=1}^n \frac{\partial V}{\partial \lambda_i} (\lambda_{i+1} + \mu \eta_i (L_{g_1} V)^T + \varphi_i \theta) + \sum_{i=1}^n z_i \left(-\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \right) - P \tilde{\theta} \dot{\hat{\theta}} \end{aligned} \quad (17)$$

If we take

$$\begin{cases} L_{g_1} V = \sum_{i=1}^n \varpi_i z_i \\ \sum_{i=1}^n \frac{\partial V}{\partial \lambda_i} \varphi_i \theta = \sum_{i=1}^n \psi_i z_i \theta \end{cases} \quad (18)$$

where $\varpi_i = \eta_i - \frac{\partial \alpha_{i-1}}{\partial \lambda_{i-1}} \eta_{i-1}$, $\psi_i = \varphi_i - \frac{\partial \alpha_{i-1}}{\partial \lambda_{i-1}} \varphi_{i-1}$ and use the inequality

$$z_k z_{k+1} \leq \frac{1}{2} z_k^2 + \frac{1}{2} z_{k+1}^2, \quad k=1,2 \quad (19)$$

Equation (17) can be further written as

$$\begin{aligned} \dot{\mathcal{L}}V(\lambda) &\leq z_3 \left(v + \frac{1}{2} z_3 + \mu (\varpi_3)^2 z_3 + 2\mu (\varpi_2 z_2) \varpi_3 \right. \\ &\quad \left. - \sum_{k=1}^2 \frac{\partial \alpha_2}{\partial \lambda_k} \lambda_{k+1} - \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} + \psi_3 \hat{\theta} \right) + z_2 \left(z_2 + \alpha_2 + \mu (\varpi_2)^2 z_2 \right. \\ &\quad \left. - \frac{\partial \alpha_1}{\partial \lambda_1} \lambda_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} + \psi_2 \hat{\theta} \right) + \left(\frac{1}{2} z_1 + \alpha_1 \right) z_1 + \left(\sum_{i=1}^3 \psi_i z_i - P \dot{\hat{\theta}} \right) \tilde{\theta} \end{aligned} \quad (20)$$

Take the smooth function ϕ satisfying

$$\phi \sum_{k=1}^3 z_k = - \sum_{k=1}^2 \frac{\partial \alpha_2}{\partial \lambda_k} \lambda_{k+1} - \frac{\partial \alpha_2}{\partial \hat{\theta}} \dot{\hat{\theta}} + \psi_3 \hat{\theta} \quad (21)$$

and we have

$$\begin{aligned}
\mathcal{L}V(\lambda) &\leq z_3 \left(v + \frac{1}{2} z_3 + \mu(\varpi_3)^2 (z_3) + \sum_{k=1}^3 \Phi_k z_k \right) \\
&+ z_2 \left(z_2 + \alpha_2 + \mu(\varpi_2)^2 z_2 - \frac{\partial \alpha_1}{\partial \lambda_1} \lambda_2 - \frac{\partial \alpha_1}{\partial \hat{\theta}} \dot{\hat{\theta}} + \psi_2 \hat{\theta} \right) \\
&+ \left(\frac{1}{2} z_1 + \alpha_1 \right) z_1 + \left(\sum_{i=1}^3 \psi_i z_i - P \dot{\hat{\theta}} \right) \tilde{\theta}
\end{aligned} \tag{22}$$

where $\Phi_1 = \Phi_3 = \phi$ and $\Phi_2 = 2\mu\varpi_3\varpi_3 + \phi$.

If the parameter α_1 and α_2 are chosen as

$$\begin{cases} \alpha_1 = -c_1 z_1 - \frac{1}{2} z_1 \\ \alpha_2 = -c_2 z_2 - z_2 - \mu(\varpi_2)^2 z_2 + \frac{\partial \alpha_1}{\partial \lambda_1} \lambda_2 - \psi_2 \hat{\theta} \end{cases} \tag{23}$$

and the control law v is taken as

$$v = \alpha_3(\lambda) = -R(\lambda)^{-1} z_3 \tag{24}$$

where $R(\lambda) = \left(c_3 + \frac{1}{2} + \mu(\varpi_3)^2 + \sum_{k=1}^3 \frac{\Phi_k^2}{2c_k} \right)^{-1} > 0$, we have

$$\mathcal{L}V(\lambda) \leq -\frac{1}{2} \sum_{k=1}^3 c_k (z_k)^2 - \frac{1}{2} \sum_{k=1}^3 c_k \left(z_k - \frac{\Phi_k}{c_k} z_3 \right)^2 + \left(\sum_{i=1}^3 \psi_i z_i - P \dot{\hat{\theta}} \right) \tilde{\theta} \tag{25}$$

Taking the adaptive law $\dot{\hat{\theta}} = P^{-1} \left(\sum_{i=1}^3 \psi_i z_i - \hat{\theta} \right)$, it follows that

$$\begin{aligned}
\mathcal{L}V(\lambda) &\leq -\frac{1}{2} \sum_{k=1}^3 c_k (z_k)^2 + \hat{\theta} \tilde{\theta} \\
&= -\frac{1}{2} \sum_{k=1}^3 c_k (z_k)^2 + \frac{1}{2} (\theta^2 - \tilde{\theta}^2 - \hat{\theta}^2) \\
&\leq -cV(\lambda) + \Upsilon
\end{aligned} \tag{26}$$

where $\Upsilon = \frac{1}{2} \theta^2$, $c = \min\{2c_1, 2c_2, 2c_3\}$. Let $\rho = \Upsilon/c > 0$, then the inequality (26) satisfies

$$0 \leq V \leq \rho + [V(0) - \rho] e^{-ct} \tag{27}$$

It is seen from (27) that when time $t \rightarrow \infty$, V exponentially decays to ρ . Therefore $\lambda(t)$ is globally uniformly bounded and the auxiliary system (14) achieves globally asymptotically stability.

According to (10), the control law

$$\begin{aligned}
 v &= \beta \alpha_3(\lambda) = -\beta R(\lambda)^{-1} z_3 \\
 &= -\beta \left(c_3 + \frac{1}{2} + \mu(\varpi_3)^2 + \sum_{k=1}^3 \frac{\Phi_k^2}{2c_k} \right)^{-1} z_3, \beta \geq 2
 \end{aligned} \tag{28}$$

solves the inverse optimal adaptive control for system (13).

3.3. Improved PI controller for the current control of HVDC system

In the current control of HVDC system, the PI controller aims at the tracking the dc current for the reference with no steady-state error, which is essential in the stable operation of system. However, traditional PI controller lacks ideal dynamic response and cannot effectively suppress external disturbance, and may even trigger protection action. Therefore, the improvement of dynamic performance is of great concern to keep the HVDC system stay stable. In this paper, the improved PI controller for the current control based on the lead-lag link for the HVDC system is proposed to further improve system dynamic performance. The control block diagram is shown in figure 2. In the figure, the ω_i , P_{mi} , P_{Li} , P_{dc} , P_{ac} , P_u are defined in Section 2. P_0 is the rated reference transmission power of the system. U_d is the DC voltage and current. I_{max} and I_{min} are the maximum and minimum limit of the obtained current command. K_1 and K_2 are the control parameters of the PI controller. The α , α_{max} , and α_{min} Are the obtained, maximum limit and minimum limit of the firing angle.

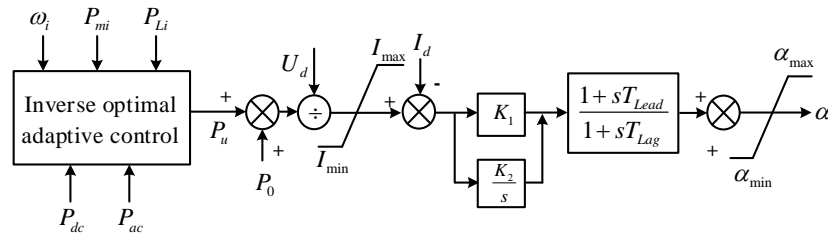


Figure 2. Control block diagram of the improved PI controller.

This link models a lead-lag function, where T_{Lead} is the lead time constant and T_{Lag} is the lag time constant. In this link, there also includes maximum and minimum output limits internally. With proper parameter design of T_{Lead} and T_{Lag} , the dynamic response of the PI controller can be realized. The control effect can be obviously seen in the case study of a typical hybrid AC/DC system in the next section.

4. Simulation results

The validation of the proposed control is tested by simulation cases on a typical hybrid AC/DC system shown in figure 3. The conventional control and our proposed control are used on this test system. The contingency of system operating conditions are simulated in this case.

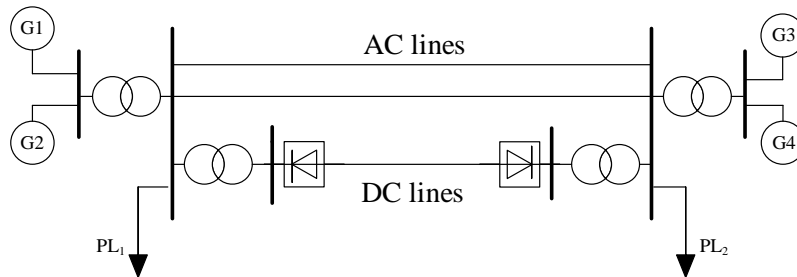


Figure 3. Typical HVDC system diagram.

In the initial steady-state operating conditions, the voltage of DC line is 800 kV and the transmission power is 5000 MW; the voltage of AC line is 500 kV and the transmission power is 2000 MW. Take equivalent time constant of the DC line is $T_d = 0.1$ s. In the improved PI controller, the T_{Lead} is set as $2e-5$ s and the T_{Lag} is set as $2e-6$ s.

Case 1:

In this case, the system operating conditions of is set as the short-circuit fault and load power fluctuations. It is supposed that the system is operating normally before 2s. At time 2s, the single-phase earth fault occurs in the receiving-end bus and lasts 0.01s. At time 2.5s, there exists 10% load fluctuation. The response curves of rotor angle and frequency deviation under the different controls are shown in figure 4.

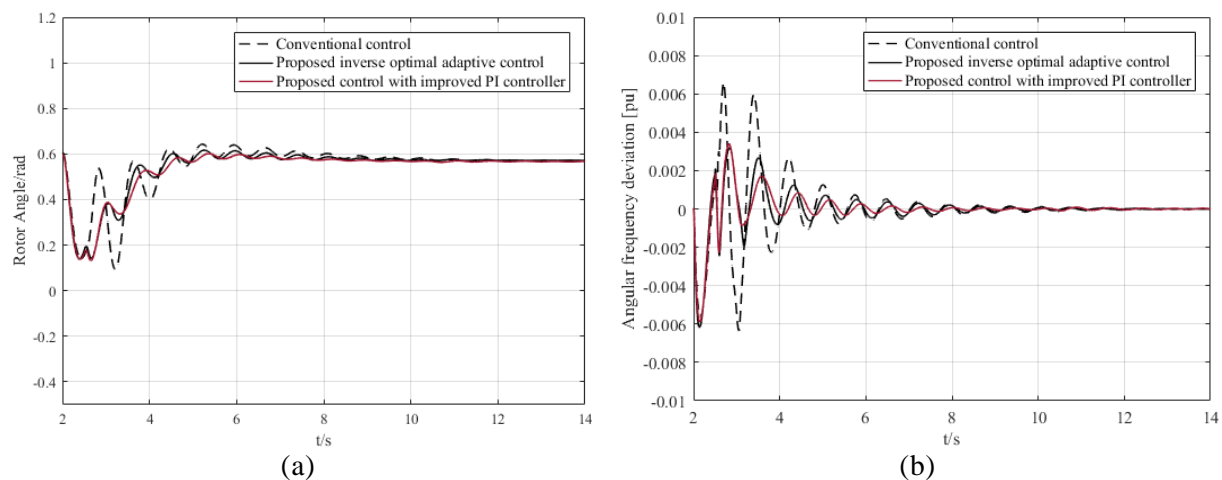


Figure 4. Response curves with different controls of case 1. (a) Rotor Angle of G1 relative to G4 and (b) frequency deviation between the sending and receiving areas of the AC line 1.

The response curves of rotor angle and frequency deviation shown in figures 4(a) and 4(b) indicate that, when facing successive and different changes of operating conditions, the overshoot of rotor angle and frequency deviation is relatively small and the recovery time is faster with the proposed control than the conventional control. From figure 4(a), the lowest point of the rotor angle response curve is improved with the proposed control while the oscillation amplitude is also mitigated. From figure 4(b), the oscillation amplitude of the frequency deviation is nearly 0.013 pu with the conventional control while it is obviously reduced under the proposed control. In addition, it can be seen that with the improved PI controller, the overshoots of both the rotor angle and frequency deviation are further reduced with shorter recovery time. Hence the proposed inverse optimal adaptive control with improved PI controller can significantly provide adequate damping and improve the dynamic response performance of the system to a great extent, which ensure the strong robustness of the system under the premise of the optimal performance of the relevant electric parameters.

Case 2:

To further test the effectiveness of the proposed inverse optimal adaptive control with improved PI controller, another case is conducted in this section. The system is operating normally before 2s and the three-phase earth fault occurs in the receiving-end bus at time 2s and lasts 0.01s. Additional, there exists 20% load fluctuation at time 2.5s. The response results of rotor angle and frequency deviation with different controls are shown in figure 5.

The response results shown in figures 5(a) and 5(b) indicate that the system loses stability with conventional control while the proposed controllers provide adequate damping and can improve the dynamic response performance of the system. It is also shown that, when facing successive and different changes of operating conditions, the overshoot of rotor angle and

frequency deviation is relatively small and the response curves reduce the amplitude faster under the proposed control with improved PI controller. Therefore, the proposed inverse optimal adaptive control with improved PI controller can significantly improve the dynamic performance of the system.

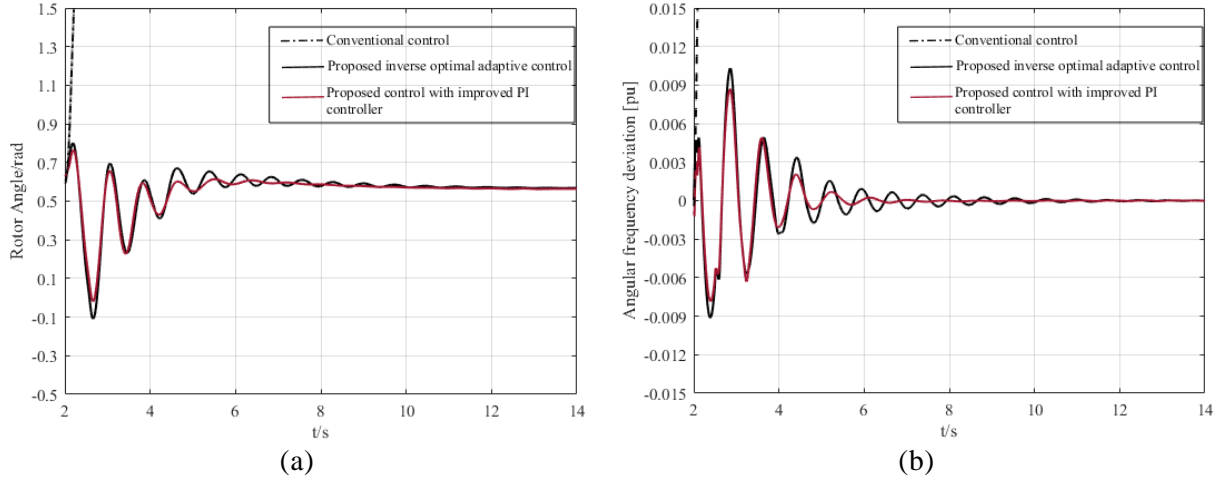


Figure 5. Response curves with different controls of case 2. (a) Rotor Angle of G1 relative to G4 and (b) frequency deviation between the sending and receiving areas of the AC line 1.

5. Conclusion

- In this paper, the control scheme based on inverse optimal adaptive control and improved PI controller is proposed to improve the dynamic performance of the hybrid AC/DC power system. The system model takes the system parameter error, external disturbance and other uncertainties random perturbation into consideration in order to strengthen the robustness of the system.
- The inverse optimal adaptive control for the hybrid AC/DC power system proposed in this paper can not only ensure the robust stability of the uncertain nonlinear system with external disturbance, but also solve the global optimization problem without solving the HJI equation, which is easy to realize.
- The improved PI controller based on the lead-lag link for the current control of the HVDC system is proposed to further improve system dynamic performance. The simulation results show that the proposed control scheme can increase oscillation damping of the system power and improve the stability of the interconnected system to a great extent.

Appendix

Definition 1 [17]: If there exists a class \mathcal{K}_∞ function γ whose derivative γ' is also a class \mathcal{K}_∞ function, positive definite function $R(\lambda, \hat{\theta})$ with $R = R^T > 0$, positive definite unbounded functions $l(\lambda, \hat{\theta})$ and $E(\lambda, \hat{\theta})$, and a feedback control law $v = \alpha(\lambda, \hat{\theta})$ with $\alpha(0, \hat{\theta}) = 0$, which minimizes the following cost functional

$$J(v) = \sup_{w_d \in D} \left\{ \lim_{t \rightarrow \infty} \left[E(\lambda, \hat{\theta}) + \int_0^t \left(l(\lambda, \hat{\theta}) + v^T R(\lambda, \hat{\theta}) v - \gamma(|w_d|) \right) d\tau \right] \right\} \quad (29)$$

where D is the set of locally bounded functions and $\hat{\theta}$ is the dynamic estimates of the unknown parameter θ , then the inverse optimal adaptive control for the nonlinear system is solvable. The cost functional $J(v)$ puts penalty on the state λ , control v and disturbance w_d so this is

the differential games of system (6), which aims to find out then optimal control under the worst disturbance.

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