Energy transfer mechanism analysis of MCR-WPT based on the theory of waves

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Abstract. In a magnetic coupling resonant wireless power transfer (MCR-WPT) system, energy is transferred between transmitter and receiver coils with same natural resonant frequency ω_0. Studying energy transfer mechanism is the theoretical basis of MCR-WPT control. This paper takes the MCR-WPT system with two coils as the research object. Firstly, the model of the system is established. Then, combined with the theory of waves, the change of the capacitor charge of the two resonant circuits is analysed, which is drawn up during the linear superposition of two different frequency modes and is also a modulation wave with a specific system resonant frequency ω_s as the carrier frequency and ω_Δ as the modulation frequency. Where ω_s is different from ω_0. Next, the energy of the two resonant circuits is analysed, which is transferred with the beat frequency $2\omega_\Delta$. Finally, ANSYS Simplorer and Maxwell co-simulation is used to verify the correctness of the theory of this paper.

1. Introduction

In recent years, WPT has been widely studied by researchers due to its longer transmission distance and higher transmission efficiency [1]. The wireless transmission of energy is mainly achieved by the resonance of two resonant bodies [2]. The study of energy transfer mechanism has significant meaning to MCR-WPT control.

Most of the existing researches on MCR-WPT use circuit theory or coupled-mode theory to model and analyse the system [3-6]. In the model of the circuit theory, using the mutual inductance model, parameters of the receiver are reduced to the transmitter. However, the process of energy transfer is not explained in detail [3-5]. In the coupled-mode theory model, each resonant unit is considered as a whole. The nature resonant frequency is modified according to the quality factor of the resonant unit, but a coupling coefficient is used to simply express the energy transfer process [6]. In [7], WPT is modeled and analyzed using the vibration theory. It mentioned that there are two modes that can be decoupled when the system works, but the subsequent analysis does not unfold in the perspective of energy. In [8], an energy model of WPT system is established, this model divides the system into multiple energy flow nodes, mainly analyzing the energy flow direction and loss. These articles focus on the analysis of system output power and transmission efficiency influenced by various parameters, while ignoring the analysis of wireless energy transfer links and what impact it will have.

To reveal the mechanism of energy transfer in the system, based on the theory of waves [9], this paper takes the capacitor charge of each resonant circuit as the research object, the model of the MCR-WPT of the two-coil structure is established. The change characteristics of the capacitor charge of each resonant circuit are first obtained, and the system resonant frequency ω_s is obtained. Next, the energy of the two resonant circuits is analyzed, which is transferred with the beat frequency $2\omega_\Delta$ between resonators. Finally, using ANSYS Simplorer and Maxwell co-simulation to verify the correctness of the theory of this paper.

2. MCR-WPT system

This article takes the two-coil MCR-WPT system as an example. The equivalent circuit is shown in figure 1. U_0 is an AC excitation. The two resonant loops have the same parameters, R, L, C represent the coil's equivalent resistance, self-inductance, and compensation capacitance, respectively. R_L represents the load of the system. The loop containing the excitation source is the transmission loop, and the loop containing the load is the reception loop. Q_a and Q_b represent the charge in the compensation capacitance of transmission loop and the receiving loop respectively. Energy is transmitted through the resonance between the two resonant loops. The mutual inductance of the coupled inductor is M.

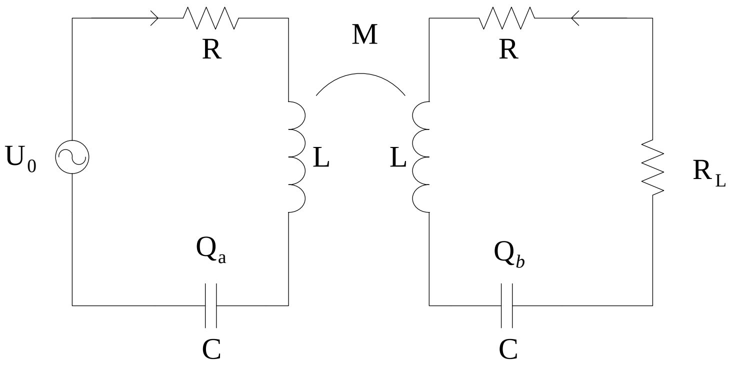

Figure 1. Equivalent circuit diagram of MCR-WPT system.

According to Kirchhoff's voltage law,

$$\begin{cases} L\ddot{Q}_a + M\ddot{Q}_b + R\dot{Q}_a + \frac{1}{C}Q_a = U \\ M\ddot{Q}_a + L\ddot{Q}_b + R'\dot{Q}_b + \frac{1}{C}Q_b = 0 \end{cases} \tag{1}$$

Where $R' = R + R_L$.

Transform formula (1) into a matrix,

$$A\ddot{Q} + R\dot{Q} + BQ = U \tag{2}$$

To simplify the analysis, let $R = R'$, so

$$A = \begin{bmatrix} L & M \\ M & L \end{bmatrix}, R = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, B = \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{C} \end{bmatrix}, U = \begin{bmatrix} U \\ 0 \end{bmatrix}, Q = \begin{bmatrix} Q_a \\ Q_b \end{bmatrix} \tag{3}$$

3. Decomposition and analysis of resonant mode of WPT system

According to the theory of waves, the system can be referred to as a two-degree-of-freedom vibration system with weak coupling. For linear equations of motion, the general motion of the system is the superposition of two simultaneous independent simple harmonic motions. Each simple harmonic motion is called a mode [9].

Assuming that the equivalent mode charge of MCR-WPT system is $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$, the expression of Q_a

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and Q_b can be obtained.

$$\begin{bmatrix} Q_a \\ Q_b \end{bmatrix} = \mathrm{P} \begin{bmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \end{bmatrix} \tag{4}$$

Where P is an orthogonal matrix, which is

$$\mathrm{P} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} \tag{5}$$

Joint formulas (4) and (5) and formula (2), an equation with a decoupled form is obtained.

$$\begin{bmatrix} L+M & 0 \\ 0 & L-M \end{bmatrix} \begin{bmatrix} \ddot{\mathrm{X}}_1 \\ \ddot{\mathrm{X}}_2 \end{bmatrix} + \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} \dot{\mathrm{X}}_1 \\ \dot{\mathrm{X}}_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{C} \end{bmatrix} \begin{bmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} U \\ \frac{\sqrt{2}}{2} U \end{bmatrix} \tag{6}$$

Where

$$P^{-1}AP = \begin{bmatrix} L+M & 0 \\ 0 & L-M \end{bmatrix}, P^{-1}RP = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}, P^{-1}BP = \begin{bmatrix} \frac{1}{C} & 0 \\ 0 & \frac{1}{C} \end{bmatrix}, P^{-1}U = \begin{bmatrix} \frac{\sqrt{2}}{2} U \\ \frac{\sqrt{2}}{2} U \end{bmatrix} \tag{7}$$

To solve formula (6), we can calculate its homogeneous differential equation first. What follows is the solution.

$$\begin{cases} X_1 = X_1 e^{-\alpha_1 t} cos(\omega_1 t + \varphi_1) \\ X_2 = X_2 e^{-\alpha_2 t} cos(\omega_2 t + \varphi_2) \end{cases} \tag{8}$$

With the attenuation coefficients of the homogeneous solution of mode 1 and 2 are

$$\alpha_1 = \frac{R}{2(L+M)}, \alpha_2 = \frac{R}{2(L-M)} \tag{9}$$

And the resonant frequencies of mode 1 and 2 are

$$\omega_1 = \sqrt{\frac{1}{(L+M)C} - \frac{R^2}{4(L+M)^2}}, \omega_2 = \sqrt{\frac{1}{(L-M)C} - \frac{R^2}{4(L-M)^2}} \tag{10}$$

The amplitude and the phase of X_1 and X_2 are determined by initial condition of the system. Which means that different modes of the system consume energy at different frequencies in the absence of motivation.

While in the presence of external incentives, the input of non-homogeneous steady-state solutions for each model is equal to its consumption [9], and the attenuation coefficient is zero. Which needs to satisfy the following formula.

$$\begin{cases} (L+M)\ddot{X}_1 + R\dot{X}_1 + \frac{1}{C} X_1 = \frac{\sqrt{2}}{2} U \cos \omega_1 t \\ (L-M)\ddot{X}_1 + R\dot{X}_1 + \frac{1}{C} X_1 = \frac{\sqrt{2}}{2} U \cos \omega_2 t \end{cases} \tag{11}$$

In the form of solution, we describe the description of amplitude and phase in terms of two amplitudes as follows.

$$\begin{cases} X_1(t) = A_1 \sin \omega_1 t + B_1 \cos \omega_1 t \\ X_2(t) = A_2 \sin \omega_2 t + B_2 \cos \omega_2 t \end{cases} \tag{12}$$

Where the four constants are

$$\begin{cases} A_1 = \dfrac{\frac{\sqrt{2}}{2}UR\omega_1}{\left[\frac{1}{C}-(L+M)\omega_1{}^2\right]^2+(R\omega_1)^2} & B_1 = \dfrac{\frac{\sqrt{2}}{2}U\left[\frac{1}{C}-(L+M)\omega_1{}^2\right]}{\left[\frac{1}{C}-(L+M)\omega_1{}^2\right]^2+(R\omega_1)^2} \\ A_2 = \dfrac{\frac{\sqrt{2}}{2}UR\omega_2}{\left[\frac{1}{C}-(L-M)\omega_2{}^2\right]^2+(R\omega_2)^2} & B_2 = \dfrac{\frac{\sqrt{2}}{2}U\left[\frac{1}{C}-(L-M)\omega_2{}^2\right]}{\left[\frac{1}{C}-(L-M)\omega_2{}^2\right]^2+(R\omega_2)^2} \end{cases} \tag{13}$$

Put formula (12) into formula (4).

$$\begin{cases} Q_a(t) = \frac{\sqrt{2}}{2}X_1(t) + \frac{\sqrt{2}}{2}X_2(t) = \frac{\sqrt{2}}{2}(A_1 \sin\omega_1 t + A_2 \sin\omega_2 t + B_1 \cos\omega_1 t + B_2 \cos\omega_2 t) \\ Q_b(t) = \frac{\sqrt{2}}{2}X_1(t) - \frac{\sqrt{2}}{2}X_2(t) = \frac{\sqrt{2}}{2}(A_1 \sin\omega_1 t - A_2 \sin\omega_2 t + B_1 \cos\omega_1 t - B_2 \cos\omega_2 t) \end{cases} \tag{14}$$

Formula (14) represents that the charge $Q_a(t)$ and $Q_b(t)$ of the two loops are linear superposition of two different frequency modes.

In addition, according to formula (10), it is calculated that $\omega_2 > \omega_1$.

Let

$$\omega_s = \frac{\omega_2+\omega_1}{2}, \omega_\Delta = \frac{\omega_2-\omega_1}{2} \tag{15}$$

Define ω_s as system resonant frequency, ω_Δ as modulation frequency.

We can sort out by formula (14).

$$\begin{cases} Q_a(t) = A_{11}\cos(\omega_\Delta t + \theta_{11})\sin\omega_s t + B_{11}\cos(\omega_\Delta t + \theta_{12})\cos\omega_s t \\ Q_b(t) = A_{12}\cos(\omega_\Delta t + \theta_{21})\cos\omega_s t + B_{12}\cos(\omega_\Delta t + \theta_{22})\sin\omega_s t \end{cases} \tag{16}$$

Where

$$A_{11} = A_{12} = [(A_1 + A_2)^2 + (B_1 - B_2)^2]^{1/2}, B_{11} = B_{12} = [(A_1 - A_2)^2 + (B_1 + B_2)^2]^{1/2} \tag{17}$$

This shows that $Q_a(t)$ and $Q_b(t)$ are modulation waves with ω_s as the carrier frequency and ω_Δ as the modulation frequency.

4. The beating nature of wireless energy transmission

Beat: Oscillation energy slowly flows between two oscillating bodies as a cycle called a beat [9]. Beat frequency is the reciprocal of the cycle.

This paper takes capacitor charge as the research object, since it is convenient to analyse the energy change of MCR-WPT system. In the process of wireless energy transfer, within each cycle corresponding to ω_s, the average energy of the received loop expression is

$$\begin{aligned} \mathrm{E(t)} = \frac{Q_b(t)^2}{2C} &= \frac{A_{12}{}^2\cos^2(\omega_\Delta t + \theta_{21}) + B_{12}{}^2\cos^2(\omega_\Delta t + \theta_{22})}{2C} \\ &= \frac{A_{12}{}^2 + B_{12}{}^2}{2C} + \frac{A_{12}{}^2\cos(2\omega_\Delta t + 2\theta_{21}) + B_{12}{}^2\cos(2\omega_\Delta t + 2\theta_{22})}{2C} \end{aligned} \tag{18}$$

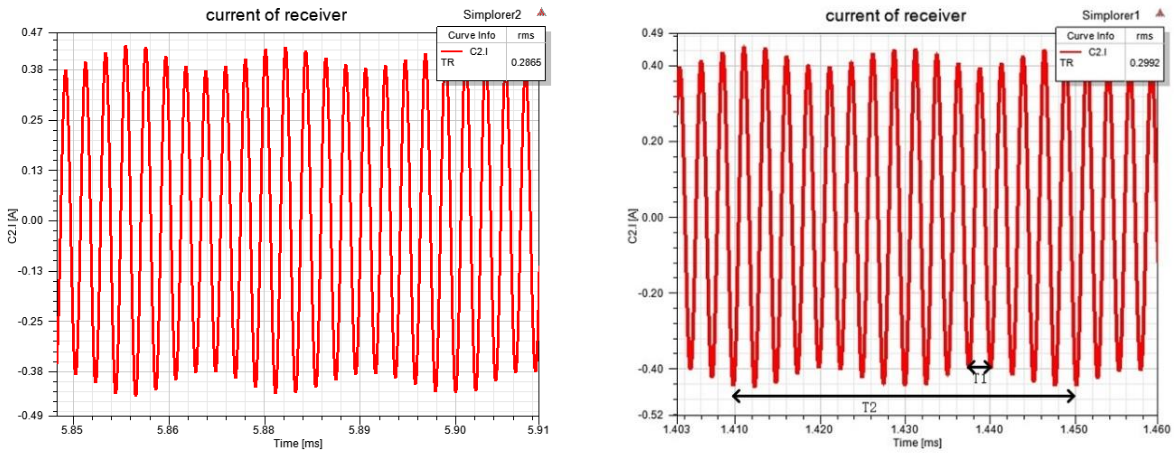
This formula shows that the energy is divided into two parts, one part of which is constant, and the other part is fluctuating at beat frequency of $2\omega_\Delta$.

5. Simulation and results

To verify the theoretical derivation of this article, ANSYS Simplorer and Maxwell co-simulation is used. ANSYS Maxwell is a software specially used for electromagnetic field analysis, this article uses it to build a model of the mutual inductance coil. ANSYS Simplorer is a circuit simulation software, where the Maxwell simulation model can be inserted as a module to realize co-simulation. The following table 1 shows the parameters of this simulation.

Table 1. Parameters of the simulation.

	Explanation	Value
L	Self-inductance of coil	34.45 μH
C	Compensation capacitor	5 nF
R	Circuit resistance	0.01 ohm
M	Mutual inductance of coil	10.293 μH
$f_0 (= \omega_0/2\pi)$	Natural resonant frequency	3.8348e+05 Hz
$f_s (= \omega_s/2\pi)$	System resonant frequency	3.9722e+05 Hz



(a) $f = f_0$ (b) $f = f_s$

Figure 2. Current waveform of receiver at different excitation frequency.

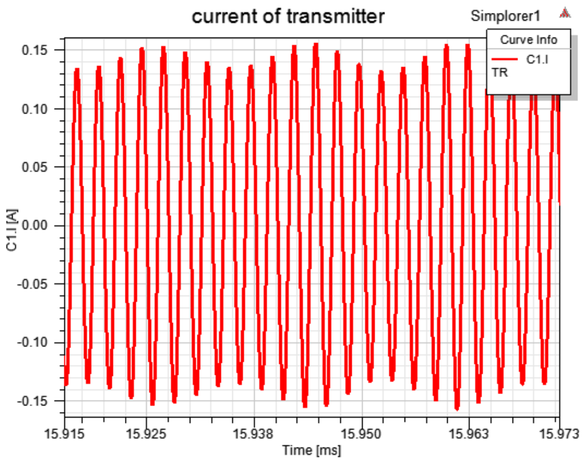


Figure 3. Current waveform of transmitter when excitation frequency is f_s.

Figure 2 illustrates how the current waveform changes when the frequencies of excitation are f_0 and f_s, respectively. When the frequency is f_0, the rms of the current in receiver is 0.2865 A, and the data for frequency at f_s is 0.2992 A. The period T1 marked in figure 2(b) is the oscillation period corresponding to ω_s, and T2 is the modulation period corresponding to ω_Δ, thus, the waveform of the capacitor charge is a modulation wave with ω_s as the carrier frequency and ω_Δ as the modulation frequency. Plus, by comparing figures 2(a) and 2(b), in terms of the effect of energy transmission, the excitation frequency of f_s is better than that of f_0. Figure 3 shows that the current waveform of transmitter is also a modulation wave. According to our analysis, such a current waveform has same features as the receiver. The result above is the basis of wireless energy transfer analysis.

Figure 4 shows the fast Fourier transform of the receiver current when the excitation frequency is f_s. There are three maximum points of frequency as the three markers (m_1, m_2, m_3) shown. The abscissa value of marker m_1 with the highest amplitude is the same as the system resonance frequency $\omega_s/2\pi$, and that of m_2 and m_3 are the same as $\omega_1/2\pi$ and $\omega_2/2\pi$, respectively.

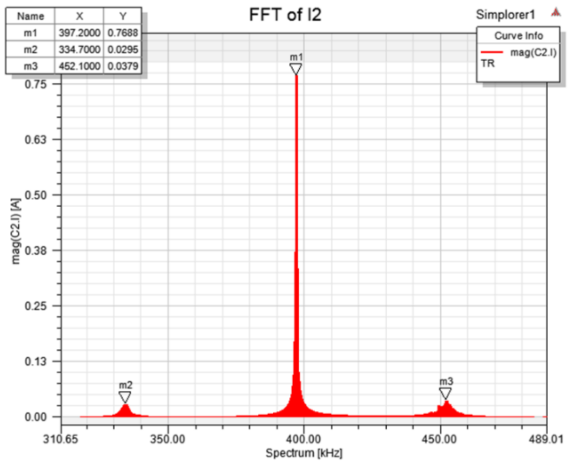
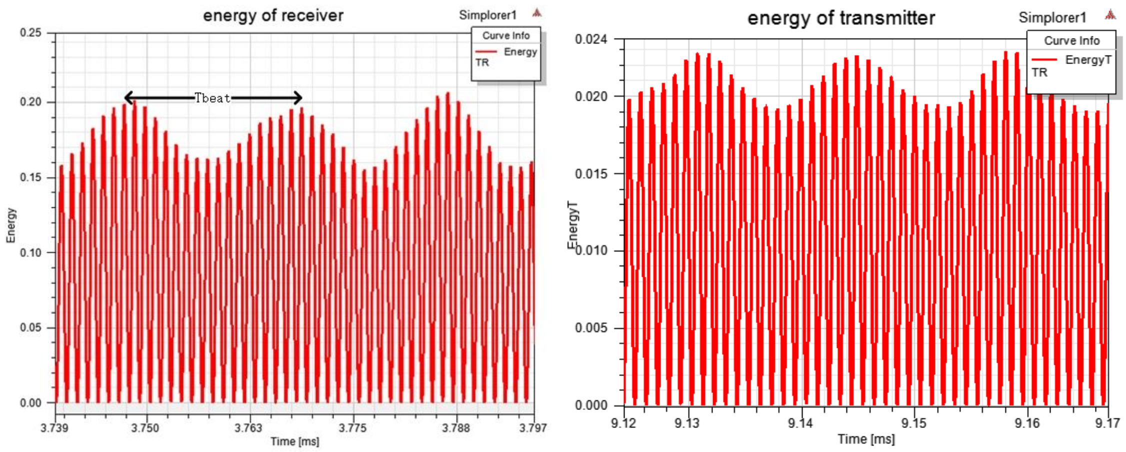


Figure 4. FFT of receiver current when excitation frequency is f_s.



(a) Receiver (b) Transmitter

Figure 5. Energy waveform when excitation frequency is f_s.

The energy waveform of receiver and transmitter are displayed in figure 5, at the excitation frequency of f_s. T_{beat} is defined as the time difference between the two energy peaks. T_{beat} is the reciprocal of beat frequency mentioned in formula (18). The energy flow of the two circuits showed a beat.

6. Conclusion

This paper takes two-coil MCR-WPT system as the research object. On the basis of waves theory, the system is modeled and analyzed. The first step is to decouple the system into two modes. Under steady-state conditions, according to the input energy of each model is equal to its consumption, to solve the differential equation group. From the solution, the capacitor charge in a single resonant loop is a linear superposition of two modes with different frequency ω_1, ω_2. It is also derived that $Q_a(t)$ and $Q_b(t)$ are modulation waves with ω_s as the carrier frequency and ω_Δ as the modulation frequency. Where ω_s is different from ω_0, in terms of the effect of energy transmission, the excitation frequency of ω_s is better than that of ω_0. Next, the energy of the two resonant circuits is analysed, which is transferred with the beat frequency $2\omega_\Delta$. Finally, simulation is to verify the correctness of the proposed theory.

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