Novel determination method of charge transfer coefficient of PEM fuel cell using the Lagrange's multiplier method

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Abstract. This paper presents an evaluation of the transfer coefficient of a reversible PEM fuel cell by using the Lagrange's undetermined multiplier technique. In a first step, the technique using the activation polarization was presented. Then we put the stress on the Lagrange's undetermined multiplier technique. The mathematical bases of this method are presented. Subsequently, an experimental determination of the transfer coefficient was carried out by using the two methods. For that purpose, a kit called Hydrocar, combining a solar panel, a reversible PEM fuel cell (which is a combination of fuel cell and electrolyser), hydrogen and oxygen storage containers, and a 3 Volts alkaline battery, was experimented. The experimental study of the solar panel was made. Some parameters of the solar panel were determined, such as the fill factor FF, the efficiency η, the short circuit current I_{SC}, the open circuit voltage V_{OC}, the maximum power P_m, and the series resistance R_s.The internal resistance R_i and the optimal resistance R_{opt} of the fuel cell were determined. Finally, experimental study of the electrolyser and the PEM fuel cell was performed. By using the activation polarization, an evaluation of the transfer coefficient was made. Then an evaluation of the transfer coefficient was made by using the Lagrange's undetermined multiplier technique which takes into account the ohmic polarization, the activation polarization and also the current-power characteristic of the fuel cell.

1. Introduction

The world energy demand is mainly satisfied today by fossil fuels such as coal, petroleum, natural gas. Fossil fuels of course meet as much as about 80% of the world energy consumption. Some major problems are linked to such energy economy. As a matter of fact, fossil fuels resources are limited and consumed at a rate that becomes faster and faster, due to the increasing demand of energy.

Moreover, fossil fuels are responsible for several environmental damages, namely global climate change, global warming of the earth, sea level rising etc...These damages are mainly caused by the great amount of greenhouse gases released in the atmosphere, such as carbon dioxide (CO_2) and methane (CH_4). Hence, the world is now looking for alternative power sources, which do not release CO_2, and are clean, renewable and sustainable. Among the new sources, hydrogen is one of the cleanest fuels. Therefore, there is a growing interest for Hydrogen Energy Systems. Through electrochemical processes, hydrogen is converted to electricity in fuel cells. Fuel cells operating on hydrogen generate zero emission of CO_2.

The proton exchange membrane (PEM) fuel cell shows several advantages. During its operating process, the PEM fuel cell only exhaust is water.

One of the most important characteristics of a fuel cell is its polarization curve. The polarization curve has several parameters. The transfer coefficient α is one of those parameters. Several works have shown that the transfer coefficient has a great influence on a fuel cell performance [1-4]. Alhassan Salam *et al* [5] investigated the effect of charge transfer coefficient (CTC) on the operating voltage of polymer electrolyte membrane (PEM) electrolyser. This study also provides an important opportunity to advance the understanding of the effect of different temperatures and pressures on the CTC. Their result successfully compared with experimental data shows that the activation overvoltage decreases when the CTC increase from 0.1 to 2.0 both at anode and cathode electrodes. They showed that, for temperatures in the range 10°C - 90°C , the CTC values range between 0.807 and 1.035 at the anode electrode, while at the cathode electrode, the range of variation is 0.202 - 0.259. Interestingly it was observed that the CTC remains the same even at balanced and unbalanced pressure. Anusree Unnikrishnan, Rajalakshmi and Janardhanan [6] presented an electrochemical model for H2 oxidation and O_2 reduction in HT-PEM fuel cells derived from multi-step single electron transfer elementary reactions. Their mechanistic model demonstrated the electrochemical charge transfer in HT-PEM fuel cells. Biaku *et al* [7] investigated the temperature dependency of the charge transfer coefficient (CTC) of the oxygen electrode of a commercial proton exchange membrane electrolyser stack. The results of this study show that at typical operating temperatures, there is a reasonable variation of the CTC from the symmetry factor. Their result indicated that within a temperature range of 20–60°C, the average CTC changes from 0.18 to 0.42.

A good estimate of the transfer coefficient is important in predicting the current–voltage characteristics of PEM fuel cell. The estimated transfer coefficient also provides information on the properties of the electrode. The purpose of this paper is to highlight the determination of charge transfer coefficient by the Lagrange's undetermined multiplier technique. The method is compared to the standard method of determination of transfer coefficient.

2. PEM fuel cell electrochemistry

2.1. Electrodes kinetics and the theoretical fuel cell potential

In a PEM fuel cell, the polymer membrane is squeezed between the two electrodes, namely the anode and the cathode. The electrochemical reactions that happen on the electrodes are the followings:

At the anode, the oxidation reaction of hydrogen is:

$$\mathrm{H_2} \rightarrow 2\mathrm{H^+} + 2\mathrm{e^-} \tag{1}$$

At the cathode, there is a reduction reaction of oxygen which is:

$$\frac{1}{2}\mathrm{O_2} + 2\mathrm{H^+} + 2\mathrm{e^-} \rightarrow \mathrm{H_2O} \tag{2}$$

The overall reaction is

$$\mathrm{H_2} + \frac{1}{2}\mathrm{O_2} \rightarrow \mathrm{H_2O} \tag{3}$$

If ΔH is the heat (or enthalpy) of the reaction (3), the Gibbs free energy ΔG is expressed as:

$$\Delta G = \Delta H - T\Delta S \tag{4}$$

The irreversible losses in the conversion of chemical energy fuel into electrical energy are due to ΔS, which is the creation of entropy.

The theoretical (ideal) fuel cell potential is expressed as:

$$E_{th} = -\frac{\Delta G}{nF} \tag{5}$$

where n = 2 is the number of electrons per molecule of H_2. F = 96485 C/mol is the Faraday's constant.

At T = 298 K, the theoretical fuel cell potential is Eth = 1.23 V

2.2. Voltage losses and fuel cell polarization curve

As a matter of fact, the cell potential E is a function of temperature and pressure. It is expressed as:

$$E = -\left(\frac{\Delta H}{nF} - \frac{T\Delta S}{nF}\right) + \frac{RT}{nF}\ln\left[\frac{P_{H_2}{P_{O_2}}^{0.5}}{P_{H_2}O}\right] \quad (6)$$

P_{H_2}, P_{O_2} and $P_{H_2}O$ are the partial pressure of hydrogen, oxygen and water, respectively. For temperatures below 100°C, equation (6) becomes [1]:

$$E = 1.482 - 0.000845T + 0.0000431T\mathrm{Ln}({P_{H_2}}{P_{O_2}}^{0.5}) \quad (7)$$

As for the relationship between the current density and potential, it is expressed by the Butler-Volmer equation [1]

$$i = i_0\left[\exp\left[\frac{-\alpha_{Rd}F(E-E_r)}{RT}\right] - \exp\left[\frac{-\alpha_{Ox}F(E-E_r)}{RT}\right]\right] \quad (8)$$

In equation (8), E_r is the reversible (or equilibrium) potential. At the cathode, and at T = 298 K, the reversible potential $E_r = 1.229\ V$. Its value is $E_r = 0\ V$ at the anode.

Equation (8) shows that the generation of current i requires an overpotential $\Delta V = E - E_r$ between the electrode potential and the reversible potential. i_0 is the exchange current.

At fuel cell cathode, the overpotential is negative, so that in equation (8), the first term is much larger than the second one. Hence, the reduction current is expressed as:

$$i = i_0\exp\left[\frac{-\alpha_{Rd}F(E-E_r)}{RT}\right] \quad (9)$$

From equation (9), one gets, for $\alpha = \alpha_{Rd}$,

$$E = E_r - \frac{RT}{\alpha F}\ln\left[\frac{i}{i_0}\right] \quad (10)$$

The potential difference $\Delta V = E_r - E$ is known as the activation polarization. It is expressed as:

$$\Delta V_{act} = \frac{RT}{\alpha F}\ln\left[\frac{i}{i_0}\right] \quad (11)$$

Moreover, when the fuel cell operates, there are some ohmic losses, due to the resistance in the flow of ions and the resistance in the low of electrons through the electrodes and electrical circuits. In this case, the potential difference is known as the ohmic polarization which is expressed as

$$\Delta V_{ohm} = iR_i \quad (12)$$

In equation (12), i is the current and R_i is the fuel cell internal resistance.

The third voltage loss is known as the concentration polarization. It is due to the fact that the reactant is rapidly consumed at the cathode at high voltage. This voltage loss due to the concentration is expressed as

$$\Delta V_{conc} = \frac{RT}{nF}\ln\left[\frac{I_L}{I_L - i}\right] \quad (13)$$

I_L is the limiting current.

From the different voltage losses, one gets the fuel cell polarization curve. The cell voltage is expressed as

$$V_{cell} = E - \Delta V_{act} - \Delta V_{ohm} - \Delta V_{conc} \quad (14)$$

Thus, the cell polarization curve is obtained from the following equation:

$$V_{cell} = E - \frac{RT}{\alpha F}\ln\left(\frac{i}{i_0}\right) - \frac{RT}{nF}\ln\left(\frac{I_L}{I_L - i}\right) - iR_i \quad (15)$$

3. Transfer coefficient determination

3.1. Method using the activation polarization

When the activation polarization is only considered, equation (15) becomes

$$\Delta V = E - V_{cell} = \frac{RT}{\alpha F}\ln(i) - \frac{RT}{\alpha F}\ln(i_0) \quad (16)$$

Equation may be expressed as

$$\Delta V = a + b\ln(i) \quad (17)$$

In equation (17), $b = \frac{RT}{\alpha F}$ is the Tafel slope; as for a it is expressed as $a = -\frac{RT}{\alpha F}\ln(i_0)$

When ΔV is plotted against ln(i), one gets a straight line with slope b and intercept a. From the knowledge of b, one gets the transfer coefficient expressed as:

$$a = \frac{RT}{bF} \quad (18)$$

3.2. Transfer coefficient determination by using the Lagrange's undetermined multiplier method

The Lagrange's undetermined multiplier technique is used when one wants to maximize (or minimize) a multivariable function f(x, y,…) subject to the constraint that another multivariable function g(x, y,…) equals a constant [8]:

$$g(x, y,\ldots) = c \quad (19)$$

A parameter λ, which is the Lagrange's undetermined multiplier (or lagrangian), is used to define a new function F(x, y… λ) expressed as

$$F(x, y, \ldots, \lambda) = f(x, y,\ldots) + \lambda(g(x, y,\ldots) - c) \quad (20)$$

The extremum (maximum or minimum) of the function f under the constraint g is obtained when the gradient of F equals the zero vector

$$\nabla F(x, y, \ldots, \lambda) = 0 \quad (21)$$

For instance, for the variable x, one gets

$$\frac{\partial f}{\partial x} + \lambda\frac{\partial g}{\partial x} = 0 \quad (22)$$

The determination of the charge transfer coefficient by using the Lagrange's undetermined multiplier technique is performed as indicated below.

If the activation polarization and the ohmic polarization are the only voltage losses considered, the cell polarization curve expressed by equation (15) becomes:

$$V = E - \frac{RT}{\alpha F}\ln(\frac{i}{i_0}) - iR_i \quad (23)$$

The output power from the fuel cell is given by the product between voltage V and current I:

$$P = V.i \quad (24)$$

Let a function g be defined as

$$g = V - E + \frac{RT}{\alpha F}\ln\left(\frac{i}{i_0}\right) + iR_i \quad (25)$$

A power W is defined as:

$$W = P + \lambda g \quad (26)$$

Hence

$$W = Vi + \lambda[V - E + \frac{RT}{\alpha F}\ln\left(\frac{i}{i_0}\right) + iR_i] \quad (27)$$

λ is the undetermined Lagrange's multiplier. For P maximum, W is maximum. The maximum value Wmax is obtained from the two following equations:

$$\frac{\partial W}{\partial i} = \frac{\partial(Vi)}{\partial i} + \lambda\frac{\partial g}{\partial i} = 0 \quad (28)$$

$$\frac{\partial W}{\partial V} = \frac{\partial(Vi)}{\partial V} + \lambda\frac{\partial g}{\partial V} = 0 \quad (29)$$

For the maximum power Pmax, $V = V_m$ and $i = i_m$. Then, from equations (25) and (28), one gets

$$V_m + \lambda(\frac{RT}{\alpha F}\frac{1}{i_m} + R_i) = 0 \quad (30)$$

From equation (30), one gets

$$V_m = -\lambda(\frac{RT}{\alpha F}\frac{1}{i_m} + R_i) \quad (31)$$

On another hand, from equation (29), one gets

$$i_m + \lambda(1) = i_m + \lambda = 0 \quad (32)$$

Hence

$$i_m = -\lambda \quad (33)$$

Using equations (31) and (33), one can calculate R_{opt} as follows

$$R_{opt} = \frac{V_m}{i_m} = \frac{RT}{\alpha F}\frac{1}{i_m} + R_i \quad (34)$$

R_{opt} is the optimum load resistance. For R_{opt}, the power transfer from the fuel cell is maximum.

Therefore, when R_{opt}, i_m and R_i are known experimentally, one can calculate the charge transfer coefficient α.

Let the thermopotential V_T be defined as $V_T = \frac{RT}{F}$. Then one gets from equation (34).

$$\alpha = \frac{V_T}{(R_{opt} - R_i)i_m} \quad (35)$$

Hence, the transfer coefficient α is the ratio between the thermopotential and the potential related to $(R_{opt} - R_i)$ at the maximum power. Relation (35) shows also a clear influence of temperature on the transfer coefficient.

For the calculation of the internal resistance R_i, the polarization curve is used. Equation (23) can be written as:

$$V = E - \Delta V_{act} - iR_i \quad (36)$$

Hence

$$\Delta V = E - V - \Delta V_{act} = iR_i \quad (37)$$

When ΔV is plotted against the current i, one gets a straight line with slope R_i

4. Results and discussion

Several studies have been performed on solar photovoltaic/electrolyser or solar photovoltaic/electrolyser/fuel cell systems [9-12]. In the present study, some experiments were made to determine the transfer coefficient. The experimental system, shown on figure 1, is a kit called Hydrocar [13], combining a solar panel, a reversible Proton Exchange Membrane fuel cell (which is a combination of fuel cell and electrolyser), and hydrogen and oxygen storage containers, a 3 Volts alkaline battery. The car is powered by electricity generated when hydrogen is consumed by the fuel cell.

Figure 1. Photograph of the Hydrocar system.

A variable resistor load was used to get the current-voltage characteristic curve of the solar panel, the current-voltage characteristic of the electrolyser and also the polarization curve of the fuel cell. Some electric lamps were used as light source for the solar panel. An Eppley type pyranometers was used for the irradiance measurements (in W/m²). Multimeters were used for current (i) and voltage (V) measurements.

4.1. Solar panel experimental study

This study was performed by using the variable resistor load and some electric lamps. Figure 2 shows the solar panel current-voltage characteristic at various irradiances (E = 105.7 W/m², E = 317 W/m², E = 672.4 W/m², E = 1056.7 W/m²).

Some parameters of the solar panel were determined, such as the fill factor FF, the efficiency η, the short circuit current I_{SC}, the open circuit voltage V_{OC}, the maximum power P_m, and the series resistance R_s.

The fill factor is expressed as [14,15]

$$FF = \frac{P_m}{I_{SC}V_{OC}} \tag{38}$$

where $P_m = V_m I_m$ is the maximum power.

Figure 2 shows that the short circuit current I_{SC} which is the current for the voltage V = 0, increases for increasing solar irradiances. As for the open circuit voltage V_{OC}, it decreases for high irradiances

As for the efficiency η, it is expressed as [14,15]

$$\eta = \frac{P_m}{P_{in}} \tag{39}$$

P_{in} is the input solar power expressed as:

$$P_{in} = SE \tag{40}$$

where S (m²) is the surface of the solar panel and E (W/m²) the irradiance.

Table 1 shows the parameters obtained for various irradiances.

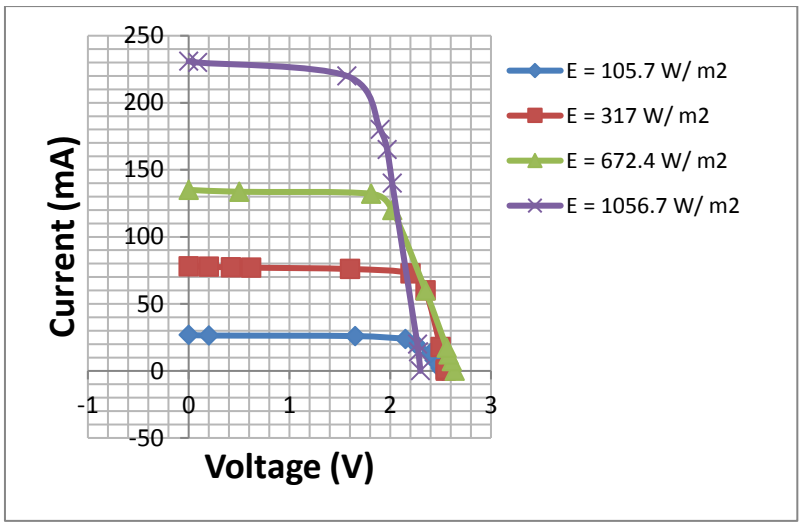

Figure 2. Current-voltage characteristic of the solar panel for various irradiances.

Table 1. Some parameters of the solar photovoltaic panel.

E (W/m^2)	Isc (mA)	Voc (V)	P_m (W)	FF	R_S (Ω)	η(%)
105.7	26.6	2.52	0.0512	0.76	3.7	11
317	78	2.55	0.16	0.8	3.78	11.7
672.4	135	2.62	0.245	0.69	3.9	8.4
1053.7	231	2.30	0.35	0.66	3.8	7.7

Table 1 shows that I_{SC} increases with for increasing irradiances; V_{OC} slightly increases with increasing irradiance then decreases for high irradiances; the fill factor FF also slightly increases with increasing irradiance then decreases for high irradiances. As for the series resistance R_s, it does not vary a lot when the irradiance is increased. The average value of R_s is $\overline{R_S} = 3.795\ \Omega$. It is noticed that the efficiency η obviously decreases for high irradiances

The current-voltage characteristic of the solar panel is linked to some of those parameters. It is of course expressed as [16]

$$I = I_{sc} - I_0\left[\exp\left(\frac{q(V+R_sI)}{nKT}\right) - 1\right] - \frac{V+R_sI}{R_{sh}} \tag{41}$$

In equation (41), n is the ideality factor. Its average value was found to be $\bar{n} = 2.7$.

4.2. Experimental study of the electrolyser

The variable resistor load was used for this experimental study. The electrolyser was connected to the 3 V battery as shown on figure 3.

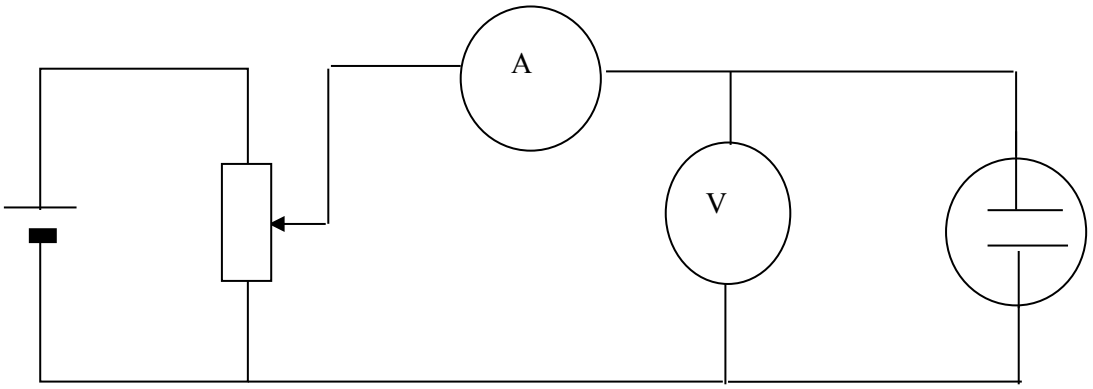

Figure 3. Experimental electric circuit for the electrolyser current-voltage characteristic determination.

For different values of the voltage applied to the electrolyser, the current was measured. The current –voltage characteristic of the electrolyser is plotted in figure 4.

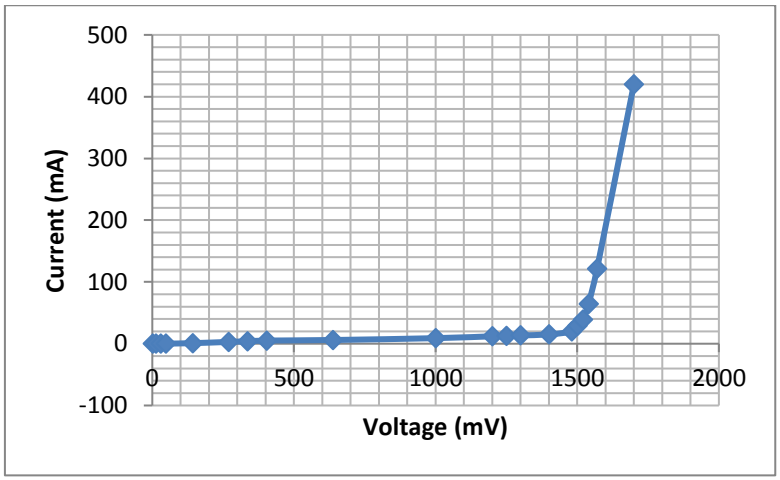

Figure 4. Current-voltage characteristic of the electrolyser.

The curve shows that the current becomes noticeable only when a voltage, which is the decomposition voltage Vd is reached. It was found that Vd = 1.50 V. When Vd is reached, the electrolyser starts working to split water into oxygen and hydrogen. It is found that Vd is greater than the theoretical decomposition voltage which is 1.23 V.

The hydrogen and oxygen produced by the electrolyser are stored in the storage containers of the system. Figure 5 shows plotting of the I-V characteristic of the solar panel for E = 672.4 W/m² (green points), together with its power curve (red points), and also the current-voltage characteristic of the electrolyser (blue points), as well as the curve showing the maximum power of the solar panel for E = 672.4 W/m² (violet points).

Figure 5 shows that the maximum power of the solar panel is 0.242 W for E = 672.4 W/m². Even for the irradiance E = 1056.7 W/m², the maximum power of the solar panel is 0.35 W .As for the I-V curve of the electrolyser as shown on figure 5, it shows that the maximum power consumption of the electrolyser is gotten for I = 0.42 A and V = 1.7 V. Hence, the maximum power consumption is 0.714 W. Obviously, this power cannot be supplied by the solar panel. As a matter of fact, the 3 V battery was used for the determination of the electrolyser current-voltage characteristic.

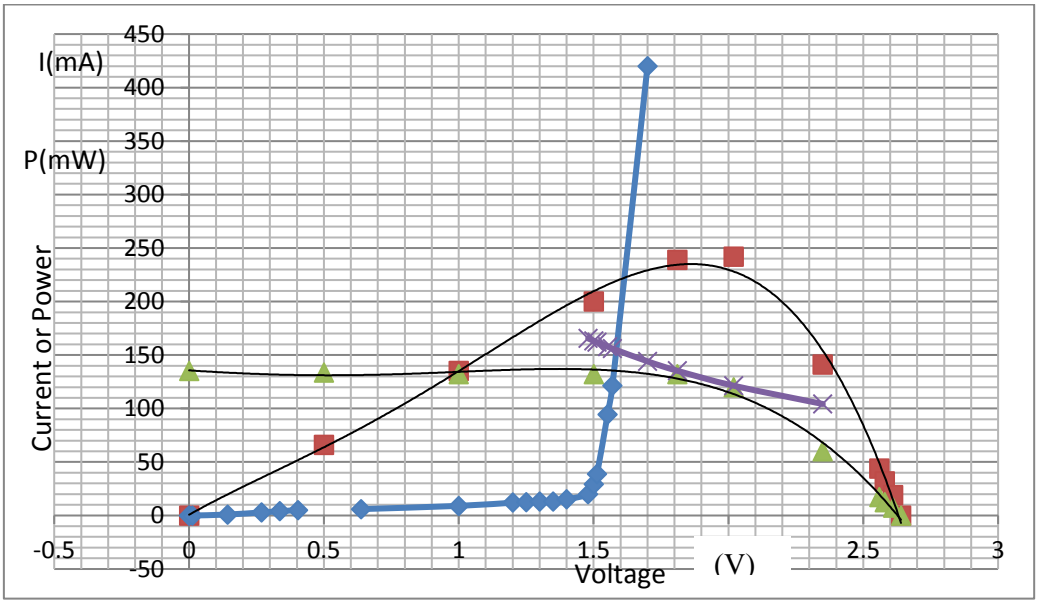

Figure 5. Current-voltage characteristic of the solar panel for E = 672.4 W/m² (green points); Power - Voltage curve of the solar panel for E = 672.4 W/m² (red points); Current-voltage characteristic of the electrolyser (blue points); Maximum power curve showing the maximum power of the solar panel for E = 672.4 W/m² (violet points).

The energy efficiency of the electrolyser was also calculated. It is defined as:

$$\eta_{energy} = \frac{H^0 V_{H_2}}{VIt} x100 \quad (42)$$

where $H^0 = 10800\ kJ/m^3$ is the hydrogen lower heating value. V_{H_2} is the volume of hydrogen (m^3) produced; V is the voltage at time t; I (A) is the current; t(s) is the time.

Figure 6 shows a plotting of the energy efficiency against ambient temperature T (°C). It shows that the energy efficiency increases linearly with temperature.

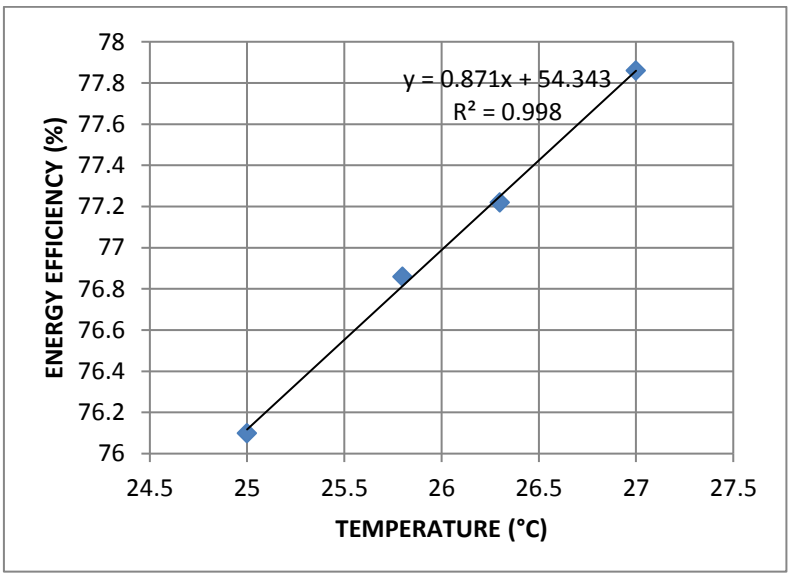

Figure 6. Energy efficiency as a function of temperature.

The Faraday efficiency ($\eta_{Faraday}$) was also calculated and plotted against temperature on figure 7. It is defined as:

$$\eta_{Faraday} = \frac{\text{Experimebtal } V_{H_2}}{\text{Theoretical } V_{H_2}} \tag{43}$$

The theoretical volume V_{H_2} is expressed as:

$$V_{H_2} = \frac{RTIt}{2F\,P_{H_2}} \tag{44}$$

Figure 7 shows a decrease of the Faraday efficiency for increasing temperatures

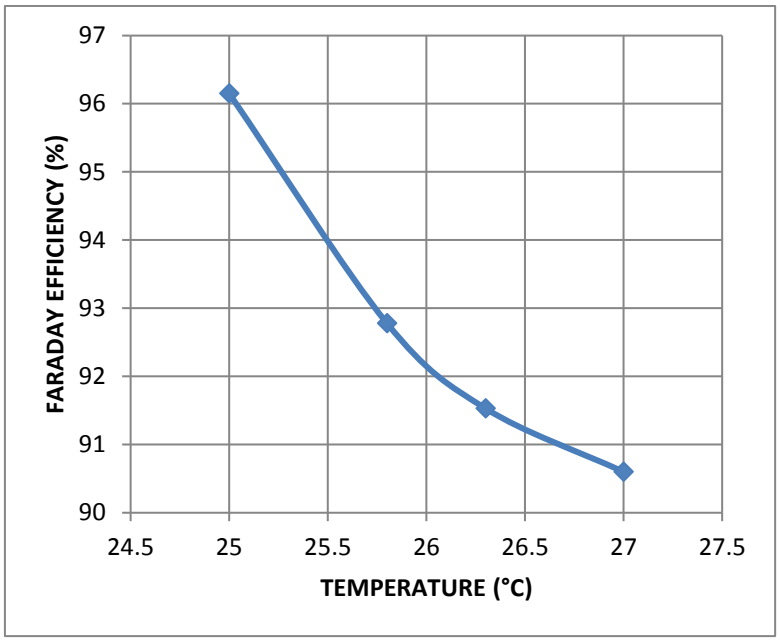

Figure 7. Faraday efficiency as a function of temperature.

4.3. 4.3. Experimental study of the PEM fuel cell

4.3.1. Experimental determination of the polarization curve. The oxygen and hydrogen produced by the electrolyser were stored in the storage containers. They were used to operate the PEM fuel cell. Experiments were made for various temperatures. The experimental circuit is shown in figure 8.

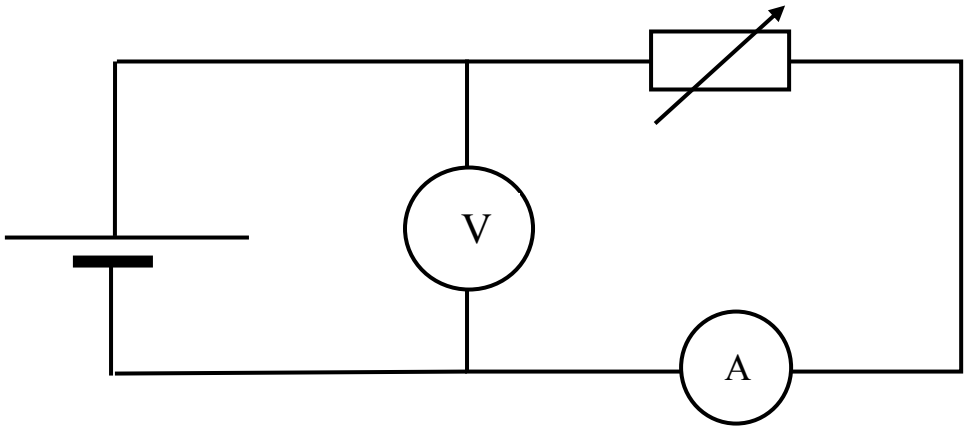

Figure 8. Experimental electric circuit for the current-voltage characteristic determination.

A variable resistor load was used to vary current I and voltage V. For each temperature, the current-voltage characteristic of the fuel cell was plotted. The experimental value of R_{opt} was also determined. R_{opt} is the value of the resistance for which the greatest energy output is obtained from the fuel cell. Figure 9 shows the plotting for T = 24.4°C, T = 25°C, T = 25.1°C and T = 25.2°C.

The Hydrocar system is made to operate at ambient temperature. So, for the present study, we did not make experiments with higher temperatures. In a future study, experiments will be carried out with higher temperatures, in order to investigate the influence of temperature on the charge transfer coefficient.

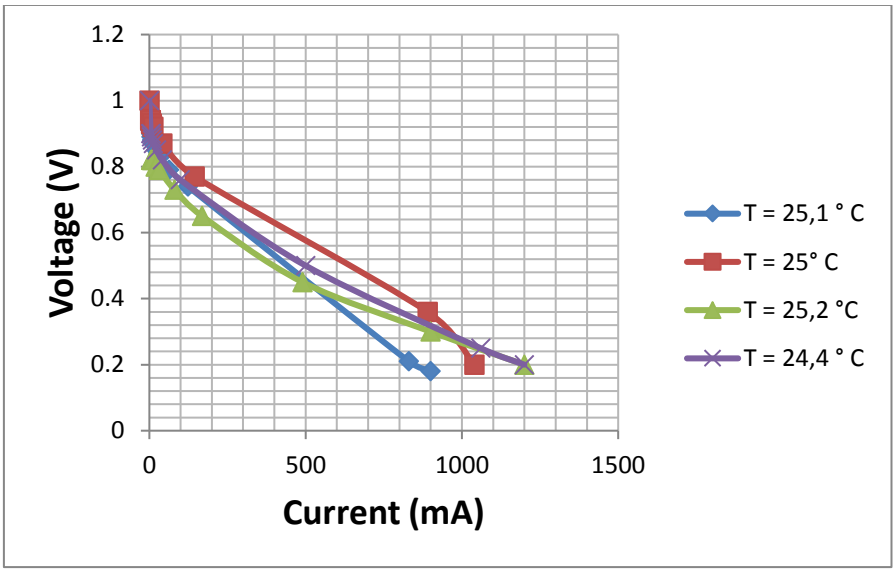

Figure 9. The PEM fuel cell polarization curve at various temperatures.

4.3.2. Experimental determination of the transfer coefficient by using the activation polarization. The current-voltage characteristic obtained for T = 25.1°C was used for this study. It is shown on figure10.

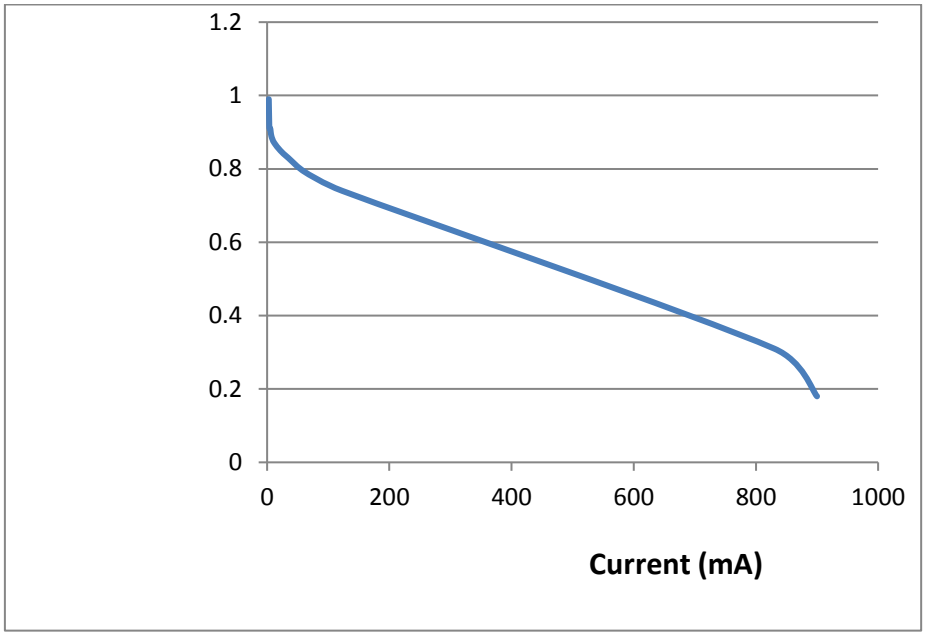

Figure 10. The PEM fuel cell polarization curve at T = 25.1°C.

For this temperature, we got $R_{opt} = 0.7\ \Omega$. Equation (17) was used to estimate the transfer coefficient

Let X and Y be defined as

$$X = \ln(i);\ Y = \ \Delta V$$

The plotting of Y against X is shown in figure 11.

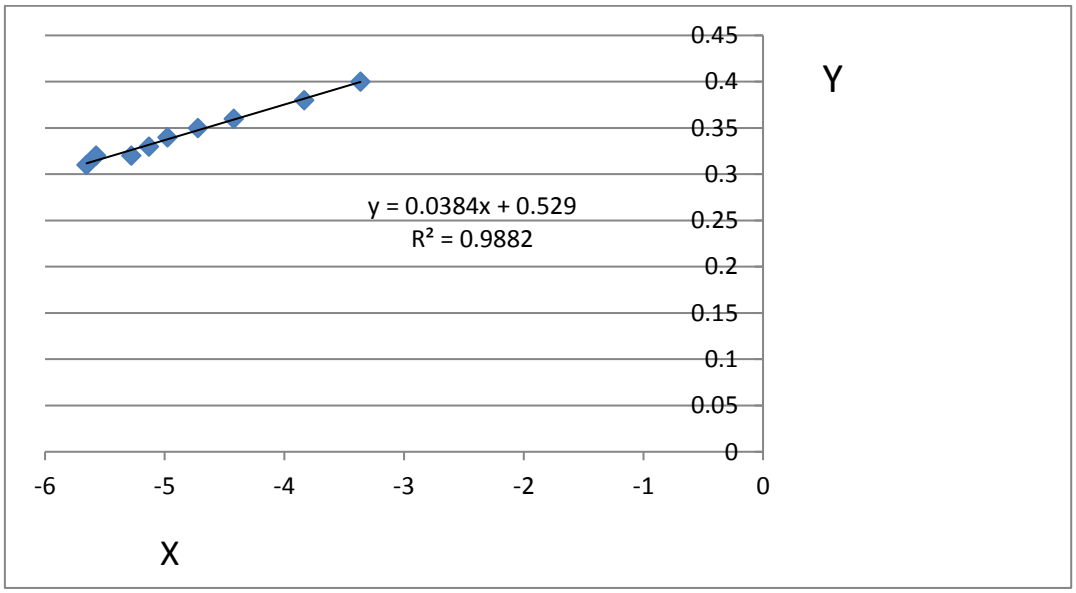

Figure 11. Plotting of Y against X for the transfer coefficient determination.

One gets

$$y = 0.0384x + 0.529 \quad (45)$$

Figure 11 shows a good linear correlation with a determination coefficient $R^2 = 0.9882$

Therefore, the Tafel slope $b = \frac{RT}{\alpha F} = 0.0384$. The ideal gas constant R = 8.314 J/mol.K.

One gets, for the temperature T = 298.25 K and for F = 96 486 C/mol, the following value of the transfer coefficient: $\alpha = 0.669$.

Moreover, $a = -\frac{RT}{\alpha F}\ln(i_0) = 0.529$

As a result, one gets, for the exchange current

$$i_0 = 1.045x10^{-6}A$$

4.3.3. Determination of the transfer coefficient by using the Lagrange's undetermined multiplier technique. Equation (35) was used. The first step was the determination of the internal resistance R_i and i_m, which is the current for which the maximum power is obtained.

The determination of R_i was made by using the current-voltage characteristic at T = 25.1°C. Equation (37) was used. Let X and Y be defined as

$$X = i;\ Y = \Delta V = E - V - \Delta V_{act}$$

Figure 12 shows the plotting of Y against X. The following equation was found

$$Y = 0.5002X - 0.0172 \quad (46)$$

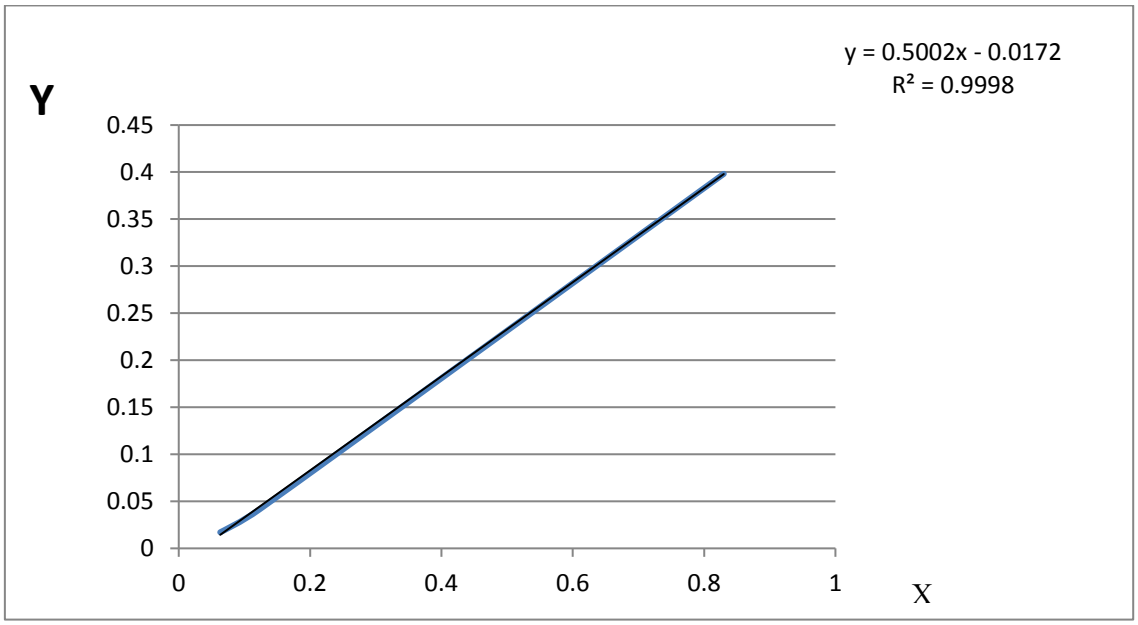

Figure 12. Plotting of Y against X for the internal resistance determination.

From equations (37) and (46), one gets $R_i = 0.5\ \Omega$

Then, from the current-voltage characteristic of the fuel cell at T = 25.1°C, the current-power characteristic was plotted on figure 13. The power P is expressed as P = V.i

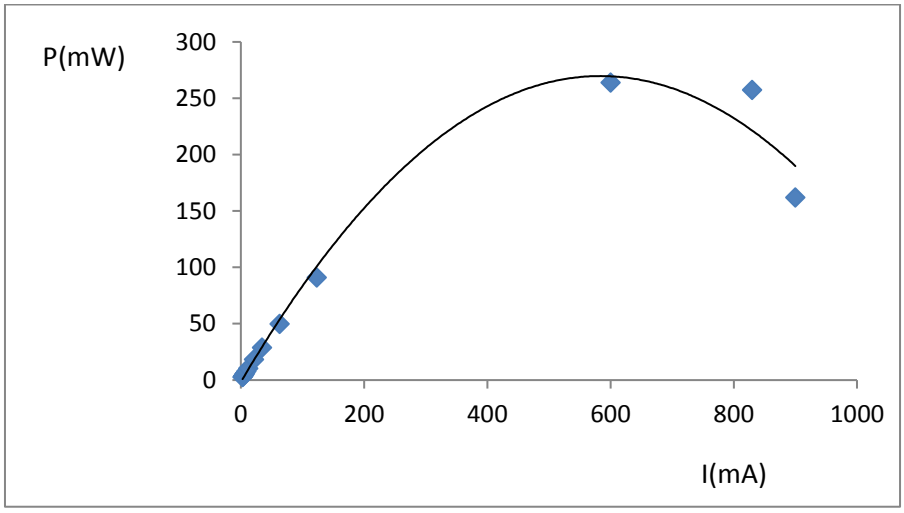

Figure 13. Current-power characteristic.

From the curve of figure 13, the value of i_m, the current at the maximum power, was found to be $i_m = 0.58$ A

Then for T = 298.25 K, $R_{opt} = 0.7\ \Omega$, $R_i = 0.5\ \Omega$, one gets the value of the transfer coefficient, by using equation (35): α = 0.222

According to some authors [11], the charge transfer coefficient α is always experimentally between 0 and 1. Ryan O'Hayre *et al* [17], indicate that α may range from about 0.2 to 0.5. According to Larmine and Dicks [18], quoted by [1], α = 0.5 for the hydrogen fuel cell anode, in which two electrons are involved and α = 0.1 to 0.5 for the cathode. As for Newman [19], quoted by [1], α may be in the range between 0.2 and 2. So according to literature, a very large range is indicated for the charge transfer coefficient

In the present work, the value of α found by applying the Lagrange's multiplier technique, α = 0.222, is of course between 0.2 and 0.5. As for The value of α found by using the activation polarization, α = 0.669, it is slightly greater than 0.5.

The advantage of the Lagrange's multiplier technique used in this work, is the fact that it takes into account the ohmic polarization, the activation polarization and also the current-power characteristic of the fuel cell. Relation (35), found by applying the Lagrange's multiplier technique, gives a clear relation between the transfer coefficient and temperature. As for α = 0.669, it takes into account only the activation polarization. Hence, the Lagrange's multiplier technique may be a better estimate of the charge transfer coefficient.

5. Conclusion

The study made on the kit called Hydrocar allowed us to determine the main parameters of the solar panel. The maximum power output P_m from the photovoltaic solar cell depends on the solar irradiance E. For E = 1056.7 W/m² for instance, one gets $P_m = 0.350$ W. A decrease of the solar panel efficiency is observed for increasing solar irradiances.

As for the PEM reversible fuel cell, for the experiments made, the maximum output power found was $P_m = 0.268$ W. The current-voltage characteristic of the fuel cell was used to determine the internal resistance. It was found $R_i = 0.5\ \Omega$. This value is lower than the optimum load resistance R_{opt}. It was found $R_{opt} = 0.7\ \Omega$. The transfer coefficient α was first evaluated from the current-voltage characteristic of the fuel cell, by using only the activation polarization. It was found α = 0.669. Then the Lagrange's undetermined multiplier technique was used to evaluate the transfer coefficient. It was found α = 0.222. The Lagrange's multiplier technique takes into account the ohmic polarization, the activation polarization and also the current-power characteristic of the fuel cell may

be a better estimate of the charge transfer coefficient.

Appendix

Nomenclature

E	Cell voltage (V)
E	Solar irradiance (W/m^2)
F	Faraday constant (C/mol)
G	Gibbs free energy (J/mol)
H	Enthalpy (J/mol)
H^0	Hydrogen lower heating value (kJ/m^3)
i	Current (A)
I_0	Dark saturation current (A)
i_0	Exchange current (A)
n	ideality factor
P	Pressure (Pa)
R	Ideal gas constant (J/mol.K)
R	Resistance (Ω)
V	Voltage (V)

Greek Symbols

α	transfer coefficient
η	efficiency
λ	Lagrange's multiplier

Subscripts

d	decomposition
i	internal
m	maximum
opt	optimum
r	reversible
s	series
sc	short circuit
sh	shunt

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