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Reliability model of the symmetric dual-node data processing system with functional redundancy

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Abstract. This scientific paper deals with the reliability model of the symmetric system based on two data processing nodes with the functional redundancy. The simplified reliability model of the system and the advanced model offered by the authors considering nonzero time of activation of the node primary functions, nonzero times for node failure detection and switching to the function backing-up mode are presented. The formulas for calculation of stationary availability factor, mean time to failure and mean time to repair are also discussed. Finally, the examples of reliability indices calculations according to the simplified and advanced reliability models are also given.

1. Introduction

Nowadays not only the specifications concerning computational performance are important for the distributed data processing systems [1], but there are also important the reliability aspects, on which the safety and stability of information processes of the modern enterprises directly depends. Respectively, development of reliability models of data processing systems is rather urgent task.

There is a set of generalized reliability models and specialized reliability models of data processing systems which are discussed in the literature [2-5] based on the Markov chain models [6, 7]. However, these models do not take into account nonzero time of activation of the primary and backup functions of the data processing nodes in the specialized systems with the functional redundancy [8] used for information systems in the modern enterprises.

Within the research work the authors developed reliability models for the data processing systems with structural redundancy taking into account nonzero time of node activation [9, 10]. Respectively, within this scientific paper the authors developed the reliability model of symmetric dual-node data processing system with the functional redundancy taking into account time of activation of the primary and backup functions of nodes.

2. Simplified reliability model of symmetric dual-node system with functional redundancy

Let us overview a symmetric data processing system (Figure 1) with functional redundancy based on two identical nodes. The system interacts with the outside world via local area network obtaining source data from it and delivering processed data into it.

Data processing nodes have identical computational power and reliability parameters. In order to provide bi-directional control, the nodes also exchange the service data about their state using the internal heartbeat network.

Each node by default performs only the primary function, but in case of failure of the neighbor node also performs the function of failed node in addition. After recovery of the failed node, the node performs only its primary function again. The functions have different meaning and purpose, but for simplification we will assume that from the reliability viewpoint they create identical load on data processing nodes. Data processing system is considered to be operable only in the case of availability of the both functions.

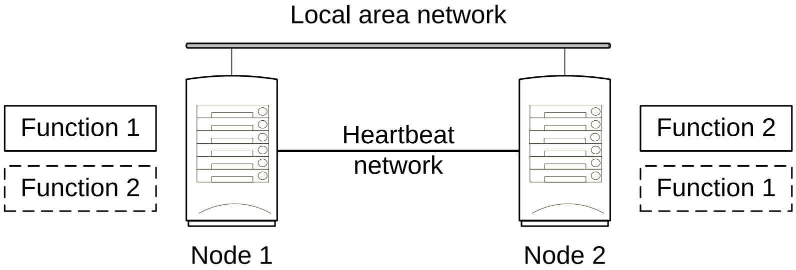

Figure 1. Symmetric system with two data processing nodes and functional redundancy.

In the simplified reliability model we will not take into account time necessary for the primary function activation, time of down-state detection and time of the neighbor node recovery, as well as entrance of the node into the backing-up mode and time of exit from the backing-up of the neighbor node function.

Each of nodes by default performs only its primary function, and in this mode its failure rate is λ_A. In case of failure of the other node it can also perform function of the neighbor node, and in this mode its failure rate is λ_B.

The nodes can be recovered independently (unlimited repair). Node repair rate is μ_N.

Then, taking into account symmetry of the system, let us consider the following set of system states and graphs of transitions from one state to another (Figure 2):

- State AA (Active / Active) – both nodes are in upstate, and every node performs only its own function.
- State FB (Failed / Backup) – one node is in downstate; the other node performs as its own function, so the function of the failed node.
- State FF (Failed / Failed) – both nodes are in downstate.

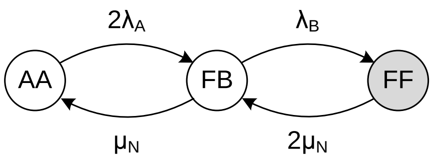

Figure 2. Simplified reliability model of the symmetric system of the dual-node system with functional redundancy.

In accordance, the stationary system of the Kolmogorov-Chapman equation is as follows:

$$\begin{cases} P_{AA} + P_{FB} + P_{FF} = 1; \\ 0 = -2\lambda_A P_{AA} + \mu_N P_{FB}; \\ 0 = 2\lambda_A P_{AA} - (\mu_N + \lambda_B) P_{FB} + 2\mu_N P_{FF}; \\ 0 = \lambda_B P_{FB} - 2\mu_N P_{FF}. \end{cases} \quad (1)$$

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Then, with due regard that only in the states AA and FB the system is operable and provides the both functions, its stationary availability factor can be calculated as the sum of probabilities of the states AA and FB:

$$K_{\mathrm{SYS}} = P_{\mathrm{AA}} + P_{\mathrm{FB}} = \frac{\mu_{\mathrm{N}}(\mu_{\mathrm{N}} + 2\lambda_{\mathrm{A}})}{\mu_{\mathrm{N}}(\mu_{\mathrm{N}} + 2\lambda_{\mathrm{A}}) + \lambda_{\mathrm{A}}\lambda_{\mathrm{B}}}. \tag{2}$$

Hereafter, the mean time to failure of the system can be calculated as ratio of the sum of probabilities of up-states AA and FB to the probability of border up-state FB, multiplied by the rate λ_{B} of transition into down-state FF:

$$T_{\mathrm{FSYS}} = \frac{P_{\mathrm{AA}} + P_{\mathrm{FB}}}{\lambda_{\mathrm{B}} P_{\mathrm{FB}}} = \frac{\mu_{\mathrm{N}} + 2\lambda_{\mathrm{A}}}{2\lambda_{\mathrm{A}}\lambda_{\mathrm{B}}}. \tag{3}$$

At last, the mean time to repair of the system can be calculated from a formula of the reliability theory, which binds the stationary availability factor with mean time to failure and mean time to repair via simple equation $K = T_{\mathrm{F}} / (T_{\mathrm{F}} + T_{\mathrm{R}})$:

$$T_{\mathrm{RSYS}} = \frac{1 - K_{\mathrm{SYS}}}{K_{\mathrm{SYS}}} T_{\mathrm{FSYS}} = \frac{1}{2\mu_{\mathrm{N}}}. \tag{4}$$

3. Advanced reliability model of symmetric dual-node system with functional redundancy

In the advanced reliability model, we will consider data processing nodes as a complex of hardware-software systems which require nonzero time for activation (transition from passive to active state after completion of the software initialization) of its primary function.

Also, we will take into consideration the nonzero time of detection of neighbor node failure and entrance into the backing-up mode of its function, as well as nonzero time of detection of neighbor node recovery and exit from the backing-up mode of its function.

Now let us consider the following set of states (Figure 3) for a node as a part of the dual-node data processing system with functional redundancy:

- State P (Passive) – node is in upstate but it is still passive, does not perform any function and waiting for completion of the software initialization.
- State A (Active) – node is in upstate and performs its primary function.
- State B (Backing-up) – node is in upstate and performs as its primary function, so the function of the failed neighbor node.
- State F (Failed) – node is in downstate.

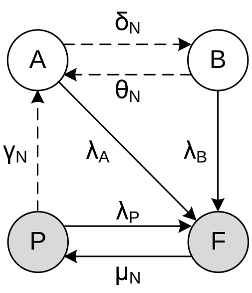

Figure 3. Reliability model of the data processing node within the symmetric dual-node system with functional redundancy.

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Where,

λ_P – node failure rate in passive state;

λ_A – node failure rate in active state;

λ_B – node failure rate in backing-up state;

μ_N – node repair rate;

γ_N – rate of node primary function activation;

δ_N – rate of entrance into the mode of backing-up of the neighbor node function;

θ_N – rate of exit from the mode of backing-up of the neighbor node function.

It should be noted, that node cannot be in state B, if another node is in states A or B, thus the nodes cannot perform any of the functions simultaneously, and from this viewpoint they are interdependent. Now taking into account this interdependence of the nodes and symmetry of the system (identical nodes from the reliability viewpoint), we consider the following set of states of the dual-node data processing system with functional redundancy (Figure 4):

- State PP (Passive / Passive) – the both nodes are in up-state, but still passive, do not perform their functions and are waiting for completion of the software initialization.
- State AP (Active / Passive) – the both nodes are in upstate, one of them is still passive and does not perform its function, the other one is active and performs its own function.
- State PF (Passive / Failed) – one node is in downstate; the other one is in upstate, but passive and does not perform its function.
- State AF (Active / Failed) – one node is in downstate; the other one is in upstate, active and performs its own function.
- State FF (Failed / Failed) – the both nodes are in downstate.
- State PB (Passive / Backup) – both nodes are in upstate, one of the nodes is passive, the other one is in backing-up mode and performs as its own function, so the function of the passive neighbor node.
- State AA (Active / Active) – both nodes are in upstate and each of them performs only its own function.
- State FB (Failed / Backup) – one of the nodes is in downstate; the other one performs its own function and function of the failed neighbor node.

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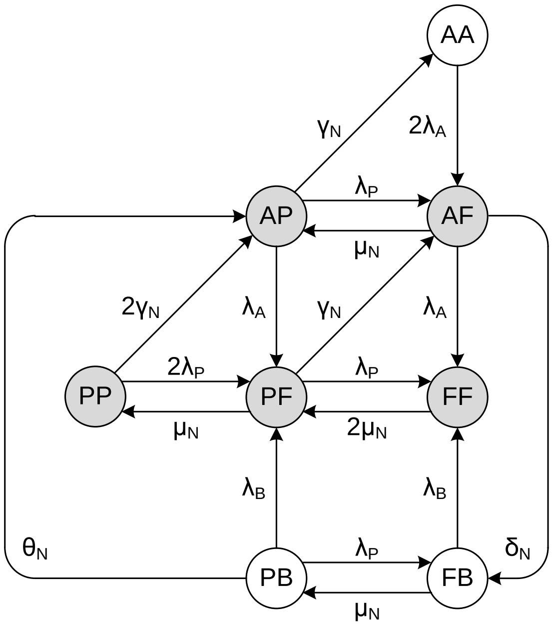

Figure 4. Advanced reliability model of the symmetric system with two nodes of data processing with functional redundancy.

Accordingly, the stationary system of the Kolmogorov-Chapman equation for the aforesaid Markov chain is as follows:

$$\begin{cases} P_{\mathrm{PP}} + P_{\mathrm{AP}} + P_{\mathrm{PF}} + P_{\mathrm{AF}} + P_{\mathrm{FF}} + P_{\mathrm{AB}} + P_{\mathrm{FB}} + P_{\mathrm{AA}} + P_{\mathrm{PB}} = 1; \\ 0 = -(2\lambda_{\mathrm{P}} + 2\gamma_{\mathrm{N}})P_{\mathrm{PP}} + \mu_{\mathrm{N}} P_{\mathrm{PF}}; \\ 0 = 2\gamma_{\mathrm{N}} P_{\mathrm{PP}} - (\lambda_{\mathrm{P}} + \lambda_{\mathrm{A}} + \gamma_{\mathrm{N}})P_{\mathrm{AP}} + \mu_{\mathrm{N}} P_{\mathrm{AF}} + \theta_{\mathrm{N}} P_{\mathrm{PB}}; \\ 0 = 2\lambda_{\mathrm{P}} P_{\mathrm{PP}} + \lambda_{\mathrm{A}} P_{\mathrm{AP}} - (\mu_{\mathrm{N}} + \lambda_{\mathrm{P}} + \gamma_{\mathrm{N}})P_{\mathrm{PF}} + 2\mu_{\mathrm{N}} P_{\mathrm{FF}} + \lambda_{\mathrm{B}} P_{\mathrm{PB}}; \\ 0 = \lambda_{\mathrm{P}} P_{\mathrm{AP}} + \gamma_{\mathrm{N}} P_{\mathrm{PF}} - (\mu_{\mathrm{N}} + \lambda_{\mathrm{A}} + \delta_{\mathrm{N}})P_{\mathrm{AF}} + 2\lambda_{\mathrm{A}} P_{\mathrm{AA}}; \\ 0 = \lambda_{\mathrm{P}} P_{\mathrm{PF}} + \lambda_{\mathrm{A}} P_{\mathrm{AF}} - 2\mu_{\mathrm{N}} P_{\mathrm{FF}} + \lambda_{\mathrm{B}} P_{\mathrm{FB}}; \\ 0 = -(\lambda_{\mathrm{P}} + \lambda_{\mathrm{B}} + \theta_{\mathrm{N}})P_{\mathrm{PB}} + \mu_{\mathrm{N}} P_{\mathrm{FB}}; \\ 0 = \gamma_{\mathrm{N}} P_{\mathrm{AP}} - 2\lambda_{\mathrm{A}} P_{\mathrm{AA}}; \\ 0 = \delta_{\mathrm{N}} P_{\mathrm{AF}} - (\lambda_{\mathrm{B}} + \mu_{\mathrm{N}})P_{\mathrm{FB}} + \lambda_{\mathrm{P}} P_{\mathrm{PB}}. \end{cases} \quad (5)$$

The authors obtained the analytical solution of the system of equations (5). Taking into account that only in states PB, AA and FB the system is operable, when each node performs its own function or one node performs the both functions (its own and neighbor's), the following analytical formulas for calculation of stationary rate of the system availability, mean time to failure and mean time to repair were obtained by the authors:

$$K_{\mathrm{SYS}} = P_{\mathrm{PB}} + P_{\mathrm{AA}} + P_{\mathrm{FB}} = \frac{M_5 + M_6 + M_7}{1 + M_1 + M_2 + M_3 + M_4 + M_5 + M_6 + M_7}. \quad (6)$$

$$T_{\mathrm{FSYS}} = \frac{P_{\mathrm{PB}} + P_{\mathrm{AA}} + P_{\mathrm{FB}}}{(\lambda_{\mathrm{B}} + \theta_{\mathrm{N}})P_{\mathrm{PB}} + 2\lambda_{\mathrm{A}} P_{\mathrm{AA}} + \lambda_{\mathrm{B}} P_{\mathrm{FB}}} = \frac{M_5 + M_6 + M_7}{(\lambda_{\mathrm{B}} + \theta_{\mathrm{N}})M_5 + 2\lambda_{\mathrm{A}} M_6 + \lambda_{\mathrm{B}} M_7}. \quad (7)$$

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$$T_{\mathrm{RSYS}} = \frac{1-K_{\mathrm{SYS}}}{K_{\mathrm{SYS}}} T_{\mathrm{FSYS}} = \frac{1+M_1+M_2+M_3+M_4}{(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}})M_5+2\lambda_{\mathrm{A}}M_6+\lambda_{\mathrm{B}}M_7}. \tag{8}$$

Where,

$$M_1 = \frac{2\gamma_{\mathrm{N}}}{\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}}}\left(1+\frac{1}{\Psi}\left(\begin{array}{c}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})\times\\ \times(\lambda_{\mathrm{B}}\lambda_{\mathrm{P}}+(\lambda_{\mathrm{B}}+\mu_{\mathrm{N}})(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}})+\theta_{\mathrm{N}}\delta_{\mathrm{N}})\end{array}\right)\right);$$

$$M_2 = \frac{2\lambda_{\mathrm{P}}+2\gamma_{\mathrm{N}}}{\mu_{\mathrm{N}}};\ M_3 = \frac{2\gamma_{\mathrm{N}}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\lambda_{\mathrm{B}}\lambda_{\mathrm{P}}+(\lambda_{\mathrm{B}}+\mu_{\mathrm{N}})(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}}))}{\mu_{\mathrm{N}}\Psi};$$

$$M_4 = \frac{\lambda_{\mathrm{P}}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})}{\mu_{\mathrm{N}}^2}\left(1+\frac{\gamma_{\mathrm{N}}}{\lambda_{\mathrm{P}}\Psi}\left(\begin{array}{c}(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})\times\\ \times\left(\begin{array}{c}\lambda_{\mathrm{A}}(\lambda_{\mathrm{B}}\lambda_{\mathrm{P}}+(\lambda_{\mathrm{B}}+\mu_{\mathrm{N}})(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}}))+\\ +\lambda_{\mathrm{B}}\delta_{\mathrm{N}}(\lambda_{\mathrm{P}}+\lambda_{\mathrm{B}}+\theta_{\mathrm{N}})\end{array}\right)\end{array}\right)\right);$$

$$M_5 = \frac{2\gamma_{\mathrm{N}}\delta_{\mathrm{N}}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})}{\Psi};$$

$$M_6 = \frac{\gamma_{\mathrm{N}}^2}{\lambda_{\mathrm{A}}(\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})}\left(1+\frac{1}{\Psi}\left(\begin{array}{c}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})\times\\ \times(\lambda_{\mathrm{B}}\lambda_{\mathrm{P}}+(\lambda_{\mathrm{B}}+\mu_{\mathrm{N}})(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}})+\theta_{\mathrm{N}}\delta_{\mathrm{N}})\end{array}\right)\right);$$

$$M_7 = \frac{2\gamma_{\mathrm{N}}\delta_{\mathrm{N}}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\lambda_{\mathrm{P}}+\lambda_{\mathrm{B}}+\theta_{\mathrm{N}})}{\mu_{\mathrm{N}}\Psi};$$

$$\Psi = \left(\begin{array}{c}\lambda_{\mathrm{A}}(\mu_{\mathrm{N}}+\lambda_{\mathrm{A}}+\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}}+\delta_{\mathrm{N}})(\lambda_{\mathrm{B}}\lambda_{\mathrm{P}}+(\lambda_{\mathrm{B}}+\mu_{\mathrm{N}})(\lambda_{\mathrm{B}}+\theta_{\mathrm{N}}))+\\ +\delta_{\mathrm{N}}\lambda_{\mathrm{B}}(\lambda_{\mathrm{P}}+\gamma_{\mathrm{N}})(\lambda_{\mathrm{P}}+\lambda_{\mathrm{B}}+\mu_{\mathrm{N}}+\theta_{\mathrm{N}})\end{array}\right).$$

4. Examples of reliability indices calculation

A symmetric system with functional redundancy based on two identical data processing nodes with the following initial reliability parameters is given:

- Node failure rate in passive state: $\lambda_{\mathrm{P}} = 1/8760$ hour^{-1} (on average one failure per year).
- Node failure rate in active state: $\lambda_{\mathrm{A}} = 2/8760$ hour^{-1} (on average 2 failures per year).
- Node failure rate in backing-up state: $\lambda_{\mathrm{B}} = 3/8760$ hour^{-1} (on average 3 failures per year).
- Node repair rate: $\mu_{\mathrm{N}} = 1/12$ hour^{-1} (on average one recovery per 12 hours).
- Node activation rate: $\gamma_{\mathrm{N}} = 20$ hour^{-1} (on average one activation per 3 minutes).
- Rate of entrance into the mode of backing-up of the neighbor node function: $\delta_{\mathrm{N}} = 20$ hour^{-1} (on average one entrance into the backing-up mode per 3 minutes).
- Rate of exit from the mode of backing-up of the neighbor node function: $\theta_{\mathrm{N}} = 20$ hour^{-1} (on average one exit from the backing-up mode per 3 minutes).

In order to estimate reliability indices in accordance with the simplified reliability model, we calculate stationary availability factor, mean time to failure and mean time to repair of the system using the formulas 2, 3 and 4. To estimate reliability indices in accordance with the advanced reliability model, we use the formulas 6, 7 and 8, correspondingly.

Results of the reliability indices calculations are given in table 1.

Table 1. Results of reliability indices calculations for the symmetric dual-node data processing system.

Reliability model	K_{SYS}	T_{FSYS} (hours)	T_{RSYS} (hours)
Simplified	0.999988802	535820	6
Advanced	0.999943315	1101	0.0624

5. Conclusion

Results of reliability indices calculations of dual-node data processing system with functional redundancy show that though the values of availability factor, obtained from the both reliability models, are close to each other, the value obtained from the advanced model is lower as far as it takes into consideration nonzero time of activation of the nodes primary function, detection of downstate of neighbor node and entrance into the backing-up mode of its function.

As for values of mean time to failure and mean time to repair, there are essential distinctions between the values obtained from the simplified and advanced reliability models.

The simplified model shows that the fault-tolerant system with two nodes possesses very long time before failure and it corresponds more to the model of ideal duplex system in which it is considered that system failure comes only after a failure of the both nodes.

The advanced reliability model shows that its mean time to failure is very short and high value of the availability factor is reached due to the short mean time to repair. So, this model more corresponds to real dual-node system with functional redundancy. The real system during most of the time is in the state when the both nodes are active, and each of them performs only its primary function. Failure of any of them results in short unavailability time of one of the system functions, and consequently in short downstate time of the system in general. However, due to the quite fast failure detection of the neighbor node and switching of its function to the remained operable node, the availability factor of the system remains to be rather high.

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