

On modeling of non-stationary two-phase filtration

V A Korotenko, S I Grachev, N P Kushakova, S A Leontiev, M I Zaboeva and M A Aleksandrov

Industrial University of Tyumen, 38 Volodarskogo st., Tyumen, 625000, Russia

E-mail: nel56-33@mail.ru

Abstract: Based on the basic mechanics equations of continuous medium, the author's model of non-stationary two-phase oil filtration and water pumped into the reservoir was developed, based on the dependence of the water saturation coefficient on the pressure of water entering the reservoir. The substantiating for applying the proposed dependence was given. To solve the system of non-linear differential equations, the integral relations method was used, which allowed obtaining an approximate analytical solution of the water pressure disturbance distribution in the two-phase filtration area. The radius of the perturbation area (front of the displacement of oil by water) was defined, and the boundaries of the perturbation region were determined for the given boundary conditions at the bottom of the injection well. Examples allowing the quantify assessment the application of the model proposed by the authors were considered.

1. Introduction

The justification of the methods for extracting residual mobile oil is an urgent problem of the development of oil deposits. Realization of the paper results of N.N. Lisovsky, R.Kh. Muslimov and other researchers devoted to the problems of the application of hydrodynamic methods of increasing oil recovery, allowed to extend the terms of field development. N.N. Mikhailov studied the problem of deteriorating the properties of the previously considered recoverable part of the reserves in the process of long-term development [1]. For this, it is necessary to know the nature of the distribution of mobile oil reserves in the pore volume of the deposit and the degree of its mobility. The implementation of the ongoing development system has a significant impact on the ratio of non-recovered oil reserves and the amount of water pumped into the reservoir. In order to bring project and actual production data and to extract the unreached by approved development system of residual oil reserves, enhanced recovery methods are used. The choice is influenced by geological, technological and economic factors.

2. Research

By constructing the computational models based on known physical fluid properties and reservoir porosity and permeability, a search for the areas of existence of unrecovered oil is made [1, 2]. When the pressure and temperature of the oil change, an associated hydrocarbon gas can release and one-, two- and three-phase filtration areas arise in the reservoir. For the construction of physical models of two and three phase filtration, the basic mechanics equations of continuous medium are used - equations of continuity, motion and rheological equations. After substituting the motion equations into the continuity equations, we have a system of differential equations describing the processes of multiphase filtration, including the displacement of oil by water. Equations describing non-stationary



three-phase filtration in a porous and fractured-porous medium were considered in papers [3-12]. Differential equations include unknown pressures as well as water and oil saturation coefficients, respectively. It is believed that oil and water do not mix in reservoir conditions.

3. Results and discussion

Let us consider a two-phase filtration water-oil at a pressure above the saturation pressure [5-12]: For isothermal two-phase filtration, the equations of continuity and motion have the form [5-12]:

$$\frac{\partial}{\partial t}(m\rho_i s_i) + \operatorname{div}(\rho_i \bar{v}_i) = 0, \quad \bar{v}_i = -\frac{k_0 k_i^*}{\mu_i} \operatorname{grad} p_i, \quad i=1-b, 2-n \text{ (water, oil)}, \quad (1)$$

where k_i^* – relative phase permeability, k_0 – coefficient of absolute permeability, μ_i – dynamic viscosity; v_i – filtration rates, m – coefficient of porosity, ρ_i – phase densities, p_i – current phase pressures.

To solve the equations system (1), rheological equations are needed, which construction is based on the experimental study of reservoir fluids and media. In underground hydromechanics, parameters determined from rheological equations, as a rule, are functions of pressure and temperature. If in the downhole reservoir area the pressure value becomes lower than the saturation pressure value, the rheological equations of liquids change. As the pressure decreases from the initial pressure to the saturation pressure, the density and viscosity of the oil first decrease, and when the pressure drops below the saturation pressure, these parameters begin to increase. The reverse pattern is observed when the volumetric oil ratio is changed. The amount of gas dissolved in oil is accounted for by the gas content factor therefore the composition of oil and the content of light hydrocarbons significantly affect the shape and boundaries of the application of rheological equations. When pumping water, the pressure, the density and viscosity increase. Dependences of the rheological parameters of porous media saturated with liquids on temperature are mainly needed when designing the application of thermal methods for enhancing the oil recovery. For isothermal processes, rheological equations which parameters obey an exponential law have the form:

$$\rho_i = \rho_{i0} \exp(\alpha_{ip} \Delta p_i), \quad m = m_0 \exp(\alpha_m \Delta p_i), \quad \mu_i = \mu_{i0} \exp(\alpha_{\mu i} \Delta p_i), \\ k_i(p_i) = k_{i0} \exp(\alpha_{ki} \Delta p_i), \quad (2)$$

where α_{ip} – coefficients of fluid compressibility, $\Delta p_i = p_i - p_0$ – pressure change (depression or repression). It is usually assumed that $\alpha_m = \beta_c$ is the compressibility factor of the reservoir, the skeleton. $\alpha_{\mu i}$ – are the coefficients of viscosity depending on pressure, α_{ki} – are the coefficients of permeability change depending on pressure, $i=1$ or b , 2 or n are water and oil.

A necessary condition for solving the system (1) is the correspondence of the number of equations to the number of unknown parameters. To solve the direct problem of two-phase filtration required unknowns are the pressures p_i and the saturation coefficients of phases s_i . By substituting the motion equations into the system continuity equations (1), we obtain two partial differential equations for the determination of four unknowns. Therefore, for a correct solution, two more closing equations are needed, connecting the unknowns p_i and s_i . The volumetric content of oil and water in the reservoir is characterized by the coefficients of oil saturation s_n and water s_b , which are interrelated by the ratio

$$s_n = 1 - s_b. \quad (3)$$

The residual mobile oil saturation can be estimated from the change in the water saturation coefficient in various parts of the deposit. Changes in the saturation coefficients are possible as a result of a decrease in the volume of fluids in the pore space and due to a change in the volume of the pore space itself. And, if it is talked about mobile oil reserves, then in the two-phase filtration area, the saturation coefficients vary within

$$s_0 < s_b < s^* < 1 \text{ and } 1 - s^* < s_n < 1 - s_0, \quad (4)$$

where s_0 is the residual water saturation; s^* is the limiting value of the water saturation coefficient at which oil filtration ceases.

Another closing equation is the relationship between the phase pressures taken into account by the capillary pressure p_k , which is determined experimentally.

$$p_1 = p_2 \pm p_k(s_1). \quad (5)$$

It should be noted that (5) is valid for the stationary state equilibrium of the medium, but in the absence of a better one, it is also used to describe dynamic processes (an analog of Coulomb's law for dry friction in theoretical mechanics.)

Let us consider some models of two-phase filtration of oil displacement by water.

The Buckley-Leverett model is one of the most common physical models describing non-piston oil displacement by water [6, 7, 11].

It is believed that the density of water, oil and porosity are constant. The rheological equations (2) are simplified (in fact they are equations of state), capillary pressure is neglected, the closing equation (5) takes the form $p_1 = p_2$. It follows from the continuity equations that the total phase velocity is only a function of time. After substituting the motion equations and the Buckley-Leverett function, which characterizes the fraction of water in the total fluid flow, in the continuity equations one partial differential equation, connecting the derivative of the Buckley-Leverett function, coordinates and time is obtained. For constant injective capacity (flow rate of injected water) Q after integration and transformations for flat-radial filtration, the following relation is satisfied

$$t = \frac{\pi m h r^2}{Q f'(s_f)}, \quad (6)$$

where s_f is the water saturation at the displacement front, determined graphically for the known values s_0 and s^* , $f(s)$ is the derivative of the Buckley-Leverett function, Q is the water flow rate (injection capacity), m is the porosity coefficient, and h is the effective thickness of the reservoir. The water cut coefficient and the current oil recovery index are determined by the Buckley-Leverett function and its derivative. On the displacement front, the derivative of the Buckley-Leverett function has a jump. From our point of view, jump occurs depend on the used hypothesis. Therefore, despite the elegance and mathematical beauty of the solution, this model has physical drawbacks due to the chosen rheological equations.

In the Muskat-Mires model [5, 7, 8], when describing the rheological properties of a porous medium, the following hypotheses are accepted: the incompressible collector, the porosity coefficients m and the absolute permeability k are constant, the change in density ρ_i of the liquid phases is expressed in terms of the volume coefficients b_i . Rheological equations in this case have the form:

$$b_i = \frac{V_i}{V_{i0}}, \quad i=b,n \text{ (water, oil)}, \quad \rho_i = \frac{\rho_{i0}}{b_i},$$

where V_i is the volume of fluids with dissolved gas under the reservoir conditions, V_{i0} is the fluid volume under normal conditions (degassed liquids), ρ_{i0} is the density of fluids under normal conditions. The volume coefficients are determined experimentally. If it is assumed that the pressures in the phases are equal (capillary pressure is not taken into account), then the closing equation (5) takes the form $p = p_i$, $p_k = 0$.

Substituting the motion equations into the continuity equations, we obtain two differential equations for the two-phase oil-water filtration without gas in the free state

$$\frac{m}{k_0} \frac{\partial}{\partial t} \left(\frac{s_i}{b_i} \right) + \operatorname{div} \left(\frac{k_i^*}{\mu_i b_i} \operatorname{grad} p \right) = 0 \quad (7)$$

Two equations (7) and relation (1) form a closed system of equations for determining unknowns p , s_n , and s_b . The solution of the system of differential equations system (7) is carried out by numerical methods. Here, as in the previous Buckley-Leverett model, the basic position of continuum mechanics is used, that in an infinitesimal volume of a porous formation, there are simultaneously movements (filtration) of two fluids due to a single pressure gradient. The specified rheological equations do not take into account the deformation of the porous medium, which can lead to a mismatch between calculated and actual development indices, for example, in high-compressible bituminous reservoirs of the Bazhenovskaya suite.

More general is the model of two-phase filtration of compressible and deformable media, considered in the papers of V.N. Nikolayevsky, K.S. Basnieva, A.T. Gorbunov and other researchers [9-10]. As a result of substitution of the equations of motion and rheological relations into the continuity equations, the following system of differential equations is obtained, taking into account the deformation of the solid phase, compressibility of fluids, porous media and capillary pressure.

$$\frac{\partial}{\partial t} [s_i \varphi_i(s_i) \exp(\beta_i(p_i - p_0))] = D_i^2 \operatorname{div} [F_i(s) \operatorname{grad}(\exp(\alpha_i(p_i - p_0)))], \quad (8)$$

$$\varphi_i(s_i) = \exp[\alpha_m s_i p_k(s_b)], \beta_i = \alpha_m + \alpha_{ip}, D_i^2 = \frac{k_0}{\mu_{i0} m_0 \alpha_i}, p_1 = p_2 \pm p_k(s), \quad (9)$$

$$F_i(s) = k_i^*(s) \exp[\alpha_i s_i p_k(s)], \quad \alpha_i = \alpha_k + \alpha_{ip} - \alpha_{ui}.$$

In the derivation of equations (8), it was assumed (hypotheses) that the rock pressure is constant, and the changes in permeability and porosity are functions of the effective pressure $P = p_1 s_1 + p_2 s_2$. From the condition (hypothesis) of the conservation of the mass of one of the phases in an elementary volume, if the flow of this phase is zero, the closing equation will take the form

$$(1-s)\varphi_2(s) \exp[\beta_2(p_2 - p_0)] = A(x), \quad s = s_1, \quad (10)$$

where $A(x)$ is the function determined by the initial distribution of the second phase in the porous medium, that is, the distribution of the saturation coefficient as a function of the coordinate. The closing equation (10) relates the pressure to the water saturation coefficient and is based on the assumption that the flow rate of one phase in the elementary volume is zero, that is, the filtration is not carried out. The solution of the equations system (8) is performed by numerical methods. It was found in the paper [9] that, the solution of system (8) reduces to solving of a non-linear differential equation. In contrast to the Muscat-Mires equations (7), the system of three equations (8), (10) takes into account the deformations of the collector, changes in permeability, porosity and capillary pressure. Hence, it is a more accurate model for describing the two-phase filtration. Thus, the differential equations system (8) in the presence of the closing equations (9), (10) makes it possible to determine the distribution of phase pressures and saturation coefficients in the two-phase filtration area. It is believed that the filtering-capacitive parameters are functions of effective pressure, which, in turn, is a function of unknown pressures and phase saturations. The two-phase filtration model described by the system (8) is more informative, in comparison with the Muscat-Mires model.

The model proposed by the authors of the paper, as well as the previous one, is constructed on the basic equations of continuum mechanics. An element of the operational object is considered in which two-phase filtration occurs as a result of the injection well operation. Changes in phase pressures and saturation coefficients in the field of non-piston oil displacement by water are carried out under the following assumptions: the increase in water saturation occurs due to water entering the reservoir,

hence, it is a function of water pressure; the phase pressure is greater than the saturation pressure; the two-phase filtration region is determined by the radius of the displacement front.

As a closing equation, we take the dependence of the water saturation coefficient on repression at the downhole of the injection well Δp_c and the current repression in the reservoir $\Delta p_b = p(r, t) - p_0$ in the area of oil displacement by water [13].

$$\begin{aligned} s_b(r, t) &= s_0 + (s^* - s_0) \frac{\Delta p_b}{\Delta p_c} = s_0 + s_w(r, t), \\ s_w(r, t) &= (s^* - s_0) \frac{\Delta p_b}{\Delta p_c}, \quad 0 \leq s_w(r, t) \leq s^* - s_0. \end{aligned} \quad (11)$$

The coefficient $s_w(r, t)$ is the increment of the water saturation coefficient $s_b(r, t)$ as a result of water pumped into the reservoir. Equation (11) is obtained under the following boundary conditions: at the downhole of the injection well, the water saturation coefficient $s_b(r, t)$ takes the value s^* , and at the front of oil displacement by the injected water it is equal to the residual water saturation s_0 . Equation (11) is valid for both plane-radial and linear filtration.

After substituting the motion equations and the closing equation (11) into the continuity equations of the system (1), we obtain two partial differential equations

$$\begin{aligned} \frac{\partial \Phi_{1i}}{\partial t} &= \frac{a_{0i}}{\beta_{2i}} \operatorname{div} \Phi_{2i}, \\ \Phi_{1i} &= s_i \exp(\beta_{1i}(p_i - p_0)), \quad \Phi_{2i} = k_i^*(s_i) \operatorname{grad} \exp(\beta_{2i}(p_i - p_0)), \\ a_{0i} &= \frac{k_0}{m_0 \mu_{0i}}, \quad \beta_{1i} = \alpha_m + \alpha_{\rho i}, \quad \beta_{2i} = \alpha_{\rho i} + \alpha_k - \alpha_{\mu i}. \end{aligned} \quad (12)$$

where a_{0i} is a coefficient which takes into account the initial physical properties of the reservoir and fluid. Omitting the index $i=1=b$ of the water phase, for flat-radial filtration, the first equation of system (12) takes the form:

$$\frac{\partial \Phi_1}{\partial t} = \frac{a_0}{\beta_2} \frac{1}{r} \frac{\partial(r \Phi_2)}{\partial r}, \quad r \frac{\partial \Phi_1}{\partial t} = \frac{a_0}{\beta_2} \frac{\partial(r \Phi_2)}{\partial r}. \quad (13)$$

To solve the equation (13), we use the G.I. Barenblatt's method of integral relations [10]. The functions entering into the differential equation (13) depend on the water pressure p . Moreover, the function Φ_2 is the product of two functions of scalar $k^*(s(p))$ and vector $\operatorname{grad} \exp(\beta_2(p - p_0))$. After substituting the functions Φ_1 and Φ_2 in (13) and transformations, we obtain a non-linear partial differential equation related to p . For the function p , we set the following boundary conditions: on the displacement front, the pressure gradient is zero, the pressure is constant

$$\left. \frac{\partial p}{\partial r} \right|_{r=\rho(t)} = 0, \quad p(\rho(t)) = p_0 \quad (14)$$

Restricting ourselves to the three terms of the series, we seek the solution in the form

$$p = c_0 + c \ln \frac{r}{\rho(t)} + c_k \frac{r^k}{\rho^k(t)}.$$

Here k is a number, not necessarily an integer, determined from the comparison of fishing data with the calculated indicators; c_0 and c_k are the parameters determined from the boundary conditions. For the boundary conditions (14), we obtain

$$p(r, t) = p_0 - \frac{c}{k} \left(k \ln \frac{\rho(t)}{r} + \frac{r^k}{\rho(t)^k} - 1 \right) = p_0 - \frac{c}{k} f_k(r, \rho(t))$$

$$f_k(r, \rho) = \left(k \ln \frac{\rho(t)}{r} + \frac{r^k}{\rho(t)^k} - 1 \right) \quad (15)$$

We integrate the equation (13) with respect to r from r_c to $\rho(t)$, taking into account the boundary conditions at the displacement front and the bottom of the injector set for the pressure p . We obtain a differential equation for determining the radius of the displacement front $\rho(t)$

$$A_k \frac{d}{dt} \left[\rho^2(t) \left(\frac{\Delta s}{f_k(r_c, \rho(t))} + \beta_1 s_0 \frac{c}{k} + \beta_1 \frac{\Delta s}{f_k(r_c, \rho(t))} \frac{c}{k} \frac{D_k}{A_k} \right) \right] = -a_0 k_m^* r_c \left. \frac{\partial p}{\partial r} \right|_{r=r_c}, \quad (16)$$

$$A_k = \frac{k^2}{4(k+2)}, \quad D_k = \frac{k^5 + 7k^4 + 20k^3 + 24k^2 + 16k + 8}{4(k+1)(k+2)^2}.$$

To determine the parameter c , a third boundary condition is necessary, depending on the choice of the operating mode of the well. As a rule, two boundary conditions at the downhole of the injection well are considered: with constant pressure at the bottom or with constant injection capacity.

Let us consider the case when the discharge pressure at the bottom is constant, $p(r_c) = p_c$, where $\Delta p_c = p - p_0$ is the repression at the bottom of the injection well. And p_c should not exceed the ultimate strength of the rock, otherwise the rock will break down, remove mechanical impurities, increase the radius of the well, etc. The parameter c is equal

$$c = -k \frac{p_c - p_0}{f_k(r_c, \rho)} = -k \frac{\Delta p_c}{f_k(r_c, \rho)}. \quad (17)$$

The right-hand side of equation (16) in this case, is equal $a_0 k_m^* \frac{k \Delta p_c}{f_k(r_c, \rho)}$, k_m^* – is the maximum value of the coefficient of relative phase permeability of the reservoir on water, determined experimentally.

The radius of the displacement front $\rho(t)$ is determined by integrating the differential equation (16)

$$A_{k1} \rho^2 \left(1 - \frac{1}{\rho^2} J_1(\rho) - \frac{2D_{k1}}{A_{k1} \rho^2} J_2(\rho) + \frac{2D_{k1}}{A_{k1} \rho^2} J_3(\rho) \right) = k_m^* a_0 k \Delta p_c t,$$

$$A_{k1} = A_k (\Delta s - \beta_1 s_0 \Delta p_c), \quad D_{k1} = D_k \Delta s \beta_1 \Delta p_c, \quad (18)$$

$$J_1(\rho) = \int_{r_c}^{\rho} u^2 \frac{\partial f_k(u)}{\partial u} \frac{1}{f_k(u)} du, \quad J_2(\rho) = \int_{r_c}^{\rho} u \frac{1}{f_k(u)} du, \quad J_3(\rho) = \int_{r_c}^{\rho} u^2 \frac{\partial f_k(u)}{\partial u} \frac{1}{f_k^2(u)} du.$$

It follows from (18) that the radius of the displacement front depends on the values of the water saturation coefficients s_0 and s^* , which determine the area of two-phase filtration, and also from the given boundary condition Δp_c . For single-phase filtration, when determining the radius of influence of a well, the effect of the change in pressure at the bottom of the disturbing well, established by V.N. Shchelkachev, was considered in detail in the papers of A.P. Telkova and S.I. Grachev [12, 14].

The pressure of the injected water in the region of two-phase filtration is determined by the relation

$$p(r,t) = p_0 + \frac{\Delta p_c}{f_k(r_c, \rho)} f_k(r, \rho(t)), f_k(r, \rho) = \left(k \ln \frac{\rho(t)}{r} + \frac{r^k}{\rho(t)^k} - 1 \right). \quad (19)$$

The distribution of the water saturation coefficient in the injection region is determined from the closing equation (11), which, taking into account the obtained results, is written in the form

$$s_b(r, t) = s_0 + (s^* - s_0) \frac{f_k(r, \rho(t))}{f_k(r_c, \rho(t))}. \quad (20)$$

It follows from (20) that the dependence of the water saturation coefficient on the boundary conditions is realized implicitly in terms of the displacement radius $\rho(t)$, determined from the solution of equation (18).

Example. Let us consider how the radius of the displacement front depends on time for different values of the parameter k .

Assumed:

$p_0=22\text{MPa}$, $\Delta p_c=4\text{MPa}$, $k_0=250\text{mD}$, $k_m^*=0.6$, MPa , $\alpha_m=10^{-4}1/\text{MPa}$, $\alpha_\mu=10^{-4}1/\text{MPa}$, $\alpha_\rho=2 \cdot 10^{-4}1/\text{MPa}$, $m_0=0,2$, $Q_0=300\text{m}^3/\text{day}$.

Table 1. Changing in the radius of the displacement front depending on time.

$\rho(t)$, m	t, days			
	k=1	k=1.5	k=2	k=3
5	0.003	0,004	0.005	0.007
10	0.013	0.018	0.02	0.03
50	0.4	0.5	0.5	0.7
100	1.5	1.9	2.2	2.9
400	24	31	36	46
1000	150	194	226	289
2000	604	779	909	1157
3000	1362	1757	2051	2603
4000	2426	3130	3653	4628
5000	3797	4897	5716	7231

From Table 1 it follows that at the initial period of operation of the injection well, the time for reaching the radius $\rho(t)=100$ meters does not exceed 3 days. And for $\rho(t)=400$ meters, which corresponds to the distance between the injection and production wells, the displacement time varies from 24 to 46 days. Therefore, the choice of the parameter k , determined by comparing the actual and calculated parameters, significantly affects the rate of propagation of the water flooding area. It should be noted that with increasing k , the propagation velocity of the flooding area slows down.

Let us compare the obtained results with the piston displacement of oil by water. Before the displacement front, the water saturation coefficient is s_0 , and behind the displacement front it is $s=s^*$. The radius of the displacement region is determined in this case from equation

$$\rho^2(t) \left(1 - \frac{J_1}{\rho^2(t)} \right) = \frac{k_m^* a_0 k}{\beta_1 s^* A_k} t \quad (21)$$

The results of calculations using formula (21) are shown in Table 2. From a comparison of the results of the calculations of Tables 1 and 2, it is clear that in this case the propagation time of the flooding area is much less than for non-piston displacement. As the parameter k grows, the time increases. If we neglect the integral in (21), then the displacement time remains practically unchanged.

Table 2. Change in the radius of the displacement front depending on time for piston displacement.

t,m	t, days			
	k=1	k=1.5	k=2	k=3
5	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000
50	0.0004	0.0005	0.0006	0.0007
100	0.002	0.002	0.002	0.003
400	0.027	0.034	0.040	0.048
1000	0.168	0.217	0.254	0.305
2000	0.677	0.873	1.019	1.225
3000	1.53	1.97	2.30	2.76
4000	2.72	3.51	4.10	4.92
5000	4.26	5.49	6.41	7.70

Unlike the model of Muskat-Mires and the deformation model, the author's version allows to obtain an analytical solution of the required problem.

The fraction of water in the total liquid flow at identical phase-pressure gradients corresponds to the Buckley-Leverett function

$$f(s) = \frac{k^*(s)}{k^*(s) + \mu_0 k_n^*(s)} \quad (22)$$

It follows from (22) that the watering of the production well located at a distance $r=r_f$ from the injection well begins at $s>s_0$, that is, the water saturation coefficient depends on time and coordinates. According to the author's model, from the closing equation (20) it follows that the growth in the water saturation coefficient at the bottom of the production well increases gradually over time. Therefore, the water cut of the production well also increases gradually and depends on the specified boundary conditions and the numerical parameter k .

The fraction of water f in the total flow of the filtering liquid, taking into account the capillary pressure, is determined by the formula given in [15]

$$f = \frac{k^*(s)}{k^*(s) + \mu_0 k_n^* \left(1 + \frac{\partial p_k}{\partial r} / \frac{\partial p}{\partial r} \right)}, \quad (23)$$

here μ_0 – is the ratio of water viscosity to oil viscosity. Capillary pressure depends on the water saturation through the Leverett function $J(s)$, the wetting angle Θ and the surface tension [16]

$$p_k(s) = J(s) \frac{a}{\cos \Theta} \sqrt{\frac{m}{k_0}}.$$

The derivative of the capillary pressure along the coordinate is

$$\frac{\partial p_k}{\partial r} = \frac{\partial p_k}{\partial s} \frac{\partial s}{\partial r} = \frac{\partial s}{\partial r} \frac{\partial J(s)}{\partial s} \frac{a}{\cos \Theta} \sqrt{\frac{m}{k_0}}.$$

It follows from (21) and (25) that

$$\frac{\partial p}{\partial r} = -\frac{c}{k} \frac{\partial f_k(r, \rho(t))}{\partial r} = c \left[1 - \left(\frac{r}{\rho(t)} \right)^k \right] \quad \frac{\partial s}{\partial r} = \frac{\Delta s}{f_k(r_c, \rho)} \frac{\partial f_k(r, \rho)}{\partial r}.$$

From (23) it follows:

$$v = f = \frac{k^*(s)}{k^*(s) + \mu_0 k_n^*(s) \left[1 \mp \frac{\partial p_k(s)}{\partial s} \frac{k \Delta s}{c f_k(r_c, \rho(t))} \right]}. \quad (24)$$

Consequently, the closing equation (20) plays an essential role in determining f and the water-cut coefficient. To determine f from (24), it is necessary to have experimental dependences $p_k(s)$ or $J(s)$. The parameter c corresponds to a given boundary condition (17). Since the relative phase permeability k_n^* and k^* are functions of the water saturation s , the value of the water saturation coefficient depends on the coordinates, the choice of the boundary conditions, $\rho(t)$ and the time to reach a given point of the reservoir, that is, from the selected method of operating the injection well. For an infinite reservoir, $\rho(t) \rightarrow \infty$, $k^* \rightarrow k_m^*$, $k_n^*(s) \rightarrow 0$, the expression in square bracket (24) tends to 1 and coincides with the classical Buckley-Leverett formula (22) and the product is watered. Thus, according to the author's model of non-stationary two-phase filtration, the water cut at the downhole of the producing well increases continuously, and not abruptly as follows from the Buckley-Leverett model.

For anomalous reservoirs containing high-viscosity oil or having low filtration properties, the initial pressure gradient in the equations of system motion (1) and expression (24) should be taken into account.

4. Conclusion

- 1) The known two-phase filtration models are based on the methods of continuous medium mechanics and reduce to a numerical solution of a system of nonlinear differential equations. The process of non-piston oil displacement by water depends on the selected rheological equations and the given boundary conditions. For a correct solution of a system of nonlinear differential equations, a closing equation is needed, which construction is based either on experimental studies or on accepted hypotheses which do not contradict physical sense. Each of the considered models has its advantages and disadvantages, which reliability can be confirmed only by comparing the calculated indicators with the actual ones.
- 2) The proposed author's model is based on the fact that when the reservoir is flooded, the change in saturation caused by the influx of injected water depends on the change in the pressure of the water pumped into the reservoir.
- 3) Unlike other models solved by approximate numerical methods, the method of integral relations of G.I. Barenblatt was used to solve the problem, by means of which an approximate analytical solution was obtained.
- 4) The closing equation (20) makes it possible to establish a relationship between the water fraction in the liquid flow and the water saturation coefficient.

References

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Information about authors

1. Korotenko Valentin Alekseevich, Candidate of Technical Sciences, associate professor of the Department of Development and operation of oil and gas fields, Industrial University of Tyumen (IUT).
2. Grachev Sergey Ivanovich, Professor, Doctor of Technical Sciences, Head of the Department of Development and operation of oil and gas fields, Industrial University of Tyumen (IUT).
3. Nelly Petrovna Kushakova, Associate Professor, Candidate of Technical Sciences, Department of Automobile transport and road and construction machines of Tyumen Industrial University (TIU).
4. Leontyev Sergey Alexandrovich, Doctor of Technical Sciences, Professor, Department of Development and operation of oil and gas fields, Industrial University of Tyumen (IUT).
5. Zaboeva Marina Ivanovna, Candidate of Technical Sciences, associate professor of the Department of Development and operation of oil and gas fields, Industrial University of Tyumen (IUT).
6. Alexandrov Mikhail Alekseevich, Candidate of Technical Sciences, Associate Professor, Industrial University of Tyumen (IUT).