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Monitoring of risk of multidimensional stochastic system as tools for a research of sustainable development of regions

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Abstract. New approach for a research of sustainable development of regions and cities is offered. It is based on the risk model of multidimensional stochastic system. In article the risk model of multidimensional stochastic system with interdependent factors is described. The hypothesis which consists that the risk can be managed by changing probabilistic properties of a component of multidimensional stochastic system is the cornerstone of the offered risk model. At the same time the multidimensional stochastic system is modeled in the form of a random vector which components in generally are mutually correlated. The questions of formation of multidimensional areas of dangerous states and calculation of risk are described. The representation of risk function is shown. For regions of the Ural Federal District on group of risk factors numerical characteristics of a multidimensional Gaussian random variable – a covariance matrix and a vector of mathematical expectations are found. Results of calculation of probability of a dangerous outcome and risk depending on the found numerical characteristics are given.

1. Introduction

The regions and the cities which are regions basis are the complex socio-economic systems consisting of huge number of the interacting elements [1]. This interaction in many respects is characterized by non-deterministic, stochastic character. Many authors point out the necessity of uses of systems approach for studying of such regional systems [2–4]. Systems approach involves the unification of all regional community in the system consisting of the interrelated elements of infrastructures [5] (environmental protection, ecology, economy, education, health, culture, policy etc.) the functioning of which is directed to achievement of sustainable development.

In this situation the multidimensional stochastic system is often modeled in the form of a random vector. Each component of this vector is the one-dimensional random value characterizing functioning of the corresponding element of the system. In [6] vector entroic model operation for the description of regional social and economic systems is used. The vector entroic model which components are the randomness and self-organization entropies allows to investigate system from the perspective of system regularities and to reflect the main trends of its development. But for development of concrete recommendations about improvement of a system condition it is necessary to add system-wide presentation with the quantitative assessment of a contribution to its deterioration in each of its infrastructures.

This problem can be solved with the help of risk analysis [7]. Some authors note that growth rates of damage considerably exceed growth rates of economy [8, 9]. It can be explained with constant increase of risk in the conditions of a scientific and technical revolution and the forced development of a technosphere [10]. Let's formulate a hypothesis: Sustainable development of territorial socio-economic system is inextricably linked with the risk of its functioning, if the risk level is lower, the development of region is steadier. Therefore diagnosis of the sustainable development of system can be carried out on the basis of a monitoring its risk.

Usually modeling a risk comes down to selection of dangerous outcomes, the quantitative assignment of consequences from their occurrence and estimation of the probabilities of these outcomes [11]. For relatively simple objects when it is possible to specify a priori all dangerous outcomes, in the presence of statistical information or expert estimates on chances of their emergence in general this approach yields the results acceptable in practice. However for many complex systems to allocate all these dangerous outcomes is not possible.

The risk model of multidimensional stochastic systems according to which the system is presented in the form of a random vector with mutually correlative components is offered in [12]. The aim of the article is the description of new approach to assessing the sustainability of regional systems on the basis of this model.

2. Model of risk analysis

Let's present a condition of the composite system in the form of some multidimensional stochastic system S. Let's mark out in this system risk factors of $X_1, X_2, ..., X_m$. The result is a representation of a system in the form of a random vector $\mathbf{X} = (X_1, X_2, ..., X_m)$ with some probability density of $p_{\mathbf{x}}(\mathbf{x})$.

Instead of the conventional selection of concrete dangerous situations we will set geometrical areas of failures. Its can look arbitrarily depending on a specific objective, and are determined on the basis of the available a priori information. For definiteness, we will describe the offered approach on the example of the common conception of dangerous conditions as large and improbable deviations of a random variable. Then we will consider dangerous situations larger and improbable deviations of selective values x_{ij} of any of the component X_j the best in sense of safety values θ_j, $j = 1, 2, \dots, m$. If the prior information about the values θ_j is absent, then we consider that they are equal to expectations $\mu_j = M[X_j]$ of random values of X_j, i.e. $\theta_j = \mu_j$, $j = 1, 2, \dots, m$. Then we will define probability of an unfavorable outcome for each of the component X_j as

$$P(D_j) = P(X_j \in D_j) = P(X_j \notin \overline{D}_j), \quad \overline{D}_j = \{x : d_j^- < x < d_j^+\},$$

where d_j^-, d_j^+ – the set left and right boundaries of admissible values ($d_j^- < d_j^+$), i.e. the area of the favorable outcomes is limited to range $(d_j^-; d_j^+)$.

Let's enter the lower b_j^- and upper b_j^+ threshold levels of permissible variations concerning values θ_j as $b_j^- = \theta_j - d_j^-$, $b_j^+ = d_j^+ - \theta_j$, at the same time the area of the favorable outcomes $\overline{D}_j$ for each component of X_j is described by range $(\theta_j - b_j^-; \theta_j + b_j^+)$.

If only the right boundary d_j^+ of admissible values is set, then we consider $d_j^- = -\infty$ and $\overline{D}_j = \{x : x < d_j^+\} = \{x : x < \theta_j + b_j^+\}$, with only a certain left boundary d_j^- we have $d_j^+ = +\infty$ and

$\overline{D}_j = \{x : x > d_j^-\} = \{x : x > \theta_j - b_j^-\}$. Expression $d_j^- = -\infty$ ($d_j^+ = +\infty$) means that, values of risk factor X_j less (more) θ_j are same safe as well as $X_j(\theta_j)$.

Now it is necessary to describe multidimensional area of dangerous situations of D, having considered the mutual influence of the component on the emergence of failures. It is equal $D = \mathbf{R}^m \setminus \overline{D}$ where $\overline{D}$ – area of admissible values of risk factors. Let's describe area $\overline{D}$. It can be done in various ways. The most justified from the geometrical point of view are represented to set it in the form of internal area of m-axis ellipsoid

$$\overline{D} = \left\{ \mathbf{x} = (x_1, x_2, \ldots, x_m) : \sum_{j=1}^{m} \frac{(x_j - \theta'_j)^2}{b_j^2} < 1 \right\}$$

with the center at point $\boldsymbol{\theta}' = (\theta'_1, \theta'_2, \ldots, \theta'_m)$, and $\forall j = 1, 2, \ldots, m$

$$\theta'_j = \begin{cases} \theta_j, & d_j^- = -\infty \vee d_j^+ = +\infty, \\ (d_j^- + d_j^+)/2, & d_j^- > -\infty \wedge d_j^+ < +\infty, \end{cases} \quad b_j = \begin{cases} (b_j^- + b_j^+)/2, & d_j^- > -\infty \wedge d_j^+ < +\infty, \\ b_j^-, & d_j^+ = +\infty, \\ b_j^+, & d_j^- = -\infty. \end{cases}$$

Then for a random vector X probability of unfavorable outcome will be equal

$$P(D) = P(\mathbf{X} \in D), \quad D = \left\{ \mathbf{x} = (x_1, x_2, \ldots, x_m) : \sum_{j=1}^{m} \frac{(x_j - \theta'_j)^2}{b_j^2} \geq 1 \right\}. \tag{1}$$

Let's notice what in (1) area D of unfavorable outcomes represents external area of m-axis ellipsoid which has semiaxes on each of coordinates are equal b_j respectively, i.e. on each j-th axis this area corresponds to a one-dimensional case of D_j. Obviously, when the outcome does not lie on one of axes, the event ($\mathbf{X} \in D$) can be implemented and in the absence of risk deviations on all component (situations $\mathbf{X} \in D$ and $\forall\, j\ X_j \notin D_j$ are possible).

Setting the function of consequences from dangerous situations (risk function) in the form of $g(x)$, we will receive model for the quantitative assessment of risk

$$r(\mathbf{X}) = \int\int_{\mathbf{R}^m} \ldots \int g(\mathbf{x}) p_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} \tag{2}$$

If in (2) to accept $g(\mathbf{x}) = 1\ \forall \mathbf{x} \in D$ и $g(\mathbf{x}) = 0\ \forall \mathbf{x} \notin D$, that $r(\mathbf{X}) = P(\mathbf{X} \in D)$, i.e. the risk is estimated as probability of unfavorable outcome. If at an early stage of system analysis is difficult to describe enough precisely the $g(\mathbf{x})$ function, then the formula (2) becomes assessment of $P(D)$ and is a convenient initial approximation of risk model.

To define a function $g(\mathbf{x})$ requires quantitative assessment of consequences for the studied system depending on values of risk factors. It demands carrying out a separate research. Let's offer one of the variants of setting risk function. Let's make the following assumptions.

1. We consider that the $g(\mathbf{x})$ function is nonnegative and the continuous everywhere on $\mathbf{R}^m$ function, and $g(\boldsymbol{\theta}') = 0$.

2. We consider $\forall \mathbf{z} \in \mathbf{R}^m$ and $\forall \alpha > 1$ $g(\boldsymbol{\theta}' + \alpha \mathbf{z}) \geq g(\boldsymbol{\theta}' + \mathbf{z})$, i.e. the $g(\mathbf{x})$ function does not decrease in any direction from point $\boldsymbol{\theta}'$.

3. We consider that on each risk factor there is information at least on one of limit values: D_j^- is more to the left of θ_j and D_j^+ is more to the right of θ_j, at which achievement consequences become

almost uncontrollable or irreversible. If $d_j^- = -\infty$ ($d_j^+ = +\infty$), then we consider that $D_j^- = -\infty$ ($D_j^+ = +\infty$).

4. $\forall D_j^- > -\infty$ $g(\theta'_1, \theta'_2, ..., D_j^-, ..., \theta'_m) = 1$ and $\forall D_j^+ < +\infty$ $g(\theta'_1, \theta'_2, ..., D_j^+, ..., \theta'_m) = 1$.

Then the risk function can be set, for example, as

$$g(\mathbf{x}) = \sum_{j=1}^{m} \alpha_j (x_j - \theta'_j)^2, \tag{3}$$

or

$$g(\mathbf{x}) = \begin{cases} \sum_{j=1}^{m} \alpha_j (x_j - \theta'_j)^2, & \mathbf{x} \in D, \\ 0, & \mathbf{x} \in \overline{D}, \end{cases} \tag{4}$$

where $\alpha_j = \begin{cases} \dfrac{1}{(D_j^- - \theta'_j)^2}, & x_j < \theta'_j, \\ \dfrac{1}{(D_j^+ - \theta'_j)^2}, & x_j \geq \theta'_j. \end{cases}$

It is apparent that if $D_j^- = -\infty$ and $x_j < \theta'_j$ or $D_j^+ = +\infty$ and $x_j \geq \theta'_j$, then $\alpha_j = 0$.

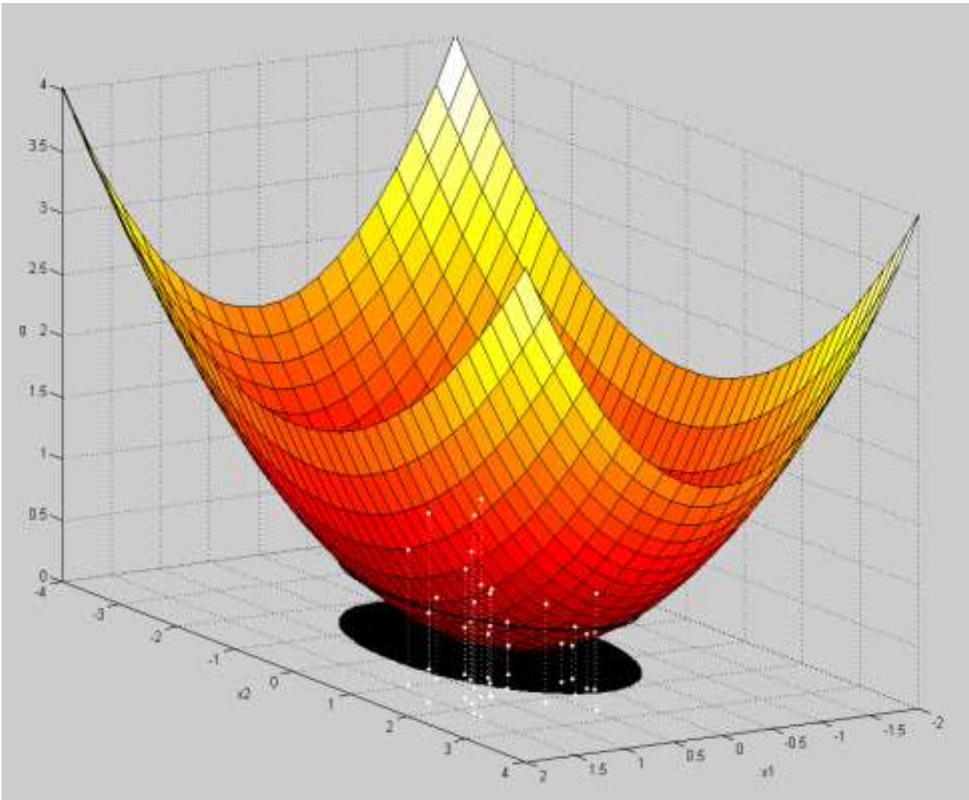

Figure 1. An example of a two-dimensional risk functions (3)

By way of illustration in figure 1 the example of the risk function set on a formula (3) for a case $m = 2$ is shown. The ellipse describing area $\overline{D}$ of admissible values of risk factors and lying on the

Ox_1x_2 (r = 0) plane is shown by black color. The paraboloid above the plane represents possible values of risk *r(**X**)*. White points on the plane are values of risk factors, to them there corresponds points on paraboloid surface which set risk values; the image of border of an ellipse $\overline{D}$ is shown in the form of the black line. All corresponding couples of points (values of risk factors and risk values) are connected among themselves by vertical dashed lines.

In the problems of risk monitoring, Along with risk assessment $r(\mathbf{X})$ on all risk factors of $X_1, X_2, \ldots, X_m$ of multidimensional system is expedient to estimate the contribution of each factor to total risk. We introduce a random vector $\mathbf{X}_k^- = (X_1,\ldots,X_{k-1}, X_{k+1},\ldots,X_m)$. Then the absolute change of risk of multidimensional system due to addition of factor X_k is equal

$$\Delta r(X_k) = r(\mathbf{X}) - r(\mathbf{X}_k^-). \tag{5}$$

Dividing $\Delta r(X_k)$ of the risk $r(\mathbf{X}_k^-)$, we will receive the relative change of risk of multidimensional system by the addition of factor X_k

$$\delta r(X_k) = \Delta r(X_k) / r(\mathbf{X}_k^-). \tag{6}$$

Let's note that along with a contribution to the common risk of one factor, formula (4) and (5) allow us to estimate influence and groups of factors.

Monitoring of risk on the basis of model (1)–(6) consists in serial estimation in time of the actual values of $r(\mathbf{X})$, $\Delta r(X_k)$, $\delta r(X_k)$, $j = 1, 2, \ldots, m$, and also dynamics of their change.

3. Practical application of the risk model of multidimensional stochastic system for monitoring of sustainable development of regions of the Ural Federal District in 1999–2016

Let's consider the most common case when **X** has joint normal distribution with a probability density

$$p_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^m |\mathbf{\Sigma}|}} \exp\left\{-\frac{1}{2}(\mathbf{x}-\mathbf{a})^T \mathbf{\Sigma}^{-1}(\mathbf{x}-\mathbf{a})\right\},$$

where $\mathbf{a} = (a_1, a_2, .., a_m)^T$ – a vector of expectations, $\mathbf{\Sigma} = \{\sigma_{ij}\}_{m\times m}$ – a covariance matrix, $\sigma_{ii} = \sigma_i^2$ – dispersion.

Use of a Gaussian random vector is based on the central limit theorem [13]. It should be noted that this simplification isn't so critical and if there are any bases to consider that density of probabilities a component of a vector of **X** have more extended tails, then this can be corrected by setting the $g(\mathbf{x})$ function accordingly.

Let's carry out approbation of the offered risk model of multidimensional stochastic system for monitoring of sustainable development of four regions (Kurgan, Sverdlovsk, Tyumen and Chelyabinsk) the Ural Federal District after the default of 1998. Consideration of regions instead of the cities is explained by more representative statistical data [14]. The general assessment of probabilities *P(D)* of macroeconomic risk factors of regions of the Ural Federal District in 2001–2015 was carried out in [15]. Below we will execute monitoring of sustainable development of these regions on dynamics of their macroeconomic risk factors, having taken as the analyzed time slice of 9 years. Let's use brought in [15] macroeconomic risk factors and threshold risk levels, having made some refinement:

1) make changes to the risk factors: remove from consideration the index of the industrial production in view of its close correlation with other factors on lesser intervals and instead of dependency ratio we use more informative indicator – unemployment rate;

2) reduce number of threshold levels of risk factors to three: K1 – an unstable state; K2 – a pre-crisis state; K3 – crisis state. These threshold levels correspond brought in [15] pre-crisis developing, crisis unstable and crisis extreme states, respectively.

Risk factors, its threshold levels and the limiting values are given in table 1

Table 1. Macroeconomic risk factors of the regions

Risk factor	Threshold levels K1	K2	K3	Extreme values
X_1 – real income movement, in % to previous year	85,9	79,93	75	$D_j^- = 50$
X_2 – the ratio of the average size of pension to subsistence minimum of pensioners	0,86	0,66	0,5	$D_j^- = 0{,}25$
X_3 – morbidity on 1000 people of the population	920	960	1000	$D_j^+ = 1500$
X_4 – mortality from external causes, number of the dead on 100000 people of the population	287,6	322,1	350	$D_j^+ = 700$
X_5 – a wear of fixed assets on the end of the year, %	66,79	71,33	75	$D_j^+ = 150$
X_6 – the volume of budget revenues per capita, in the prices of 2016, thousand rubles	29,18	21,75	15	$D_j^- = 7$
X_7 – quantum index of gross regional product, % to previous year	92,62	88,4	85	$D_j^- = 40$
X_8 – unemployment rate, in %	15	18	21	$D_j^+ = 40$

In figures 2–4 results of calculation of probability of unfavorable outcome P(D) and risk of r(X) for threshold levels of risk factors K1, K2, K3, respectively are shown. The risk function was set on a formula (4). Calculation of integral (2) was carried out by means of a method of statistical tests of Monte Carlo.

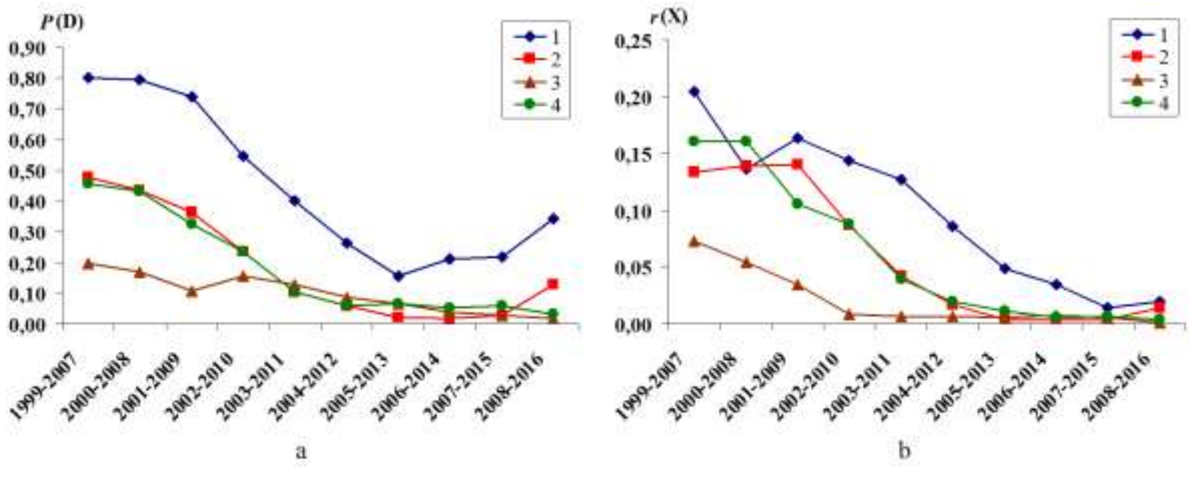

Figure 2. Estimates of P(D) and r(X) for threshold levels of risk factors K1:
1 – the Kurgan region, 2 – the Sverdlovsk region, 3 – the Tyumen region, 4 – the Chelyabinsk region

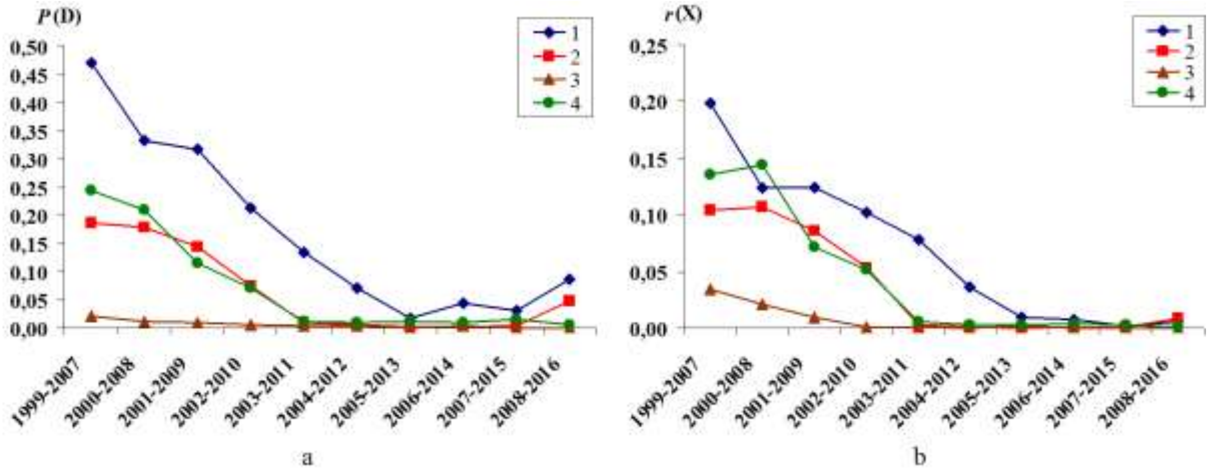

Figure 3. Estimates of P(D) and r(X) for threshold levels of risk factors K2:
1 – the Kurgan region, 2 – the Sverdlovsk region, 3 – the Tyumen region, 4 – the Chelyabinsk region

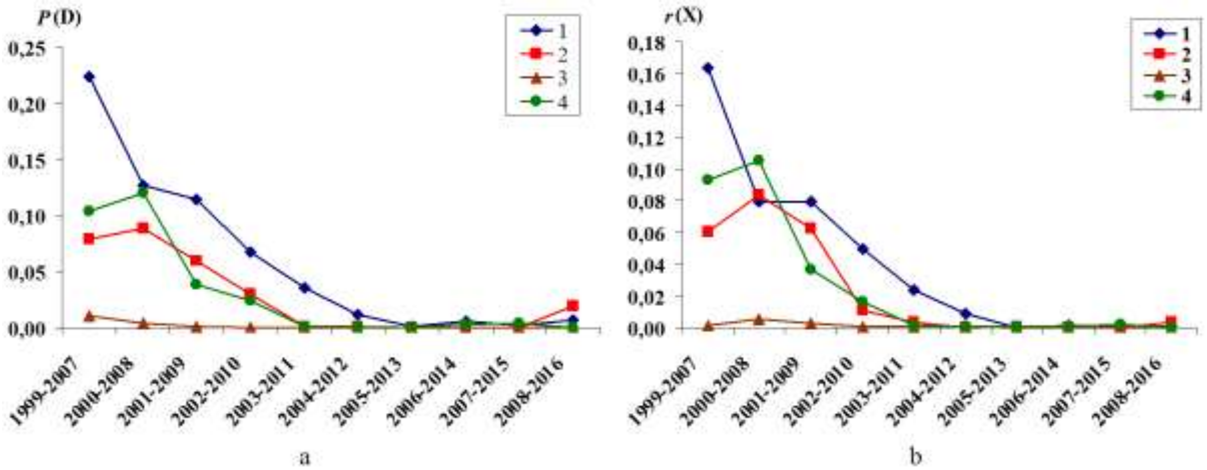

Figure 4. Estimates of P(D) and r(X) for threshold levels of risk factors K3:
1 – the Kurgan region, 2 – the Sverdlovsk region, 3 – the Tyumen region, 4 – the Chelyabinsk region

Analysis of the results of monitoring the sustainable development of regions showed the following:

1. During the initial period (1999–2007) on all four regions the greatest values of both probabilities of a failure of *P(D)*, and risk of $r(\mathbf{X})$ on all three levels of danger K1, K2, K3 were observed. It corresponds to the minimum stability level of each of the regions. The Kurgan region had the worst level of stability, and Tyumen region – the best among four regions.
2. In dynamics stability gradually increased (decrease of *P(D)* and $r(\mathbf{X})$). However in the last three years for the Kurgan and Sverdlovsk region the tendency to decrease in stability of development was outlined again.
3. Analysis on each of regions:
   3.1. In the Kurgan region the greatest contribution to instability of functioning untill 2013 was made by X_6 factor, and then the situation significantly changed – practically all factors began to exert approximately similar impact.
   3.2. In Sverdlovsk region untill 2010 the main contribution to instability of functioning was made by a factor X_6, then X_2 became such factor, and since 2015 factor X_7 began to make the main contribution to instability.

3.3. In the Tyumen region untill 2011 the greatest contribution to instability of functioning was made by factor X_2, then such factors became several (X_1, X_2, X_4, X_6), and during the last two periods X_1 and X_2 belong to such factors.
3.4. In Chelyabinsk region untill 2011 the main contribution to instability of functioning was made by a factor X_6, then such factors were X_2, X_6, X_7, and since 2014 the main contribution to instability began to make factor X_7.

4. Conclusion
1. A new approach to risk analysis of the complex systems is offered. It is based on modeling the system as a multidimensional random variable, which components are risk factors.
2. Two options of the risk analysis are considered. In the first case evaluated the probability of dangerous states of the system, and the second – directly the risk based on the risk function.
3. The carried out approbation of the offered risk model of multidimensional stochastic system on actual data showed its adequacy for monitoring of sustainable development of regions. The received results are well interpreted and correspond to the actual situation in general.

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