

Predator-prey model of exploited fish population Arowana (*Scleropages* spp.)

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Abstract. Interactions in the ecosystem are inseparable from predator-prey interactions. Population model of two species that the predator interacts with its prey for its survival has been developed. The growth rate of a predator depends on predation on its prey. The model used to describe this predation is the Holling Tanner II function. This model also includes a catch variables of prey and predator populations. This formulation provides more realistic picture of exploited fisheries, such as *Scleropages* spp. (arowana) that prey on small fish and shrimp. The purpose of this study is to analyze the model of predator-prey and to interpret on the model based on simulation. High of exploitation rate will lead to rapid population decline. Exploitation rate ranges from $0 \leq h \leq h_{max}$, where h_{max} is the maximum exploitation rate. Results of these simulation show that population of the predator fish will become extinct if exploitation rate exceeds the maximum exploitation rate.

1. Introduction

There are two types of fisheries in Indonesia, namely marine fisheries and inland fisheries. Exploitation activities are carried out on resource populations in those two ecosystems. Exploitation carried out continuously regardless of sustainability, can lead to the extinction of fish resources. To overcome this, the management steps need to be applied to maintain the sustainability of fishery resources. Management that has been done so far is to set the maximum capture or often called the maximum sustainable yield (MSY). However, on MSY based management it is based solely on a single species without considering the interaction of species within it [1].

Several previous studies related to prey-predation models by including MSY in them include research on MSY in fisheries with prey-predator structure [2,3,4,5]. [6] conducted research on Michaelis-Menten prey-predator models with constant harvesting in the prey population. Then research on stability analysis of prey-predator models with constant harvesting on prey fish [7,8]. The studies discussed the stability of the prey-predator models with harvesting in the form of function in the population of prey fish and constant harvesting. Effects of harvesting on various types of prey-predator models have been considered by many researchers [9,10,11,12,13], and [14] focused on the harvesting in predator prey interaction of seals, steelhead trout and herring MSY formulation shows that the highest level of harvesting of predator fish occurs when the growth of prey fish is on its maximum level [15]. Continues harvesting until it passes the limit causes the extinction of these resources [16]. Therefore, it is necessary to restrict the catch to maintain the sustainability of these resources.

Arowana fish is one of economical fishery commodities, usually exported to foreign countries such as Singapore, Hongkong, Taiwan, Japan, Brunei Darussalam and Malaysia [17]. This fish is classified as a predator in inland waters. Population density of arowana fish is very low, and the number of adult population become declined [18]. Some that threaten the life of arowana fish are caused by low



fecundity, the habit in accommodating tillers in the mouth and spawning in open water [19]. The intensive level of exploitation shown by an increase in the number of exports from 1995-2001, that also become a factor that threatens the arowana population [17]. This paper aims to analyze the stability and make simulations of the prey-predator model by harvesting on arowana fish as the development of existing models. The results of the study are expected to provide an overview and information in arowana resource management so that it is sustained and maintained.

2. Mathematical models

The model formed in this study is the interaction of prey-predation with harvesting on predator species, namely the interaction between arowana fish with small fish. Arowana fish is a type of predatory fish and the main prey is small fish [20]. The other assumptions used in this model include:

- Small fish in one population are prey with logistic growth.
- Arowana fish are predatory fish that depend on the existence of small fish. Small (juvenile stage) arowana feeds insect & invertebrate; while the larger one (adult) feeds on small fishes and invertebrate. To simplify the model, the study focused on the predatory prey interaction between arowana as a predator and small fish as a prey.
- In conducting predation there is a limit of saturation prey so that the function of predator follows Holling Tanner II Models.
- Only Arowana is exploited, and henceforth, exploitation is called harvesting.
- No environmental changes/predator or prey responding independently to the environmental factors.

The predator-prey interaction is shown in figure 1.

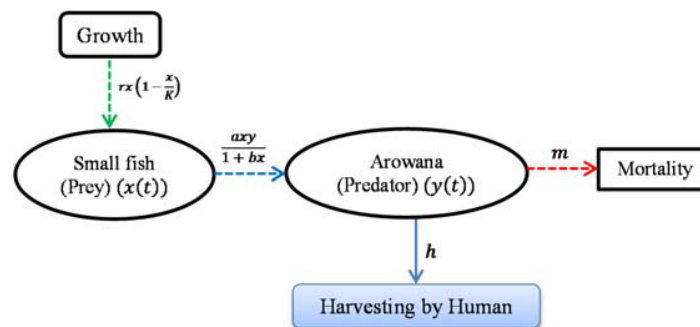


Figure 1. Interaction diagram between arowana and small fish with harvesting rate.

From the interaction and assumptions of variables above, the model is formulated as below:

$$\begin{aligned} \frac{dx(t)}{dt} &= rx(t) \left(1 - \frac{x(t)}{K} \right) - \frac{ax(t)y(t)}{1+bx(t)} \\ \frac{dy(t)}{dt} &= \frac{cax(t)y(t)}{1+bx(t)} - my(t) - hy(t) \end{aligned} \quad (1)$$

r is the intrinsic growth rate of small fish, K is carrying capacities, m is mortality rate coefficient of arowana, a coefficient small fish predation by arowana, c conversion factor from the number of calories required by the new arowana for each prey small fish caught, b is the half saturation of predation on small fish co-efficient and h are harvesting rate of arowana.

3. Equilibrium and stability analysis

To obtain the equilibrium point, let the system (1) if $\frac{dx(t)}{dt} = 0$ and $\frac{dy(t)}{dt} = 0$. Meanwhile, to know the stability of the system (1), is used Jacobi Matrix and finding the negative real parts of the eigenvalues evaluated at each equilibrium point. The Jacobi Matrix of system (1) is:

$$J_E = \begin{bmatrix} \frac{-2rx(1+bx)^2 + K(r(1+bx)^2 - ay)}{K(1+bx)^2} & -\frac{ax}{1+bx} \\ \frac{acy}{(1+bx)^2} & -h - m + \frac{acx}{1+bx} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (2)$$

The result of equilibrium point analysis shows there are three points. Point $E_1(0,0)$, which means that in the ecosystem between arowana fish and its prey are extinct. The Jacobian matrix evaluated at E_0 gives, $J_{E_1} = \begin{bmatrix} -r & 0 \\ 0 & -h - m \end{bmatrix}$ and the eigenvalues of J_{E_0} are $\lambda = r > 0$ and $\lambda = -h - m < 0$. The next equilibrium point is $E_2(K,0)$, which means that arowana fish is extinct so that small fish as arowana prey grow until reaching the carrying capacity. The Jacobi Matrix of these point is

$$J_{E_2} = \begin{bmatrix} -r & -\frac{ak}{1+bk} \\ 0 & -h - m + \frac{ack}{1+bk} \end{bmatrix} \text{ and the eigenvalues is } \lambda = -r < 0 \text{ and } \lambda = ack - (1 + bk)(h + m).$$

The third equilibrium point is $E_3\left(\frac{h+m}{ac-bh-bm}, \frac{rc(Kac-Kbm-Kbh-m-h)}{(ac-bm-bh)^2K}\right)$, which is the expected condition, because with those condition the ecosystem will remain maintained. This condition implies that between prey and predator populations live together. Existence of this will be valuable biologically if it has a non-negative value, so the requirement for point E_3 is biologically significant if $\frac{b(h_y+m)}{ac} < 1$ and $\frac{(m+h_y)(Kb+1)}{Kac} < 1$.

4. Maximum harvesting

Maximum harvesting is the maximum allowable harvest to maintain the sustainability of a population. In the fisheries is usually called the maximum sustainable yield (MSY). Model (1) implements harvesting of arowana population in the form of parameter h . It is known that the maximum growth value of the prey fish population is half of its capacity $\left(\frac{K}{2}\right)$ [8]. Based on the assumptions given for harvesting rate ie $0 \leq h \leq h_{maks}$, where h_{maks} is the maximum harvest rate. Then the maximum harvest rate in model (1) is :

$$h_{max} = \frac{Kac - Kbm - 2m}{Kb + 2} \quad (3)$$

5. Simulation and discussions

This paper studied the dynamical behaviors of one prey one predator system. The interaction between the prey and the predator is assumed to be established by a Holling type II response function because the predator needs sufficient handling time for other prey and there is saturation. A good example of a prey and predator system is arowana (predator) and small fish as a prey. Existing problem is the real world data are not available for this model. Therefore, we using parameters from [7][8]] to illustrate the results numerically, e.i. $r = 0.8$; $K = 100$; $b = 0.1$; $c = 0.75$; $m = 0.001$ and the initial value $x(0) = 80$ and $y(0) = 20$, whereas $a = 0.01$, these value are small because arowana feeds insect & invertebrate. By using those parameters and harvesting rate at $h = 0.02$, system (1) has three equilibrium points, i.e $E_1(0,0)$; $E_2(100.0)$ and $E_3(4.107)$. The result show that all of equilibrium points are unstable (figure 2).

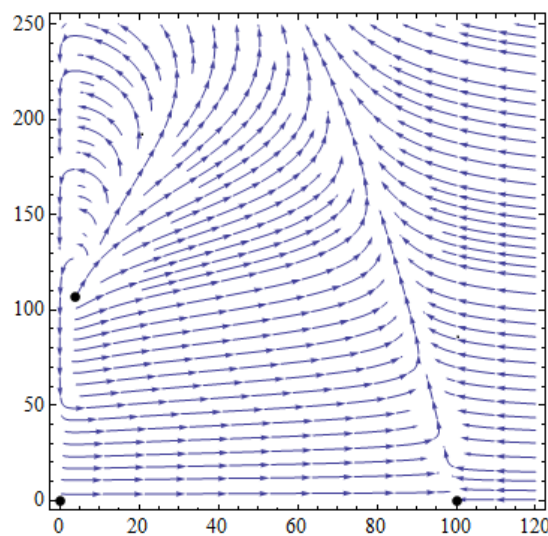


Figure 2. Phase potrait of prey x and predator y , with $h = 0.02$.

Figure 2 describe the points of $E_1(0,0)$ indicated that arowana and small fish are extinct, $E_2(100,0)$ indicated that arowana extinct and the growth of small fish reach to the carrying capacity on the ecosystem. The points of $E_3(4,107)$ means that arowana and small fish can coexist in an ecosystem with a conserved individual population of 4 for small fish and 107 for arowana.

One of the causes of population extinction is the high predation rate of prey and the low level of prey growth or the low initial population of the prey population [21] [22]. Harvesting the predator population may control those populations that can lead to the extinction of prey populations. Conversely, by controlling the predator populations to avoid those extinction, harvesting restrictions will be imposed on predator populations. The restriction of harvesting on predatory fish in this case is arowana fish, because of the exploitation that has been done since 1970 and the number of exports of these fish continues to increase [17].

Based on (3), the maximum harvesting limit is 0.0615. The study of [23] states that the exploitation of arowana fish does not exceed of 50%. To determine the effect of harvesting on the population of prey and predator is done simulation with three treatments, when $h < h_{max}$ (harvesting less than maximum harvesting), $h = h_{max}$ (harvesting and maximum harvesting are equal) and $h > h_{max}$ (harvesting more than maximum harvesting). In prey–predator system, maximum harvesting that only apply on the prey population will tend to extinction of the predator population [4] and/or other species [3]. The fact is both predator and prey has the opposite condition, when predator is decreased conversely with prey, or vice versa. In this condition, standing stock of the predator are still high when the condition of the effort is small, and the yield becomes again lower when the effort is larger [3]. In order to keep the sustainability of prey and predator populations, a maximum harvesting is required, in accordance with the statement [3] that in the Holling–Tanner prey–predator system with any type of response function, fishing of the predator species at the MSY level is a safe harvesting policy for the coexistence of the species. Harvest restrictions need to be done because of the population declining. On the other hand, these species is listed as threatened under CITES Appendix II, which mean that this fish must be protected [17].

5.1. Conditions when $h < h_{max}$

First simulation is done if harvesting less than maximum harvesting, by taking the value of $h = 0.02$. The simulation results show that when $h < h_{max}$, population of arowana will increase and population of small fish tend to decrease. This is due to the increasing predation of the small fish by arowana. This condition can be interpreted with less harvesting rate than maximum harvesting, arowana fish

population and its prey can coexist and mutual source of both stay maintained (figure 3). In fact this condition does not occurred in arowana fishery, because arowana has an economic value. The high demand for arowana fish is a threat to those population [17].

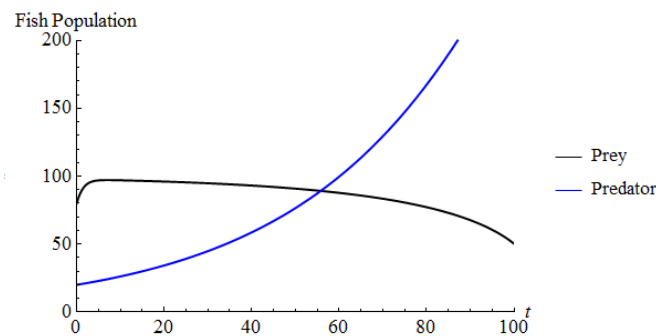


Figure 3. Effect of harvesting of arowana when $h < h_{max}$ on ecosystem.

5.2. Conditions when $h = h_{max} = 0.0615$

The second simulation, harvesting rate is equal to the maximum harvest level. In this case, the harvesting rate of 0.0615 is the maximum level of harvesting that can be done to maintain resource sustainability and give the maximum catch based on model (1). The result shows that small fish as a prey with initial value of individual is 80 tend to unchange. As for arowana fish population, with initial value of 20 tend to increase but the increase was not as significant in the first simulation ($h < h_{max}$). This can be interpreted that, arowana fish population and small fish can still coexist and its existence is maintained in ecosystem if harvesting done maximally (figure 4).

Maximum harvesting is the concept of Maximum Sustainable Yield (MSY). The concept of MSY aims to maintain the size of the fish population at its maximum point and provide economic benefits to society [24]. Management of arowana fisheries that is already done is based on number of capture quotas. For example, the capture of Arowana in Kumbe River reach to 50% of the total production of arowana in these region [25]. Maximum fishing on this predator is one of the strategy in maintaining the sustainability of resources within an ecosystem, however both prey and predator can coexist under this harvesting policy. Maximum harvesting that only apply on the prey population will tend to extinction of the predator population, while maximum harvesting on both prey and predator will cause a single isolated population (composed of prey only, i.e., the predator population goes to extinct) [26].

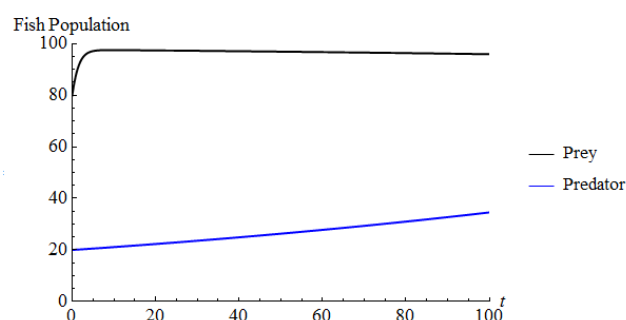


Figure 4. Effect of harvesting of arowana when $h = h_{max}$ on ecosystem.

5.3. Conditions when $h > h_{max}$

Harvesting is done continuously and exceeds the limit will cause the extinction of a resource [16]. High level of harvesting will lead to a decrease in the number of fish populations in an ecosystem. Those condition is reflected in figure 5, when the harvesting rate of arowana is added to be $h = 0.1$, the arowana fish population will decrease, even at a time this fish population will be extinction. The declined of predatory fish population shows that the predation rate of small fish is reduced so after that the population of small fish will increase it reaches carrying capacity condition. Different condition will occurred when harvesting is done on prey fish, which is high harvesting of prey fish populations will decrease even to the extinction condition. This condition will be followed by the extinction of predatory fish populations respectively [8]. Harvesting rate that exceeds the maximum limit is similar to over exploitation. over exploitation of fish resources causes unsustainability [27]. Legality is applied to avoid those condition. Therefore, the Government has issued a regulation No.7/1999, Decree of the Minister of Forestry No.209/kpts-II/2001 renewed by Regulation of the Minister of Forestry Number P.12/Menhut-II/2005 to protect arowana fish population.

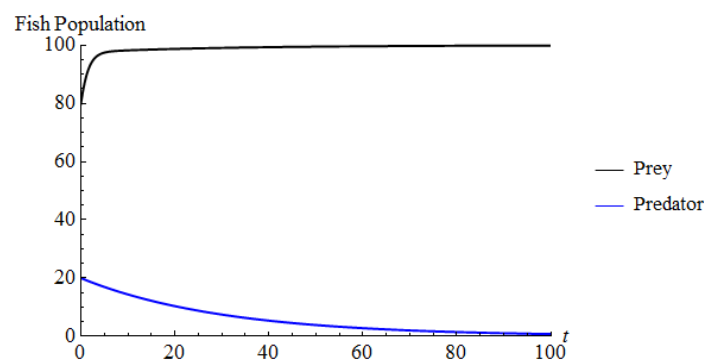


Figure 5. Graphs of stability of prey x and predator y , with $h > h_{max}$.

6. Conclusions and suggestion

6.1. Conclusion

The results of the analysis conducted on the prey-predator model with harvesting on predators, obtained a maximum harvest rate h_{maks} is 0.0615. The model produces three equilibrium points, namely the condition when all populations become extinct, in the ecosystem only the remaining fish are prey and both populations between prey and predator coexist. The simulation indicates that it is necessary to restrict the exploitation of predatory fish to maintain its sustainability. High exploitation coefficient will cause the decrease or extinct of predator population.

6.2. Suggestion

In the next study, the prey-predator model will conduct harvesting treatment of both species, also by giving a treatment of existed conservation area.

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