

Development of an Algorithm for Calculation and Application of Conformal Mapping Methods on the Calculation of Hydrodynamic Coefficients

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Abstract. The multipole method was first developed by Ursell [1]. His method consists of the superposition of potential functions that satisfy the Laplace equation, the free-surface boundary condition, and the condition at infinity. The potential functions represent a source and horizontal dipole at the origin, which give the radiated waves at infinity, and a series of multipoles that die off rapidly as one moves away from the origin. The strengths of the source, dipole and multipoles are all determined so that the body boundary condition is met. Ursell used this method to solve the problem of a heaving circular cylinder. For sections that are not circular in shape, conformal mapping is used.

In the multipole method, the mapping function that transforms the ship section into a semi-circle is found. The mapping function can then be used in conjunction with Ursell's known solution for a circular cylinder to find the solution for the actual ship section.

The difficulty in the technique is to determine the proper mapping function for each cross section. Various methods have been proposed to find this mapping function. The most common mapping uses the so-called Lewis-forms [2], [3] and [4].

It is recognized that this method gives smooth solutions over all frequency range (no irregular frequencies). On the other hand, sharp corners are not well represented, and sections with very low sectional area coefficient may not be well represented as well. Even though, for first rough estimates in initial stages of ship design, this method may give results that agree reasonably well with the other more computational demanding methods, in terms of order of magnitude and trend.

Considering the difficulty to find good charts of the Lewis forms data – to the best of our knowledge, the best known are those published by Bhattacharyya [5] - and even more difficult to find the data in digital form. The main purpose of this work is to present a computational method of computing the data given through the *Lewis forms* and apply it to naval ship sections in order to find rough estimates of the hydrodynamic coefficients in heave.

1. Introduction

One of the areas of Ocean Engineering and Naval Architecture of extreme importance is *seakeeping*, as well as its ability to maneuver in the aquatic environment, allowing us to obtain a forecast of the ship's hull behavior in waves in its coupled six degrees of freedom. As a consequence of this, the study of ship dynamics has been traditionally separated into two main areas [6]:

- *Manoeuvring* or controllability in calm water and
- *Seakeeping* or vessel motion in a seaway.



With regard to *seakeeping*, there are a huge number of methods that can be applied, ranging from simple two-dimensional linear numerical methods in frequency or time domain, such as *Strip Theory*, to advanced nonlinear methods such as *Computational Fluid Dynamics (CFD)*. Both methods have advantages and disadvantages. Although the *Strip Theory* is simpler and considers certain assumptions (e.g., the flow is *2D* and the fluid is inviscid), it continues to have results with good precision enough for initial stage design and in which the computational effort is not so demanding, being possible to obtain them with computers with less effort and faster [7].

The *2D* methods using a *Strip Theory* to find the ship's response in seaway need as one of the inputs the *hydrodynamic coefficients* to solve the equations of motion. From the several methods available today, the most used is the panel method *2D/3D*. However, other less computational, time a cost demanding methods are still possible to use, at least for first rough estimates.

The multipole method was first developed by Ursell [1]. His method consists of the superposition of potential functions that satisfy the *Laplace equation*, the free-surface boundary condition, and the boundary condition at infinity, and the body boundary condition. The potential functions represent a source and horizontal dipole at the origin, which give the radiated waves at infinity, and a series of multipoles that die off rapidly as one moves away from the origin. The strengths of the source, dipole and multipoles are all determined so that the body boundary condition is met. Ursell used this method to solve the problem of finding the *hydrodynamic coefficients* for a *heaving* circular cylinder. For sections that are not circular, *conformal mapping* has been used.

In Ursell's multipole method, a mapping function that transforms the ship section into a semi-circle is found, which can then be used in conjunction with Ursell's known solution for a circular cylinder to find the solution for the actual ship section.

The difficulty in the technique is to determine the proper mapping function for each cross section. Various methods have been proposed to find this mapping function. The most common mapping uses the so-called *Lewis forms* [2], [3] and [4].

It is recognized that this method gives smooth behaviour of the solutions over all frequency range. On the other hand, sharp corners are not well represented, and sections with very low sectional area coefficient may not be well represented as well. Even though, for first rough estimates in initial stages of ship design, this method may give results that agree reasonably well with the other more computational demanding methods, in terms of order of magnitude and trend.

Nowadays it is difficult to find good published charts with Lewis form's data. The best known are those published by Bhattacharyya [5], which are difficult to read and convert reliably.

2. Ship Dynamics

Regarding the dynamic analysis of *seakeeping*, certain simplifications need to be accounted: the ship will be considered a rigid body with small amplitudes motions.

2.1. Motions and Reference Frame

It is necessary to predict the vessel's translational (*surge*, *sway* and *heave*) and rotational (*roll*, *pitch* and *yaw*) motions. These motions are considered as being six degrees of freedom, as is shown in Figure 1.

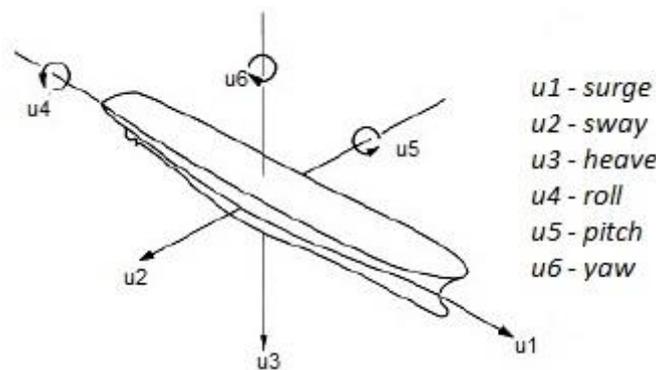


Figure 1. Six degrees of freedom of movement of the ship [6].

Unfortunately, there is no universal coordinate system accepted in the literature of the behavior of the ship at sea. Thus, taking into account the main linear numerical method in the frequency domain applied to linear waves (*Strip Theory*), two coordinate systems are normally used [8]:

- The *ship-fixed system (non-inertial system)* x, y, z , with axis pointing from amidships forwards, to starboard and towards the keel. In this system, the center of gravity of the ship is independent of the time x_g, y_g, z_g ;
- The *Earth-fixed system (inertial system)* ξ, η, ζ , which follows the constant movement of the vessel with velocity $V = \sqrt{u^2 + v^2}$, dependent on the quasi-velocities in surge – u , and sway – v .

It should be noted that, in *seakeeping*, the coordinate system used is usually the *inertial system*.

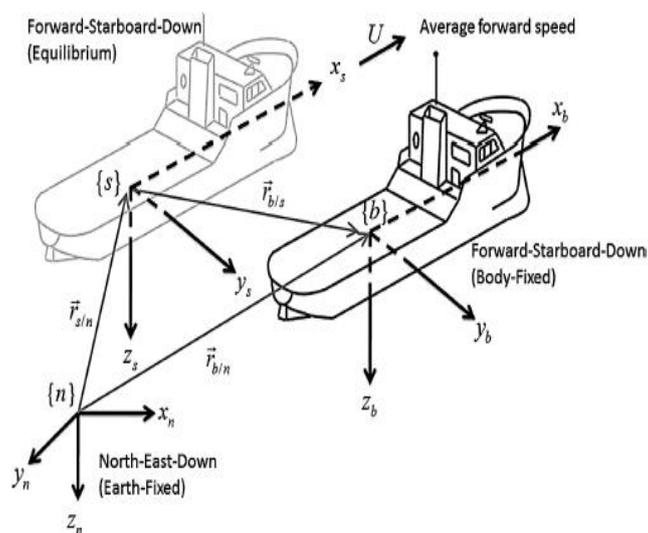


Figure 2. Seakeeping coordinate systems [9].

2.2. Linear Equations of Ship Dynamics in Regular Waves

The prediction of a ship's response in seaway, the *seakeeping*, is a complex process, involving the interactions between the ship's own dynamics and the surrounding hydrodynamic forces. Knowing the

ship's responses in regular waves for different frequencies, we can predict its behavior for several sea states.

The general form of the linearized equations of ship dynamics in the six degrees of freedom, or in other words, the *Euler equations* of ship motion used in the literature devoted to *seakeeping*, using the fixed axes on the ship can be described as follows [9]:

$$\sum_{k=1}^6 \Delta_{jk} \ddot{\eta}_k(t) = F_j(t) \quad j = 1, 2 \dots 6 \quad (1)$$

where:

Δ_{jk} – the inertia matrix components of the ship, such as mass and moment of inertia;

$\ddot{\eta}_k$ – the accelerations in mode k ;

F_j – the sum of the forces and moments acting on the body in the direction j ;

and F_j are harmonic functions in the time base.

Linearizing equation (1), certain terms in Δ_{jk} may be considered zero, as shown by [10], which for a ship with lateral symmetry [9], this equation can be reduced to the following six equations relating to the six degrees of freedom:

$$\begin{aligned} \Delta(\dot{\eta}_1 + z_c \dot{\eta}_5) &= F_1 \\ \Delta(\ddot{\eta}_2 - z_c \ddot{\eta}_4 + x_c \ddot{\eta}_6) &= F_2 \\ \Delta(\dot{\eta}_3 - x_c \dot{\eta}_5) &= F_3 \\ I_{44} \ddot{\eta}_4 - I_{46} \ddot{\eta}_6 - \Delta z_c \ddot{\eta}_2 &= F_4 \\ I_{55} \ddot{\eta}_5 + \Delta(z_c \ddot{\eta}_1 - x_c \ddot{\eta}_3) &= F_5 \\ I_{66} \ddot{\eta}_6 - I_{64} \ddot{\eta}_4 + \Delta x_c \ddot{\eta}_2 &= F_6 \end{aligned} \quad (2)$$

where:

$F_j(t), j = 1, 2, 3$ - the sum of the forces in the directions x, y, z respectively;

$F_j(t), j = 4, 5, 6$ – the sum of the moments acting on the x, y and z axes, with the positive moment following the right hand rule;

Δ - the total mass of the ship;

$I_{jj}, j = 4, 5, 6$ – the moments of inertia about the x, y and z axes, respectively;

I_{46} - the product of inertia between the degrees of freedom *roll-yaw* = I_{64} ;

$(x_c, 0, z_c)$ – the coordinates of the center of gravity of the ship in the non-inertial system x, y, z ;

$\ddot{\eta}_j(t)$ – the acceleration in the degree of freedom j , in the system with the fixed axes in the ship, referring to $j = 1, 2, 3, 4, 5, 6$ the *surge, sway, heave, roll, pitch* and *yaw*, respectively.

Comparing the equations (1) and (2) it is possible to write the inertia matrix Δ_{jk} as:

$$\begin{bmatrix} \Delta & 0 & 0 & 0 & +\Delta z_c & 0 \\ 0 & \Delta & 0 & -\Delta z_c & 0 & +\Delta x_c \\ 0 & 0 & \Delta & 0 & -\Delta x_c & 0 \\ 0 & -\Delta z_c & 0 & I_{44} & 0 & -I_{46} \\ +\Delta z_c & 0 & -\Delta x_c & 0 & I_{55} & 0 \\ 0 & +\Delta x_c & 0 & -\Delta x_c & 0 & I_{66} \end{bmatrix} \quad (3)$$

Taking into account the following assumptions [9]:

- Considering only the gravitational and fluid forces acting on the ship;
- Taking into account the linear theory, the ship's responses will be directly proportional to the wave amplitude, occurring at the frequency at which the ship suffers the incident waves;
- Considering only the ship's response in sinusoidal waves, the time-dependent responses of the vessel $\eta_j(t)$ will be sinusoidal at a given encounter frequency ω_e being represented by: $\eta_j(t) = \bar{\eta}_j e^{i\omega_e t}$ $j = 1, 2 \dots 6$;
 - where $\bar{\eta}_j$ is the amplitude of the response of the ship in the direction j .
- The pressure around the hull can be obtained from the *Bernoulli* equation;
- The hydrostatic and hydrodynamic forces acting on the ship are obtained by integrating the fluid pressure along the surface of the submerged surface of the hull of the ship S (assuming the inviscid and irrotational flow, which allows the linear theory to be applied);
- The hydrodynamic forces resulting from the radiation problem involve the *added mass* and *damping* coefficients.
- F_j^I is the complex amplitude of the excitation force component due to the incident waves, commonly called the *Froude-Krylov* excitation force;
- F_j^D is the complex amplitude of the excitation force component due to the diffracted waves, called the *diffraction* excitation force.

Then, equation (1) results in the following expression:

$$\sum_{k=1}^6 [-\omega_e^2 (\Delta_{jk} + A_{jk}) + i\omega_e B_{jk} + C_{jk}] \bar{\eta}_k = F_j(t) = F_j^I + F_j^D \quad j = 1, 2 \dots 6 \quad (4)$$

where:

A_{jk} - the *added mass coefficients* in dof j due to motion in dof k ;

B_{jk} - the *damping coefficients* in dof j due to motion in dof k ;

C_{jk} - the *hydrostatic restoring force coefficients* in dof j due to motion in dof k ;

$F_j^I + F_j^D$ - the two components of the amplitude of the excitation forces acting on the ship.

2.3. Hydrodynamic Loads

Applying *Strip Theory*, and to simplify the concepts discussed so far, the three-dimensional (3D) problem is reduced to a two-dimensional (2D) problem by dividing the hull into several two-dimensional vertical sections along the length of the ship, each strip having a constant cross-section [7] and a flow that do not interferes longitudinally with the adjacent strip flow. Subsequently, some restrictions will be presented that need to be considered in applying the *Strip Theory*.

A common approach in the calculation of hydrodynamic loads can be made by dividing the hydrodynamic problem into two sub-problems [7]:

- *Sub-problem A*: The movement of the ship when exposed to incoming waves is predicted. In this sub-problem, the *Froude-Krylov* and the diffraction forces and moments of the wave excitation are computed.
- *Sub-problem B*: The incoming waves are not considered. In this sub-problem, we postulate that the ship is moving in its six degrees of freedom at the matching frequency corresponding to the wave frequency of *sub-problem A*. Here, the *added mass coefficients* A_{jk} , the *damping coefficients* B_{jk} , and the *hydrostatic restoring force coefficients* C_{jk} are calculated.

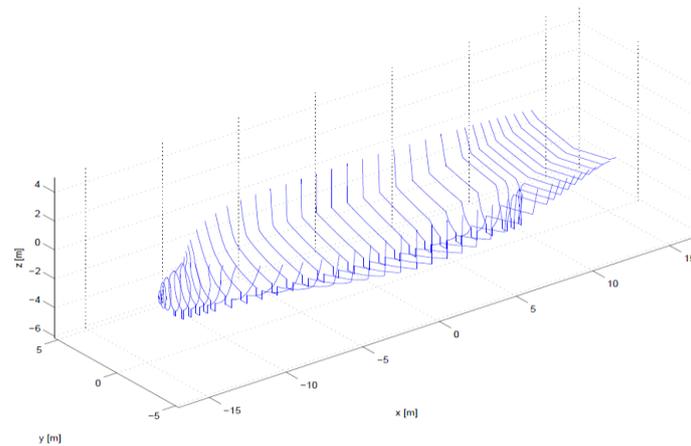


Figure 3. Strip Model of a ship. Each strip has constant cross section [5].

The decoupled equation of the ship's motion can be given by:

$$(M + A)\ddot{\eta} + B\dot{\eta} + C\eta = Fe^{i\omega et}$$

$$\text{Sub - problem A} \rightarrow Fe^{i\omega et} \quad (5)$$

$$\text{Sub - problem B} \rightarrow A\ddot{\eta} + B\dot{\eta} + C\eta$$

This work addresses the radiation problem – *sub-problem B*, that is the calculation of the *hydrodynamic coefficients* A_{jk} and B_{jk} .

3. Conformal Mapping

As started above the main principle in *Strip Theory* involves dividing the submerged part of the ship into a finite number of strips. Hence, *2D hydrodynamic coefficients* for *added mass* a_{jk} and *damping* b_{jk} can be computed for each strip and then be summed over the length of the body to yield the *3D coefficients* [11].

The *2D dynamic coefficients* can be calculated from boundary element methods or via *conformal mapping* [11].

Conformal mapping is used to transform the section into a circle, for which the form of the multipole potential is known. This representation is then transformed back into the physical plane using the derived mapping function [12].

The problem on *conformal mapping* now becomes one of determining the parameters in the transformation which map the arbitrary section to a unit circle.

The earliest two-parameter mapping technique was due to [2]. This method produces reasonable representations of conventional sections [12].

The determination of the local 2D *added mass* and *damping coefficients* were being studied by many researchers, such as Ursell [1], [13], [14], [15] and [16], using the so-called *Lewis Forms* [17].

However, nowadays most of the computing tools available make use of more complex panel methods.

3.1. Computation of the Hydrodynamic Coefficients using a two-parameter conformal method

The algorithm developed computes the local *heave added mass* a_{33} and *heave damping coefficients* b_{33} on each section of the ship (the global ship *added mass* and *damping coefficients* in *heave* using *Strip Theory* being: $A_{33} = \int a_{33} dx$ and $B_{33} = \int b_{33} dx$ respectively).

Our work was based on the formulation developed by [18]. This scientific document gives the expressions for the *added mass* and *damping coefficients* for conventional hull cross-sections related with the *Lewis forms* approximations.

3.1.1. Lewis Transformation Method

Due to the fact that ship's hulls do not have semi-circular cross-sections, *Lewis Transformation Method* is used to extend the results for the semi-circle into solutions for more realistic hull shapes. In this way, it must be considered that [17]:

- Small motion amplitudes are assumed.
- The *hydrodynamic coefficients* are calculated with the usual potential flow assumptions of:
 - Negligible viscosity;
 - Negligible compressibility;
 - No flow separation;
 - No skin friction.
- The transformation relates only the half cylinder below the free surface.
- The mapping relates only to the underwater shape of the hull cross section.

In this technique, the circle and the flow around it (stream and potential functions) are calculated in the complex z plane where:

$$z = x + iy = ire^{-i\theta} [m] \quad (6)$$

Then, these results are mapped into the flow around a hull section in the complex ζ plane (the hull cross section plane) defined as:

$$\zeta = x_{B2} + ix_{B3} [m] \quad (7)$$

These two complex planes can be related by the following transformation:

$$\zeta = f(z)[m] \quad (8)$$

It's important to understand that for each size and shape of the section of the ship in the ζ plane, the functional form of the transformation equations must be determined for every individual case.

Therefore, the transformation that will map any point on a semicircle of radius a meters in the z plane into a corresponding point on a given shape in the ζ plane (if appropriate values of the coefficients a_0, a_1, a_3 are chosen) can be referred as [17]:

$$\zeta = f(z) = a_0 a \left(\frac{z}{a} + \frac{a a_1}{z} + \frac{a^3 a_3}{a^3} \right) [m] \quad (9)$$

Lewis forms are defined by the values of:

- The section area coefficient:

$$\sigma = \frac{A}{BD} \quad (10)$$

- The beam/draft ratio:

$$H = \frac{B}{D} \quad (11)$$

where:

A – the underwater sectional area;

B – the underwater sectional beam;

D – the sectional draft.

Taking into account that the section of the ship has radius $r = a$ [m], substituting equations (6) and (7) into equation (9), and separating real and imaginary parts, we can obtain a pair of parametric equations in θ (from $\theta = \pi/2$ to $\theta = 0$) describing the shape of the *Lewis form* in the ζ plane:

$$x_{B2} = a_0 a [(1 + a_1) \sin \theta - a_3 \sin(3\theta)] [m] \quad (12)$$

$$x_{B3} = a_0 a [(1 - a_1) \cos \theta - a_3 \cos(3\theta)] [m] \quad (13)$$

The coefficients a_1 and a_3 are obtained with these equations:

$$c = 3 + \frac{4\sigma}{\pi} + \left(1 - \frac{4\sigma}{\pi}\right) \left(\frac{H-2}{H+2}\right)^2 \quad (14)$$

$$a_3 = \frac{3 - c + \sqrt{9 - 2c}}{c} \quad (15)$$

$$a_1 = (1 + a_3) \left(\frac{H-2}{H+2}\right) \quad (16)$$

It should be noted that a_0 is a scale factor governing the overall size of the *Lewis form* [12]:

$$a_0 = \frac{B_0}{1 + a_1 + a_3} \quad (17)$$

Being B_0 the half underwater sectional beam. Some examples of *Lewis forms* are presented in Figure 4.

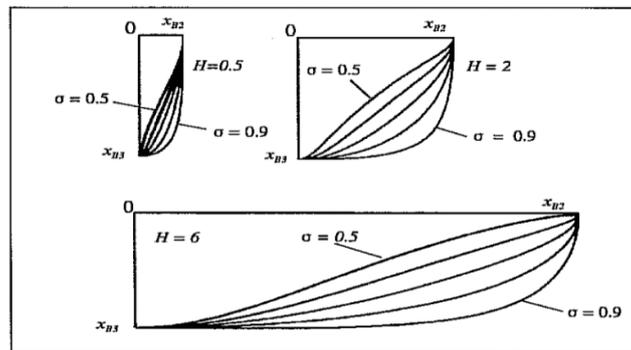


Figure 4. Examples of *Lewis forms* [11].

3.1.2. Added Mass and Damping Coefficients for a heaving *Lewis Form* – frigate cross section

Considering a cross section from a *Portuguese Navy frigate* with the following dimensions:

- $B = 13.2$ [m];
- $D = 4$ [m];
- $A = 22.4$ [m²].

It is possible to obtain the sectional area coefficient and beam/draft ratio:

- $\sigma = \frac{A}{BD} = 0.4242$;
- $H = \frac{B}{D} = 3.3000$.

By the fact that only the half section of the ship is being considered on the *Lewis Transformation Method*, then in the ζ plane x_{B2} and x_{B3} it will be:

- $x_{B2} = \frac{13.2}{2} = 6.600$ [m];
- $x_{B3} = 4.000$ [m].

The studied *frigate* cross section *Lewis form* is presented on Figure 5.

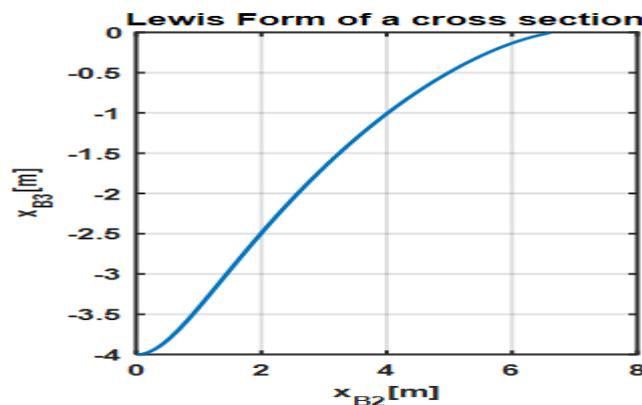


Figure 5. *Lewis form* of a cross section from *Portuguese Navy frigate*.

Considering the studies made by [18] the expressions for *added mass* and *damping coefficients* in *heave* for each section of the ship are:

$$a_{33} = \frac{\rho B^2 (A_* N_0 + B_* M_0)}{2(A_*^2 + B_*^2)} [t] \tag{18}$$

$$b_{33} = \frac{\rho B^2 \omega \pi^2}{4(A_*^2 + B_*^2)} [kN/(m/sec)] \tag{19}$$

An exhaustive description for both equations can be found in Appendix [17].

Comparing the *non-dimensional* curves of *hydrodynamic coefficients* in *heave* (*added mass and damping*) presented on page 120 [17] with the curves obtained using the algorithm written in *Matlab*®, it is possible to see that the results are similar with an approximate error of:

- $H = 2, \sigma = 1$; 2% for *added mass* and 17% for *damping*;
- $H = 4, \sigma = 1$; 4% for *added mass* and 8% for *damping*;
- $H = 8, \sigma = 1$; 8% for *added mass* and 11% for *damping*;
- $H = 1.155, \sigma = 0.5$; 5% for *added mass* and 14% for *damping*.

The respective curves are presented in Figures 6 and 7. This comparison serves the purpose of validation before making the calculations for the real cross section of the naval *frigate*.

The legend for the curves given by [17] and *Matlab*® code:

Form	Key H	σ	Symbol
Rectangle D	2	1.0	□
Rectangle E	4	1.0	▭
Rectangle F	8	1.0	▭
Triangle G	1.155	0.5	▽

—	H=2, $\sigma=1$
—	H=4, $\sigma=1$
—	H=8, $\sigma=1$
—	H=1.155, $\sigma=0.5$

1. *Added mass coefficient in heave:*

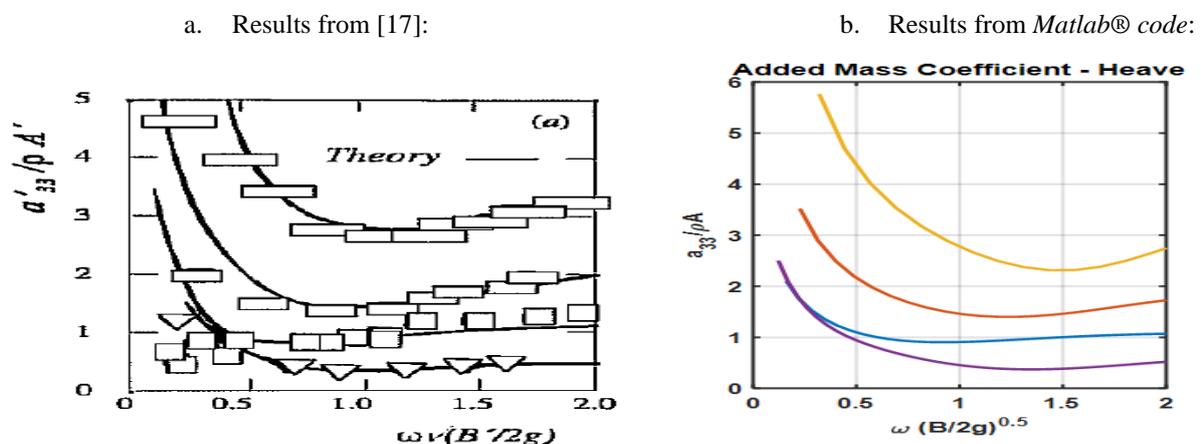


Figure 6. *Hydrodynamic coefficients - added mass*, from [17] and *Matlab*® code.

2. Damping coefficient in heave:

a. Results from [17]:

b. Results from *Matlab*® code:

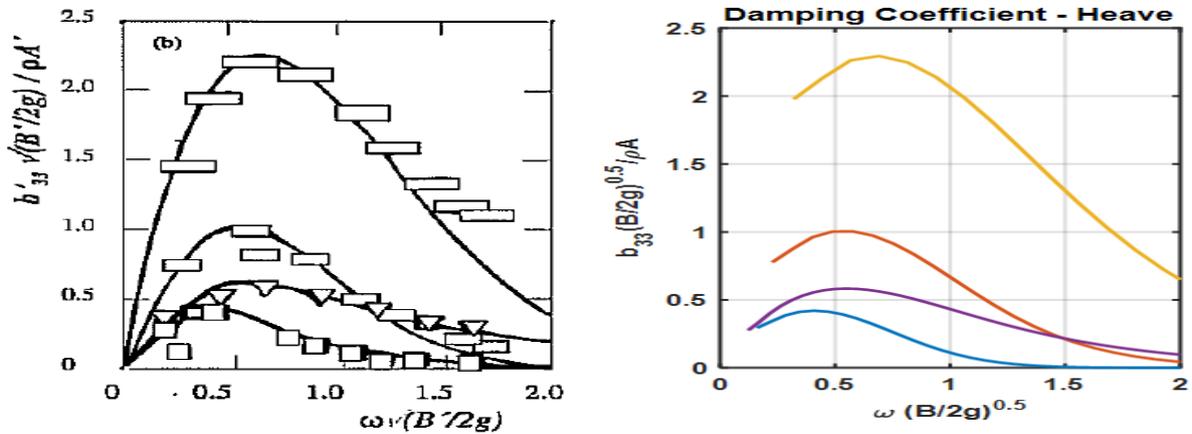


Figure 7. Hydrodynamic coefficients – damping, from [17] and *Matlab*® code.

Then the cross-sectional *non-dimensional added mass and damping hydrodynamic coefficients* for the cross section from *Portuguese Navy frigate* in heave were calculated and the results can be seen in Figures 8 and 9.

While the *damping* curve follows a expected tend, the *added mass* presents oscillations at the high frequencies that where not expected and need further study and validation.

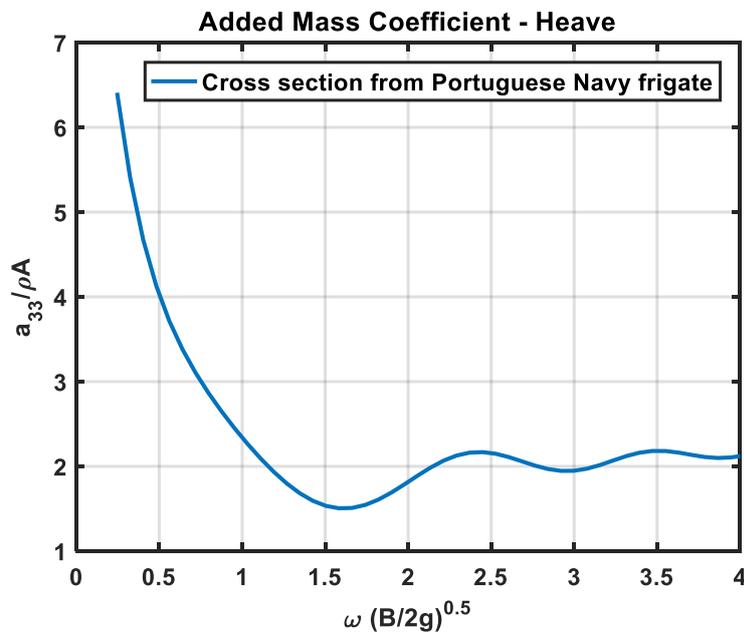


Figure 8. Added mass coefficient of a cross section from *Portuguese Navy frigate* in heave.

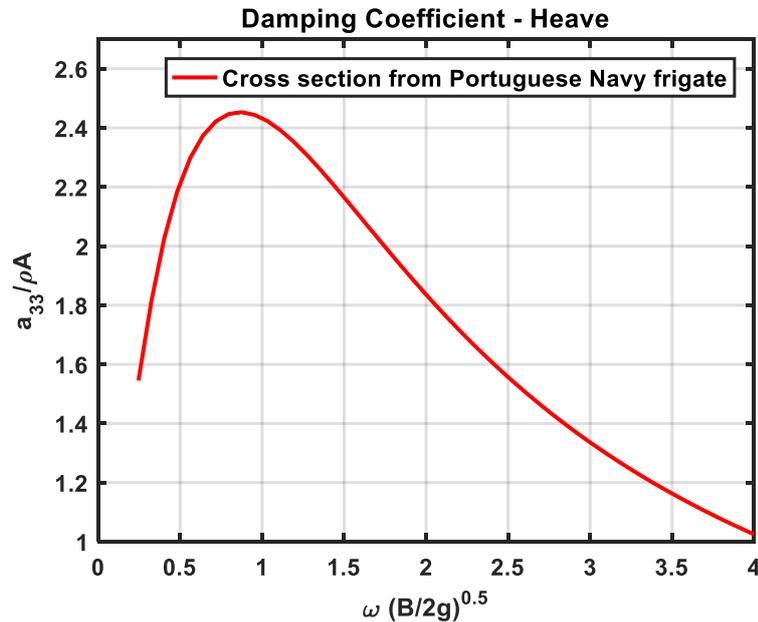


Figure 9. Damping coefficient of a cross section from Portuguese Navy frigate in heave.

4. Conclusions

This work tried to address the radiation problem in what respects the calculation of the *hydrodynamic coefficients* A_{jk} and B_{jk} in heave, A_{33} and B_{33} .

A computing code was developed to estimate the local *heave added mass* a_{33} and *heave damping coefficients* b_{33} using a two parameter *conformal mapping* method usually known as *Lewis forms*.

The code was based on the equations for the radiation problem presented by Lloyd [17].

The results in the form of *non-dimensional heave added mass* and *heave damping coefficients* were compared with those presented by de Jong [18] for several sectional forms and seem to agree fairly well, making a first validation of the code.

The code was applied to the cross section of a navy frigate and the results obtained for the *Lewis form* seem correct. The results obtained for a_{33} and b_{33} show that:

- for the *added mass* there are oscillations in higher frequencies region that need further study since they are not expected using this method which is known to be stable through all range of frequencies;
- the *damping* follows a expectable trend, however there are calculation instabilities to be solved in the very low frequency range ($\omega(B/2g)^{0.5} < 0.25$);
- for both *hydrodynamic coefficients* a validation needs to be made through comparison with other method for the same section.

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Appendix

$$a_1 = (1 + a_3) \left(\frac{H - 2}{H + 2} \right)$$

$$a_3 = \frac{3 - c + \sqrt{9 - 2c}}{c}$$

$$c = 3 + \frac{4\sigma}{\pi} + \left(1 - \frac{4\sigma}{\pi} \right) \left(\frac{H - 2}{H + 2} \right)^2$$

$$x_{B2} = a_0 a [(1 + a_1) \sin \theta - a_3 \sin(3\theta)]$$

$$x_{B3} = a_0 a [(1 - a_1) \cos \theta - a_3 \cos(3\theta)]$$

$$a_{33} = \frac{\rho B^2 (A_* N_0 + B_* M_0)}{2(A_*^2 + B_*^2)} [t]$$

$$b_{33} = \frac{\rho B^2 \omega \pi^2}{4(A_*^2 + B_*^2)} [kN/(m/sec)]$$

$$A_* = \Psi_c \left(1, \frac{\pi}{2} \right) + \sum_{m=1}^{\infty} \left[p_{2m} (-1)^{m-1} \frac{kBQ_1}{2Q_2} \right] [m^2/sec]$$

$$B_* = \Psi_S \left(1, \frac{\pi}{2} \right) + \sum_{m=1}^{\infty} \left[q_{2m} (-1)^{m-1} \frac{kBQ_1}{2Q_2} \right] [m^2/sec]$$

$$M_0 = \int_0^{\frac{\pi}{2}} \Phi_S(1, \theta) \frac{Q_3}{Q_2} d\theta + \frac{1}{Q_2} \left(\sum_{m=1}^{\infty} [q_{2m} (-1)^{m-1} Q_4] + \frac{\pi kBQ_5 q_{2m}}{8Q_2} \right) [m^2/sec]$$

$$N_0 = \int_0^{\frac{\pi}{2}} \Phi_c(1, \theta) \frac{Q_3}{Q_2} d\theta + \frac{1}{Q_2} \left(\sum_{m=1}^{\infty} [p_{2m} (-1)^{m-1} Q_4] + \frac{\pi kBQ_5 p_{2m}}{8Q_2} \right) [m^2/sec]$$

$$\Psi_c = \pi \exp(-kx_{B3}) \sin(k|x_{B3}|) [m^2/sec]$$

$$\Phi_c = \pi \exp(-kx_{B3}) \cos(kx_{B2}) [m^2/sec]$$

$$\Psi_S = -\pi \exp(-kx_{B3}) \cos(kx_{B2}) + \int_0^{\infty} \frac{\exp(-v|x_{B2}|)}{v^2 + k^2} [v \sin(vx_{B3}) + k \cos(vx_{B3})] dv [m^2/sec]$$

$$\Phi_S = \pi \exp(-kx_{B3}) \sin(k|x_{B2}|) - \int_0^{\infty} \frac{\exp(-v|x_{B2}|)}{v^2 + k^2} [v \cos(vx_{B3}) + k \sin(vx_{B3})] dv [m^2/sec]$$

$$\Psi_c(1, \theta) - \frac{Q_6}{Q_2} \Psi_c \left(1, \frac{\pi}{2} \right) = \sum_{m=1}^N p_{2m} f_{2m} [m^2/sec]$$

$$\psi_s(1, \theta) - \frac{Q_6}{Q_2} \psi_s\left(1, \frac{\pi}{2}\right) = \sum_{m=1}^N q_{2m} f_{2m} [m^2/sec]$$

$$f_{2m} = -\left(\sin(2m\theta) + \frac{kB}{2Q_2} Q_7 + \frac{kB}{2Q_2^2} (-1)^m Q_1 Q_6\right) [m^2/sec]$$

$$Q_1 = \frac{1}{2m-1} - \frac{a_1}{2m+1} - \frac{3a_3}{2m+3}$$

$$Q_2 = 1 + a_1 + a_3$$

$$Q_3 = (1 + a_1)\cos\theta - 3a_3\cos(3\theta)$$

$$Q_4 = \frac{1 + a_1}{4m^2 - 1} + \frac{9a_3}{4m^2 - 9}$$

$$Q_{5p2m} = (1 + a_1 - a_1 a_3)p_2 - a_3 p_4$$

$$Q_{5q2m} = (1 + a_1 - a_1 a_3)q_2 - a_3 q_4$$

$$Q_6 = (1 + a_1)\sin\theta - a_3\sin(3\theta)$$

$$Q_7 = \frac{\sin((2m-1)\theta)}{2m-1} + \frac{a_1 \sin((2m+1)\theta)}{2m+1} - \frac{3a_3 \sin((2m+3)\theta)}{2m+3}$$