

Using Gravity Potential Field and Inertial Navigation System in Real Time Submarine Positioning

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Abstract. Military submarines need specific skills when they operate in stealth mode, being not able to use their sonar system and need to travel in a total darkness environment. Therefore, a dead reckoning is used to evaluate the position, usually based on gyrocompass, measured speed and Inertial Navigation System (INS). This method has an acceptable distribution of errors for submarine position estimation, when it navigates in stealth mode for relative short time, but with serious errors for longer course. Our proposed method uses the Second Gradient of the Gravity (SGG) measured in real-time that is compared with a digital gravity gradient map, using an original method based on Kalman Filter. Finally, the INS estimation is improved based on correlation between the gradient map and the measured gradient value.

1. Introduction

Any military submersible embeds an inertial navigation system (INS), able to measure the boat's motion and constantly updates position, computing data from acceleration/deceleration, pitch and roll.

The main advantages of INS are:

- It does not rely on celestial sightings, GPS or radio signals;
- It allows the boat to navigate without sonar while remaining hidden under the surface.

Moreover, the estimation of position is doing with dead reckoning course information, processed up from gyrocompass information, measured speed and estimation of local ocean currents. The submarine is updating its reference position periodically, e.g. using outside navigational radio signals.

For deeper navigation is important the underwater terrain relief, usually detected with fathometer, used only in friendly water. Therefore, there is a challenging regarding positioning in stealth mode, when the sonar is not allowed. For this case, a silent measure replaces the active sonar, measuring a geophysical component that corrects the current point position by an indirect method. Matching the measured values with a previous obtained map, permits topographical coordinates extraction of the identified point on the map, which is the measuring point.

Our approach yields significant improvement, minimizing the drift tendency of the INS over time by matching the SGG (Second Gradient of the Gravity field) with gravity gradient map to determine the submarine position. Moreover, the method (especially SGG sensor) could be used on submersible drones, where the most specific characteristic is the extremely reduced size of the ship body.



2. Inertial Navigation System

A typical Inertial Navigation Systems performs navigation information from accelerometers and gyroscopes data but, for submarines, is added at least a pressure sensor for depth. The INS computer produces navigational state estimation using transformation matrix, Extended Kalman Filter and Unscented Kalman Filter. When possible, the position data is collected from GPS and INS set it as the reference position. These instruments are commonly referred as Inertial Measurement Units (IMUs).

The source of INS errors can be defined knowing the magnitude and direction of the submarine's accelerations, noted as vector $\ddot{\vec{s}}$, that is the second gradient of the submarine position \vec{s} in respect to time. Therefore, the acceleration of the vessel $\ddot{\vec{s}}$ is equal with gravitational field $\vec{g}(\vec{s})$ plus \vec{a} , the measured acceleration.

$$\ddot{\vec{s}} = \vec{g}(\vec{s}) + \vec{a} \quad (1)$$

The position of the navigation equation (1) could produce the first order approximation of the position error in submarine's acceleration $\delta\ddot{\vec{s}}$,

$$\delta\ddot{\vec{s}} = \frac{\partial \vec{g}}{\partial \vec{s}} \delta\vec{s} + \delta\vec{g} + \delta\vec{a} \quad (2)$$

Where the error in the gravitational field is $\delta\vec{g}$, $\delta\vec{a}$ is the error of measured acceleration and $\frac{\partial \vec{g}}{\partial \vec{s}}$ is the gravity gradient.

For modeling purpose, INS errors break down into three categories: inertial sensor error, alignment error and computational error. The inertial sensor error is accumulated over time into system and it isn't eliminated until the next GPS references. The other errors remain constant or oscillate over time between predictable limits.

The first order derivate equation estimates the sources of error at a given time, based on the initial sources of error and include the tilt error, heading error, velocity error, position error, gyroscope bias, and accelerometer bias. For any given time, the error may be found by multiplying a transition matrix by the assumed initial error source:

$$[\delta\vec{s}(t)] = [\Psi(t - t_0)][\delta\vec{s}(t_0)] \quad (3)$$

To improve the accuracy of the INS it should integrate the gained information (from gravity gradient instruments and map matching this aid) with the INS.

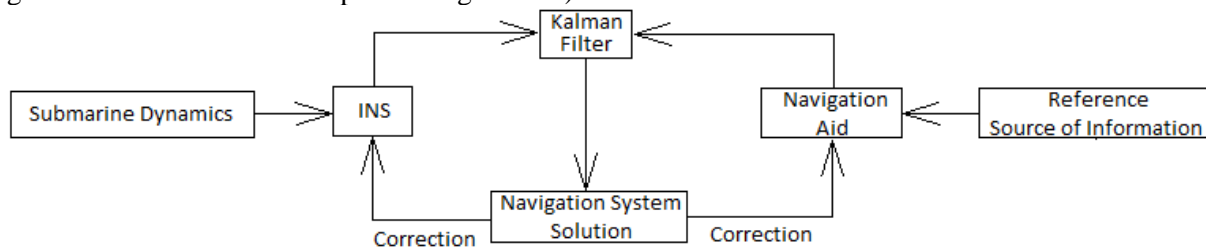


Figure 1 - Inertial Navigation System and Navigation Aid data correlation.

3. Kalman Filter

The information from navigation aids must be integrated using computer algorithms which maximize the accuracy of the navigation solution (e.g. a new position based on information from aid is updating the INS).

However, the Kalman filter is often choose as algorithm for integrating information from aids and INS. In literature it is described as a combination of two estimates of a variable to form a weighted mean, the weighting factors chosen to yield the most probable estimate. One estimate derives from INS information and the equations of motion, while the second comes from an aid, e.g. GPS.

The Kalman Filter equations (4), (5) are modeling the signal values evaluated with a linear stochastic equation (4), where \mathbf{x}_i is the state vector at the moment i , \mathbf{w}_i is the white noise process with known covariance for Φ , and Φ represents the state transition process from the state of time i to $i + 1$.

$$\mathbf{x}_{i+1} = \Phi\mathbf{x}_i + \mathbf{w}_i \quad (4)$$

To compute \mathbf{z}_i that is actual measurement of \mathbf{x} at time i , the equation of the model for the observations (5) uses a state vector and measurement vector connection \mathbf{M} without noise and specific measurement errors \mathbf{v}_i :

$$\mathbf{z}_i = \mathbf{M}\mathbf{x}_i + \mathbf{v}_i \quad (5)$$

Figure 2 presents the Kalman filter recursive algorithmic loop, where the fundamental matrix equations of a discrete, linear Kalman filter is Kalman Gain Equation (6).

$$\mathbf{\Lambda}_i = \frac{\mathbf{E}_{i|i-1}\mathbf{M}_i^T}{\mathbf{M}_i\mathbf{E}_{i|i-1}\mathbf{M}_i^T + \mathbf{N}_i} \quad (6)$$

The Update Estimate expression is:

$$\hat{\mathbf{x}}_{i|i} = \hat{\mathbf{x}}_{i|i-1} + \mathbf{\Lambda}_i(\mathbf{z}_{i|i} - \mathbf{M}_i^T \hat{\mathbf{x}}_{i|i-1}) \quad (7)$$

Error of the Update Covariance equation:

$$\mathbf{E}_{i|i} = (\mathbf{I} - \mathbf{\Lambda}_i\mathbf{M}_i)\mathbf{E}_{i|i-1}(\mathbf{I} - \mathbf{\Lambda}_i\mathbf{M}_i)^T - \mathbf{\Lambda}_i\mathbf{M}_i\mathbf{\Lambda}_i^T \quad (8)$$

The state (9) and error covariance (10) projected in $i+1$ form:

$$\hat{\mathbf{x}}_{i+1|i} = \Phi\hat{\mathbf{x}}_{i|i} \quad (9)$$

$$\mathbf{E}_{i+1|i} = \Phi\mathbf{E}_{i|i}\Phi^T + \mathbf{Q} \quad (10)$$

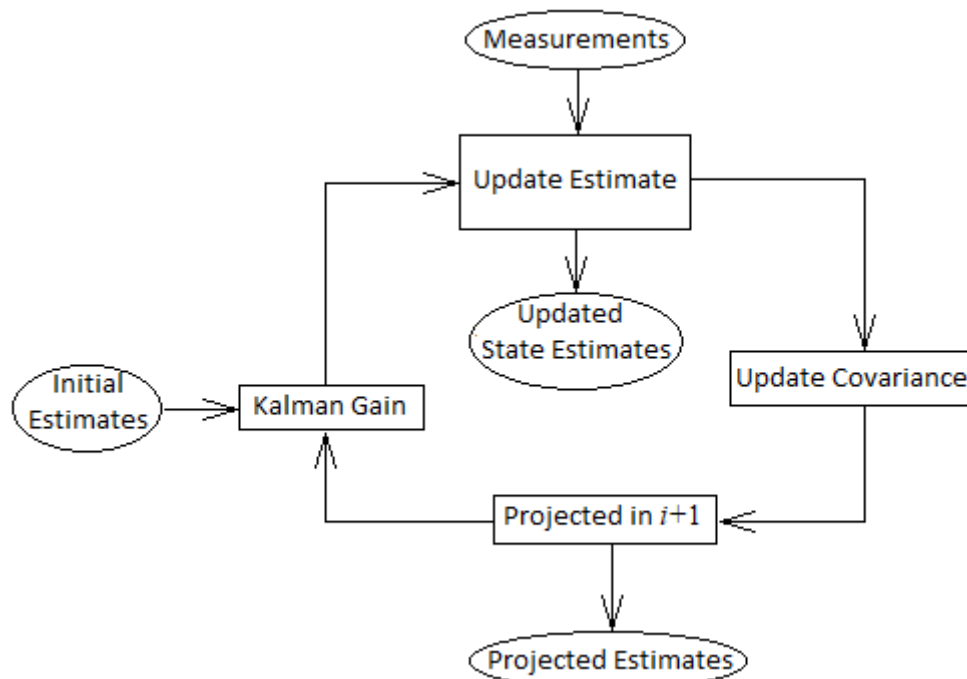


Figure 2 - Kalman Filter Algorithm. Squares represent matrix functions and ellipses - inputs/outputs.

Where $\hat{\mathbf{x}}_{i|j}$ represents the estimate of \mathbf{x} at observation time i up to but including at time $j|j \leq i$ and \mathbf{Q} represents a matrix of covariance that defines the system's noise, while the measurement noise is \mathbf{N} .

These equations assumes no correlation between the system and measurement noise and that both possess a zero mean and Gaussian distribution. In other words, the prediction process is defined by (9) and (10) and correction by (6), (7) and (8).

The most famous application of the Kalman filter occurred on Apollo mission moon flight, where it has provided corrections of the midcourse navigation.

4. Second Gradient of the Gravity Field

The vertical gravity gradient in geophysics is the rate of change of vertical gravity with high. However, in mathematical analyse it indicates the convexity/concavity (the sign of the first gradient)

and inflexion points (the solutions of the second gradient) of the gravity graphic. The geopotential expression is:

$$W = V + V_c \quad (11)$$

The potential derived from centrifugal force is:

$$V_c = \frac{\omega^2}{2} (x^2 + y^2) \quad (12)$$

And the gravitational potential expression (which is the main part of geopotential field value) is:

$$V(x, y, z) = G \iiint_v \frac{\delta(\xi, \eta, \zeta) d\xi d\eta d\zeta}{[(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2]^{1/2}} \quad (13)$$

With $P(x, y, z)$ the observation point, $\Pi(\xi, \eta, \zeta)$ the source effect, G the gravitational constant and δ the density of the source.

The gravity g is the first gradient of geopotential field, in 3D form equation:

$$g(x, y, z) = \sqrt{\left(\frac{\partial W}{\partial x}\right)^2 + \left(\frac{\partial W}{\partial y}\right)^2 + \left(\frac{\partial W}{\partial z}\right)^2} = \sqrt{g_x^2 + g_y^2 + g_z^2} \quad (14)$$

As the horizontal components are practically null, (14) becomes:

$$g(x, y, z) = \sqrt{g_z^2} \cong g_z = \frac{\partial g}{\partial z} \quad (15)$$

The vertical gradient of gravity is:

$$W_{zz} = \frac{\partial^2 W}{\partial z^2} = \frac{\partial g_z}{\partial z} = \frac{g_z(z + \frac{l}{2}) - g_z(z - \frac{l}{2})}{l} \quad (16)$$

Moreover, in General Relativity the Riemann curvature tensor (Γ) uniquely characterizes the gravity field, with corresponding gravity gradient tensor quantity in Newtonian field (V).

The gravity gradient tensor is:

$$\Gamma_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j} \quad (17)$$

Where $\Gamma_{ij} | (i = x, y, z), (j = x, y, z)$ are the elements of the symmetric gravity gradient tensor.

4.1. Gradiometer Principle

The gradiometer is an instrument for vertical gravity gradient measurement. It measures the gravity in two points (using accelerometers) on the same vertical and divides the measured difference by distance.

An inline component gradiometer makes the linear acceleration difference between two aligned masses, A and B or C and D, figure 3.

The cross-component gradiometer could combine linear accelerations on four test masses with their aligned sensitive axes. It can be also constructed by differencing signals between two concentric angular accelerometers with their arms orthogonal (figure 4).

The combination between three in line and three cross component gradiometers constitutes a tensor gradiometer. In a rotating reference frame, the gravity gradient Γ' is measured in relation with the gradient in the inertial frame by (18), where $\Psi = (\Psi_1, \Psi_2, \Psi_3)$ is the angular vector in gradiometer coordinate system.

$$\Gamma' = \begin{pmatrix} \Gamma_{11} + (\Psi_2^2 + \Psi_3^2) & \Gamma_{12} + \Psi_1 \Psi_2 & \Gamma_{13} + \Psi_1 \Psi_3 \\ \Gamma_{21} + \Psi_2 \Psi_1 & \Gamma_{22} + (\Psi_3^2 + \Psi_1^2) & \Gamma_{23} + \Psi_2 \Psi_3 \\ \Gamma_{31} + \Psi_3 \Psi_1 & \Gamma_{32} + \Psi_3 \Psi_2 & \Gamma_{33} + (\Psi_1^2 + \Psi_2^2) \end{pmatrix} \quad (18)$$

4.2. Gravity Gradient Linearization

The function of gravity gradient is nonlinear. The linearization process replaces the surface of gravity gradient $h(x, y)$ with the plane equation $f(x, y)$ defined as linear expression of gravity gradient in the nearby INS referenced position (\hat{x}, \hat{y}) using the parameter of gravity gradient linearization α , h_x and h_y :

$$f(x, y) = \alpha + h_x(x - \hat{x}) + h_y(y - \hat{y}) \quad (19)$$

Where h_x and h_y are the vertical and horizontal slope of gravity gradient and α is the referenced gravity gradient at position (\hat{x}, \hat{y}) .

There are several gravity gradient linearization algorithms: First Order Taylor Series, Nine Point Fit, Full Plane Fit and Two Subgroup Fit.

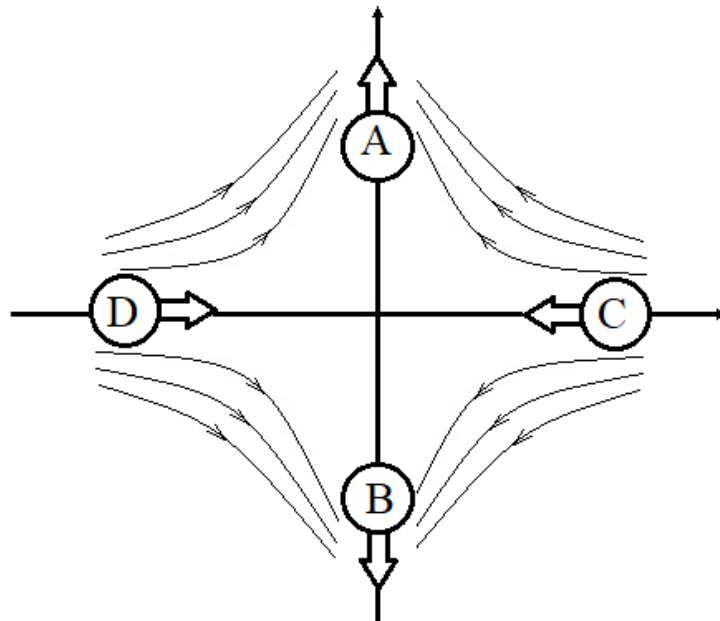


Figure 3 – In line component gradiometer

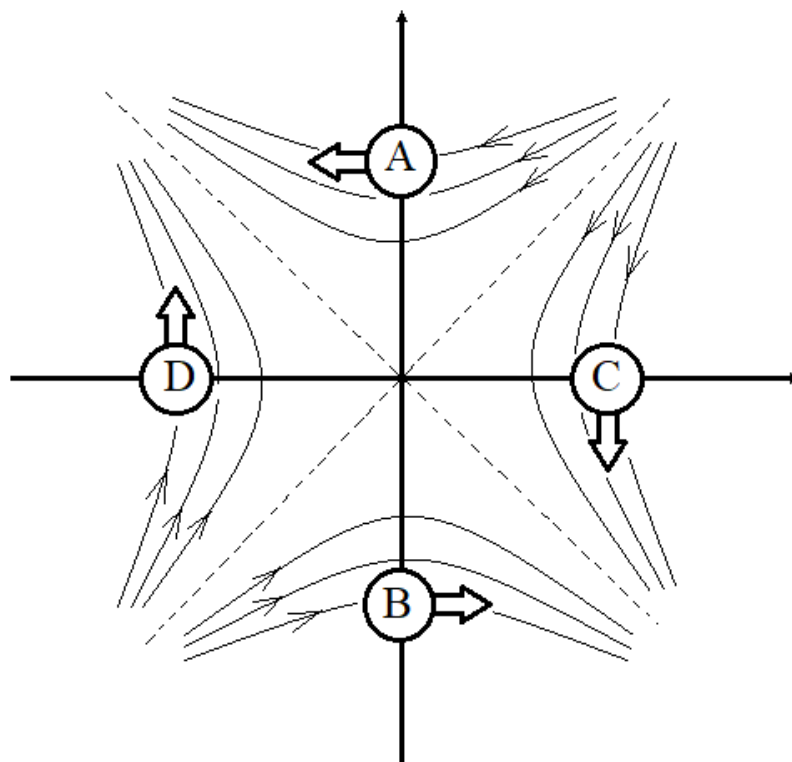


Figure 4 – Cross component type gradiometer

In the context of this research we propose to locate a submarine's position using an algorithm by matching the gravity gradient measurements profile and the digital gravity gradient map.

First are extracted the characteristic parameters of the series of measured values, then it is processed the corresponding gradient gravity map and finally is applied the map matching algorithm.

4.3. Method Description

(a) Gradiometer Measured Values

Each value measured by gradiometer is paired with a distance from the previous measurement point, approximated by Integration Navigation System along with δ , the submarine navigation direction. A model of these values is graphically represented into figure 8 a.

The graphic characteristics are defined using analytical parameters which full identify the graphical representation of the measurements series. In this work the parameters are minimum, maximum and inflexion points of the signal, where \mathbf{M} , \mathbf{m} and \mathbf{I} are maximum, minimum and inflexion point:

$$\mathbf{M} = \{p(x_i, y_i) \in \Gamma \mid y_i > y_{i+1}, y_i > y_{i-1}\} \quad (20)$$

$$\mathbf{m} = \{p(x_i, y_i) \in \Gamma \mid y_i < y_{i+1}, y_i < y_{i-1}\} \quad (21)$$

And \mathbf{I} is the inflexion points set:

$$\mathbf{I} = \mathbf{I}_{up} \cup \mathbf{I}_{down} \quad (22)$$

Where

$$\mathbf{I}_{up} = \{p_i(x_i, y_i), p_{i+1}, p_{i-1} \in \Gamma; y_i = a_i x_i + b_i \mid [y_{i-1} > a_i x_{i-1} + b_i \text{ AND } y_{i+1} < a_i x_{i+1} + b_i]\}$$

$$\mathbf{I}_{down} = \{p_i(x_i, y_i), p_{i+1}, p_{i-1} \in \Gamma; y_i = a_i x_i + b_i \mid [y_{i-1} < a_i x_{i-1} + b_i \text{ AND } y_{i+1} > a_i x_{i+1} + b_i]\}$$

As the equations (22) of the inflexion point show, the tangent parameter is enough difficult to be computed, comparing with minimum and maximum detection numerical expression, thus it is computed the second gradient of the numerical signal. As \mathbf{I} points are solutions of the second gradient of the function that modeling the graphical representation, we apply the numerical gradient function (∇F):

$$\nabla F = \frac{\partial F}{\partial x} \hat{i} + \frac{\partial F}{\partial y} \hat{j} \quad (23)$$

Each specific point of one of type \mathbf{M} , \mathbf{m} or \mathbf{I} , detected into the gradiometer measure series, is defined by type (*typ*) and evaluated distance (*dist*) from the reference point, \mathbf{P}_0 to the measuring point, and it is denoted $\mathbf{p}_i(\mathbf{type}_i, \mathbf{distance}_i)$ where the index i is increasing by the unity counter.

The specific MATLAB code segment that applies two times numerical gradient function is presented in figure 5.

```
x = [ 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 15 17 18 19 20 21 22 23 24 25 26 27 28];
y = [ 124 131 132 133 132 129 127 125 123 120 118 117 115 115 114 116 118 126 125 120 116 111 109 106 105 108 114 123 131];
grd1=gradient(y,x)
grd2=gradient(grd1,x)
id=sign(grd2)
idx=strfind(id,[-1 1])
inflexionP=x(idx+1)
```

Figure 5 – part of MATLAB code to detect inflexion points (noted *inflexionP*).

For matching process, the parameters $\mathbf{p}_i(\mathbf{type}_i, \mathbf{dist}_i)$ are gathered into a vector of pairs, \mathbf{G} :

$$\mathbf{G}(\mathbf{type}_i, \mathbf{dist}_i) = \left\{ \begin{array}{l} \mathbf{type}_i = \mathbf{type}_i \\ \mathbf{dist}_i = \mathbf{distance}_i \end{array} \right\} \mid i \text{ is index of } \mathbf{p}_i \mid (\exists) \mathbf{p}_i(\mathbf{type}_i, \mathbf{distance}_i) \in \Gamma \quad (24)$$

(b) Digital Gradient Map

The gravity gradient map data must offer specific information in terms of M , m and I parameters for the geographical area where the route of submarine is. Then, based on data collected by submarine on board gradiometer that matching the digital map, the submarine position is approximated with a minimal error.

There are several pre-processing steps:

1. Computing the first gradient of the digital map – it will give the geometrical places for minimum and maximum of the map signal (represented as isolines of zero value of first gradient map), noted Mg , mg ;
2. Computing the second gradient map that define geometrical place of the inflexion points (for isolines of zero value of second gradient map) noted Ig ;
3. Defining N directions around the INS evaluated direction, δ , using a predefined threshold to generate **Point extreme (+)** and **Point extreme (-)** around evaluated point P_e ;
4. Creates pairs $p_{k,i}(\text{type}_i, \text{distance}_i)$ where $k = 1$ to N is direction index and i is the point index of type M , m or I on the profile/direction k of the map; distance_i is the distance between the reference point P_0 and the i -th point $p_{k,i}$ of measure by type M , m or I along k direction (figure 6);

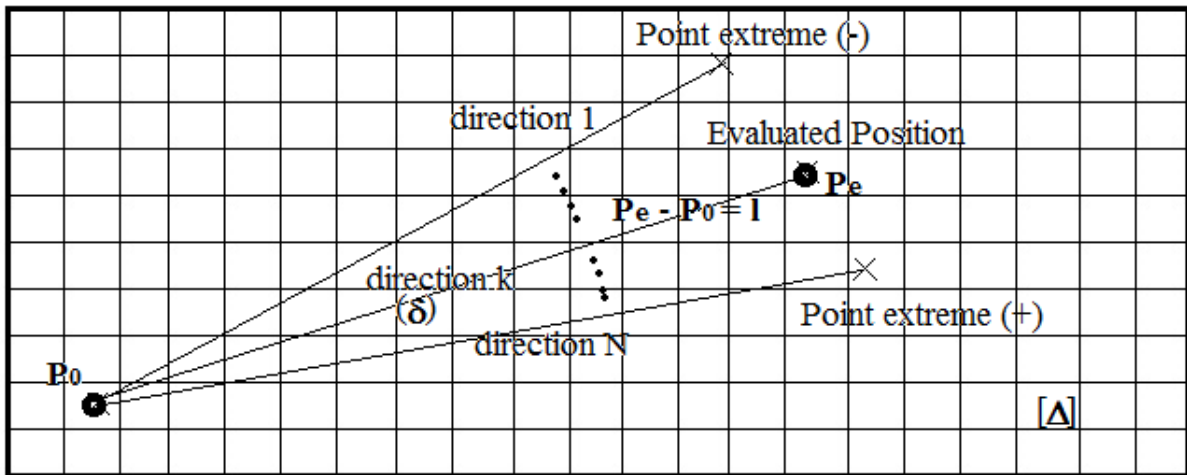


Figure 6 – Directions defined on Gravity Gradient Map, using evaluated point position and a configured threshold to define **Point extreme (+)** and **Point extreme (-)** for first and last direction.

5. Creates N vectors H_k of $p_{k,i}(\text{type}_i, \text{distance}_i)$ where $k=1$ to N , N is the maximum number of directions to be tested on the map:

$$H_k(\text{type}_i, \text{distance}_i) = \left\{ \begin{array}{l} \text{type}_i = t_i \\ \text{distance}_i = d_i \end{array} \left| \begin{array}{l} k = 1 \text{ to } N \\ i < N \\ (\exists) p_{k,i}(t_i, d_i) \in \text{direction } k \end{array} \right. \right\} \quad (25)$$

(c) Matching Algorithm

The matching algorithm searches the H_k vectors, previously generated for the specific sequence defined by typ_i of measured vector values G .

The algorithm is:

- Searching in $H_k(type_i, distance_i)$ for $type_i == typ_i$ of $G(typ_i, dist_i)$ and select the sequence number which has all $type_i == typ_i$ and memorize it as an element of the vector A :

$$A = \left\{ k_j = k \left| \begin{array}{l} k = \text{index } k \text{ of } H_k \\ type_i == typ_i \text{ for } H_k(type_i, distance_i) \text{ and } G(typ_i, dist_i) \\ j = \text{unity counter} \end{array} \right. \right\} \quad (26)$$

In other words, A will gather all k indexes of map directions that have the same ordering series of point types as the measured vector G ;

- Generating the vector of errors E_j , as distances differences of identified type points:

$$E_j = \{e_i = |distance_i - dist_i| \text{ where } H_{k_j}(type_i, distance_i) \text{ and } G(typ_i, dist_i)\} \quad (27)$$

- Choosing the minimum deviation error vector D :

$$D = \{p = k_j | j = \min_j E_j \text{ for } H_{k_j}\} \quad (28)$$

5. Model and Analysis

The matching results are improving the system position accuracy by integrating the new information via Kalman filter and Navigation System Solution with INS (figure 7).

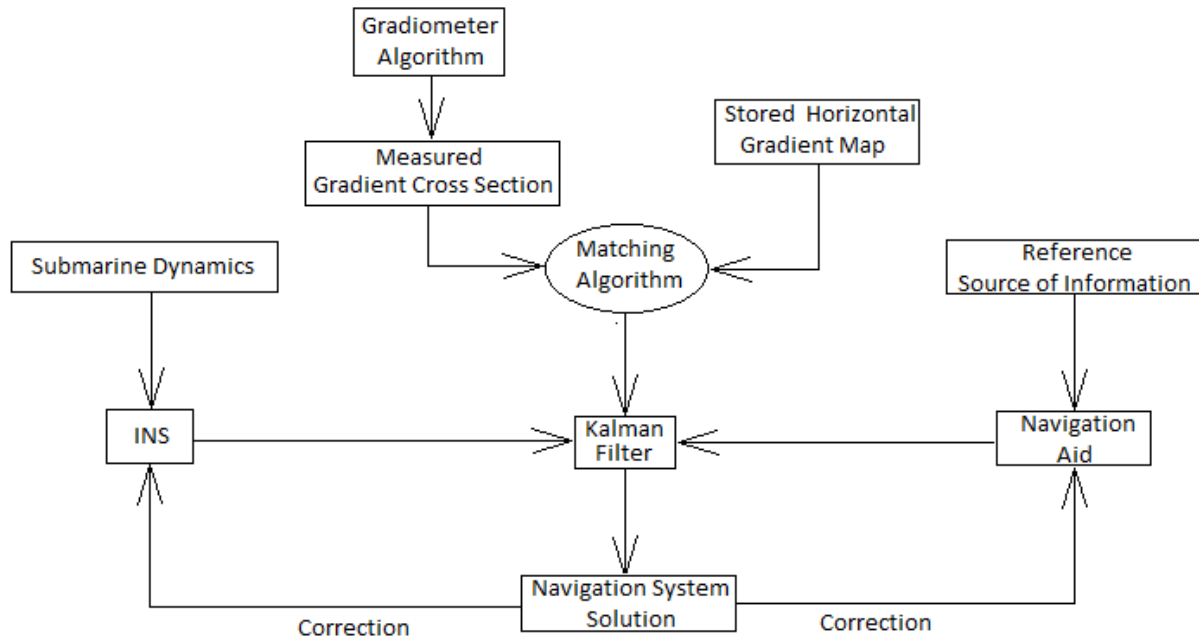


Figure 7 – INS correlated through Kalman filter with matching algorithm; squares represent matrix functions and ellipses inputs/outputs.

In equation (28) the D vector has the closest $distance_i$ (from $H_k(type_i, distance_i)$) of $dist_i$ (from $G(typ_i, dist_i)$). It means the map signal of the k direction is matching with the measured signal and the k direction is the computed navigation route closest of the real one, for our submarine.

A model representation for INS evaluation route (figure 8) and INS with Gradient information integrated (figure 9) demonstrate a reduced error of 65-80%.

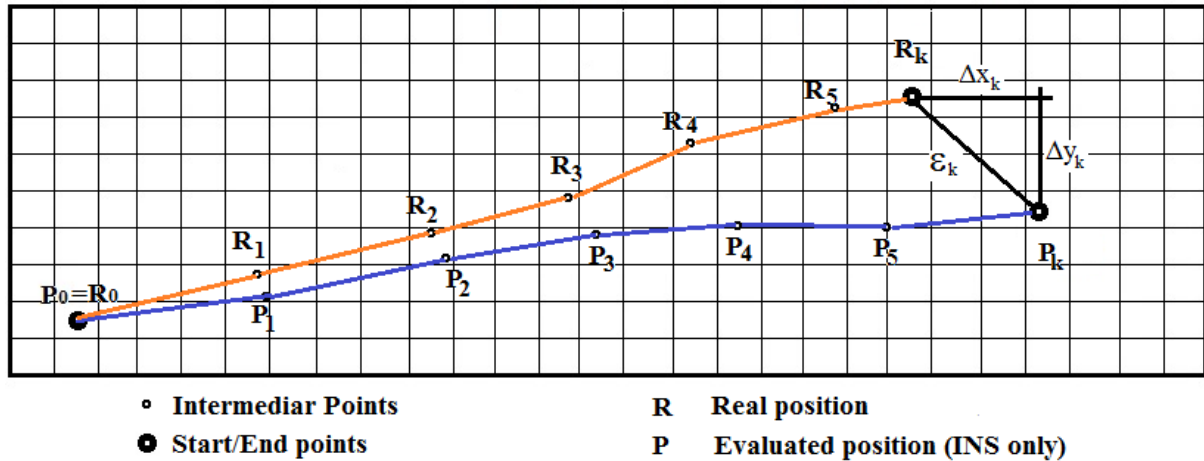


Figure 8 Error model deviations for several points for an INS evaluation without gradient matching system

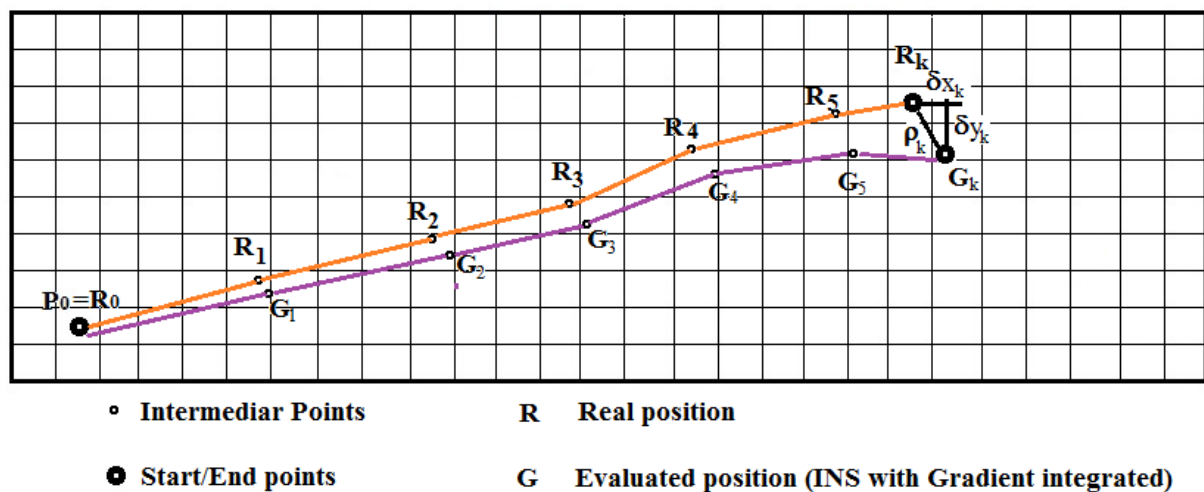


Figure 9 Deviations of intermediary navigations points for INS with gradient matching system

In figure 10 b is presented a Vertical Cross Section of Gravity Gradient Map (Δ) where the map's isolines (orange) are correlated with the cross-section representation on 29 verticals for $\delta = k$ direction. Mg , mg and Ig represents maximum, minimum and respectively inflexion points of the gravity gradient cross section function.

The alignment between figure 10 (a) and figure 10 (b) is performed on correlation between M , m and I and Mg , mg and Ig .

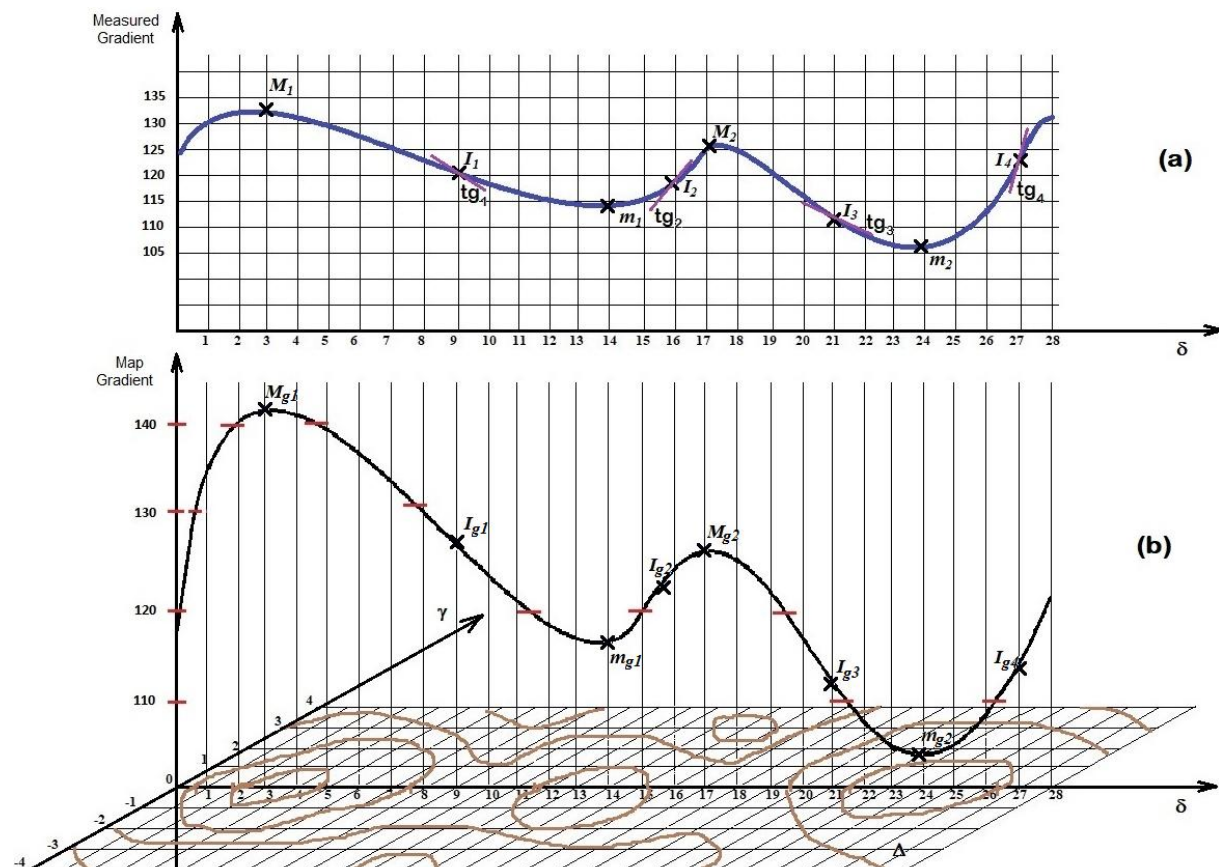


Figure 10 - (a) is a graphical representation of the measured values by gradiometer, where M , m and I represents maximum, minimum and inflexion points respectively; - (b) shows the profile on the gradient map along the $\delta = k$ direction (vector of submarine navigation), where M_g , m_g and I_g are the parameters of the profile on the gradient map.

6. Conclusions

The presented method compensates the INS deviation using a digital gravity gradient map for comparison with the measured gradient using a spatial algorithm.

The minimal analyze processing method extracts the first and second gradients from both signals and compares several alignments from the map with the measured signal – using an original method able to be easily implemented even into embedded systems.

The models analyze for errors shows minimal qualitative deviations.

However, as the parameters of the used signals are qualitative, the dependency of any measurement correction is minimal (e.g. Bouguer correction, etc.) and thus any dependency of the gradiometer type is eliminated (figure 8).

While there is still some room for improvement in both the INS and measured geophysical parameters stage, we have demonstrated promising results especially on alignment process that are unlikely to be correctly aligned by earlier approaches usually based on frequency analyze, due to lack of analytical references positioning features: minimum, maximum and inflexion points.

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