

# Research of Statistical Characteristics of Target Strength of a Single-layer Cylindrical Shell with Random Coating

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**Abstract.** That anechoic tiles fall off from the surfaces is a common phenomenon that modern submarines are frequently faced with, which has negative effects on acoustic stealthy performance and increase Target Strength(TS) levels. Based on Monte-Carlo theory, PEM calculations are made by assuming that each anechoic tile fall off randomly and their conditions are tested separately. And the statistical-based values where statistical probability density of the PEM results reaches its culmination are considered as the TS level of a structure that part of its anechoic tiles fall off. The effectiveness of the MC-PEM is validated by the calculation of plate with anechoic tiles, since it achieves good agreement with the results of analytical method. And the relations between TS and the anechoic tiles' exfoliation rate of single-layer cylinder shell are gained by MC-PEM. Finally, log-normal distribution is introduced to describe cumulative failure probability of anechoic tiles. And the dynamic characteristic of TS versus non-dimensional time suggests that it is of significance to overhaul anechoic tiles in the region of  $0.3t\sim 0.5t$  ( $t$  represents average lifespan of anechoic tiles) as to keep TS in relative low level.

## 1. Introduction

Modern submarines are generally equipped with a sound absorbing layer made of rubber and other soft materials, that is, anechoic tile.[1], which will be lost and shedding as submarines carry out mission. Figure 1(a) resembles that for the Virginia-class nuclear submarine of the United States, large-scale exfoliation of anechoic tiles result in returning to repair. Figure 1(b) shows that a typhoon-class submarine shell whose surface anechoic tiles is mostly shedding.



(a)



(b)

**Figure 1.** Figure of Anechoic Tiles' Exfoliation

On the one hand, exfoliation of anechoic tiles decreases hull fairing[2]., while increases flow resistance and flow noise, and accelerates the corrosion of the hull at the same time. On the other hand,



the intensity of the acoustic target will increase, which has a negative effect on the submarines' acoustic stealth performance.

At present, the main use of the high frequency approximation theory in engineering is to obtain the target strength under the state of full rigid surface and full coating, while the problem of the effect of the shedding of the anechoic tiles on the target strength is still lacking. Fan[3] developed the PEM(plate element method) based on the Kirchhoff high frequency approximation theory, which can be then extended to the calculation of the target strength of the immersed double-layer structures[4]. And the accuracy of PEM is consistent with the BASIS algorithm commonly used [5]. Considering the anechoic tiles, the equivalent material parameters are back calculated by simulated annealing algorithm combining the acoustic tube test data[6], and the acoustic coefficient is obtained by multilayer medium transfer matrix method[7].

The anechoic tiles' exfoliation can be attributed to the reliability issues of the product. Di YL [8] illustrates several important reliability distributions, such as lognormal, Weibull and double index distributions. Chang Lixin[9] analyzes the influence of the active phased array antenna failure on the performance of the antenna unit obeying the normal distribution. Through the simulation calculation, Niu Cf[10] analyses that when the failure rate of the phased array unit is at 10% level, and the system can still work as usual or downgrade. Zhu Fc[11] analyses the fitting accuracy of several common aviation ammunition storage failure distributions. The mentioned study is of great significance to the estimation of the exfoliation probability and lifespan of the anechoic tile.

Considerable randomness exist in the anechoic tiles' exfoliation. Monte Carlo method, also known as the statistical simulation method, has become the main tool to solve complex statistical models and high-dimensional problems[12], which has been applied in many aspects such as reliability evaluation of generation and transmission system[13], and detecting probability of submarine launched torpedo[14].

By combining the engineering algorithm predicting target strength of shell with tile, reliability theory and Monte Carlo method, this paper quantitatively analyzes the effect of the anechoic tiles' exfoliation rate on the target strength, and gives the target strength curve in corresponding with the use of time, which has a certain engineering significance for the maintenance work of submarines.

## 2. Principles

### 2.1. Monte Carlo - Plate Element Method

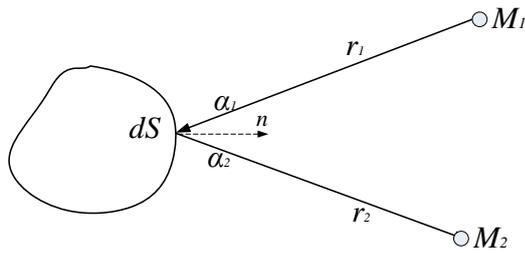
Monte Carlo-Plate Element Method (MC-PEM) is a method that combines the plate element method with the Monte Carlo method to analyze the target strength of shell after the random shedding of the anechoic tile. This method is based on the following assumptions.

- (1)The probability of falling off of the submarines' anechoic tiles in the same area is consistent.
- (2)The thickness, material characteristics and acoustic properties of the anechoic tile in the same area are in accordance with each other, and the exfoliation of any anechoic tiles is independent of each other.
- (3)The number of anechoic tiles is enough that each tile has a smaller area relative to the whole surface.

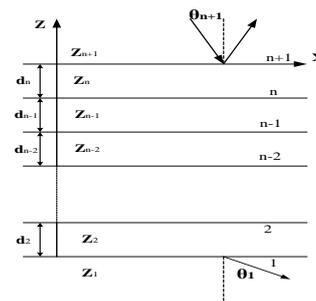
Anechoic tiles exist in two states: normal attachment or detachment. Independently judge the status of each plate, and calculate the target intensity repeatedly a number of times to obtain a series of numerical, mathematical statistics and distribution fitting, the TS values where the maximum probability density occur is obtained as final result.

The above is the core principle of the Monte Carlo algorithm. The PEM is derived from Kirchhoff's physical acoustics method, which is illustrated in Figure 2. The Kirchhoff integral formula for the scattering field of an object on any external surface is:

$$\phi_s(r_2) = \frac{1}{4\pi} \int_s \left[ \phi_s \frac{\partial}{\partial n} \left( \frac{e^{ikr_2}}{r_2} \right) - \frac{\partial \phi_s}{\partial n} \frac{e^{ikr_2}}{r_2} \right] ds \quad (1)$$



**Figure 2.** Kirchhoff Approximation Theory



**Figure 3.** Transfer Matrix of Multi-layers Medium

$\Phi_i$  is the incident wave potential,  $\Phi_i = (A/r_1) e^{ikr_2}$ ,  $\Phi_s$  is the reflected wave potential,  $V(\alpha)$  is the sound reflection coefficient,  $Z_n$  is the surface acoustic impedance, with the boundary conditions, for the far field transceiver combined situation, the following formula can be derived:

$$\phi_s = \frac{-A}{2\pi} \int_s e^{ik2r} V(\alpha) \left[ \frac{ikr-1}{r^3} \cos \alpha \right] ds \tag{2}$$

Through the establishment of three-dimensional geometric model, the surface is divided into a number of flat plate. The Gordan formula will converted integral arithmetic into algebraic operation[15], and then we can use the plate element method to calculate the program to obtain the target strength.

With regard to the local acoustic pressure reflection coefficient of the shell, the sound pressure reflection coefficient of the steel shell laying anechoic tile structure is calculated by deducing the transfer-matrix[16] relationship between the stress and displacement of each medium layer and the boundary conditions of the upper and lower interfaces which is illustrated in Figure 3.

### 2.2. Reliability Theory

Cumulative failure probability refers to the probability that a product loses its required function under specified conditions and for a specified period of time, also referred to as the unreliability function  $F(t)$ .

$$F(t) = \begin{cases} P(T < t) & t \geq 0 \\ 1 & t < 0 \end{cases} \tag{3}$$

Anechoic tile can be seen as a product, when it is closely attached to the hull surface, that is effective, when the anechoic tile off, then that failure. Lognormal distribution is an important distribution of reliability analysis, expressed as  $LN(\mu, \sigma^2)$ , whose density function is

$$f(t|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi t\sigma}} \exp\left\{-\frac{1}{2\sigma^2} (\ln t - \mu)^2\right\} \tag{4}$$

$t > 0, -\infty < \mu < \infty, \sigma > 0$

In the reliability analysis,  $t$  is life expectancy,  $\mu$  is the mean of natural logarithms of lifetime, and  $\sigma$  is the standard deviation of life logarithms. The cumulative failure probability function is:

$$F(t) = \Phi\left(\frac{\ln t - \mu}{\sigma}\right) \tag{5}$$

$\Phi(x)$  is the distribution function of normal distribution. In this paper, logarithmic normal distribution is used to characterize the shedding life distribution of anechoic tiles.

## 3. Method Validation

### 3.1. Calculation of Anechoic Tiles' Acoustic Performance

Submarines hulls are made of steel. ( $\rho = 7800 \text{kg/m}^3, E = 2.1 \times 10^{11} \text{Pa}, \mu = 0.3$ ). The surface of the anechoic tile is porous rubber, the equivalent material parameters of anechoic tile which are shown in Table 1 are obtained by simulated annealing method, and their thicknesses are shown in Figure 4.

**Table 1.** Equivalent Coefficients of Anechoic Tiles

$\rho$	$\nu$	$d$
$1140kg / m^3$	0.49	40mm

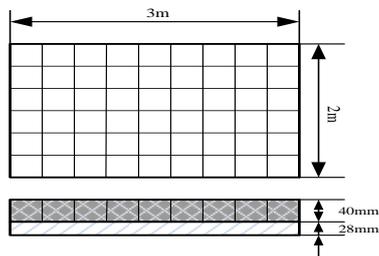
In addition, reference 5 gives anechoic tiles' Young's modulus, loss factor fitting curves.

$$E = 1.102 \times 10^7 + 5447.2f - 3.515 \times 10^{-2} f^2 \tag{6}$$

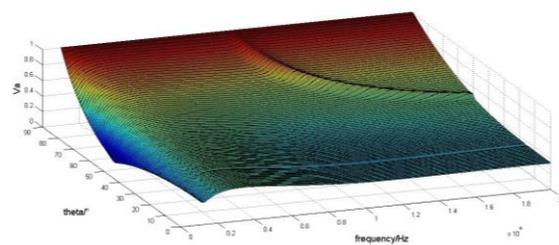
$$\eta = 0.401 + 1.21 \times 10^{-4} f - 3.29 \times 10^{-9} f^2 \tag{7}$$

$$E' = E(1 - i\eta) \tag{8}$$

The relationship between the water-anechoic tile-steel-air acoustical reflection coefficient with the frequency and incident angle which are shown in Figure 5 is obtained by using the multi-layer elastic medium transfer matrix method.



**Figure 4.** Diameter of a Panel with Anechoic Tiles



**Figure 5.** Sound Reflection Coefficients of Water-Rubber-Steel-Air System

3.2. Example for verification

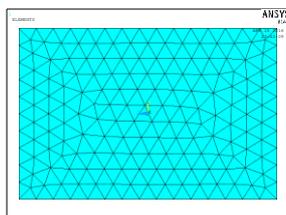
Since the normal direction of the plate is consistent, the target strength of the plate that part of its anechoic tiles have been lost can be calculated by analytical method, and the MC-PEM can be verified. The length of the plate is 3 meters, the width is 2 meters, the thickness of the steel plate is 28mm, the thickness of the anechoic tile is 40mm, the inside is air and the outside is water.

At a certain moment, the probability of failure of a single anechoic tile is assumed to be P, then the number of failure plates which are shown in Figure 6 and Figure 7 obeys the binomial distribution:

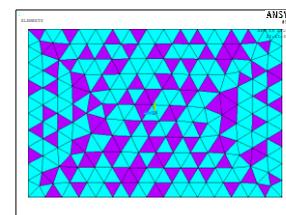
$$p(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} = b(k; n, p) \tag{9}$$

$$(k = 0, 1, \dots, n) \tag{10}$$

$$E\xi = np$$



**Figure 6.** Figure of Panel Meshing



**Figure 7.** Anechoic Tiles' Falling off (p=0.3)

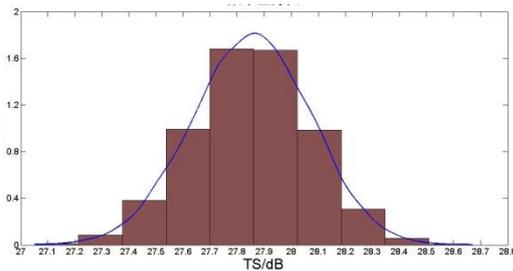
MC-PEM is a numerical method. When the frequency is 10KHz, the sound wave is perpendicular to the plane of incidence, and the calculation times are 50, 100, 200, 2000, 10000 respectively, the statistical results are shown in Table 2.

**Table 2.** Statistical Results of MC-PEM Calculation

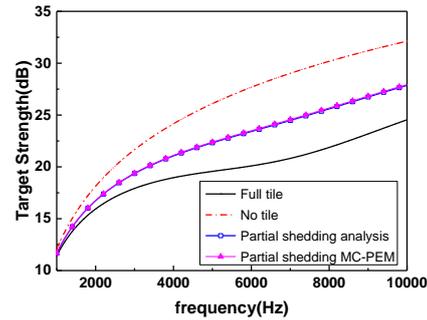
N	$\mu$ /dB	$\sigma$ /dB
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N	$\mu$ /dB	$\sigma$ /dB
50	27.850	0.2262
100	27.866	0.2185
200	27.864	0.2215
2000	27.847	0.2178
10000	27.854	0.2179

As can be seen from the frequency histogram (Figure 8), the discrete TS value has a typical normal distribution, i.e.  $TS \sim N(\mu, \sigma^2)$ . The results were normal fitted to the fitted mean value as part of anechoic tiles fall of via which the target strength are obtained.



**Figure 8.** Frequency Histogram (N=10000)



**Figure 9.** Comparison of the Results of Analytical Method and MC-PEM

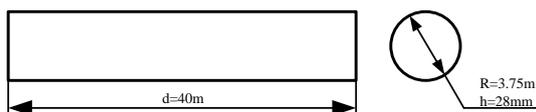
For the cladding flat steel plate, the target intensity at the partial shedding can be linearly interpolated between the target intensity value  $TS_{coat}$  of the full-blasted tile state and the target intensity value  $TS_{rigid}$  of the total reflection state because the incident angles of the acoustic waves are the same.

$$TS = 10 * \log_{10} \left( (1 - P) * 10^{\frac{TS_{coat}}{10}} + P * 10^{\frac{TS_{rigid}}{10}} \right) \tag{11}$$

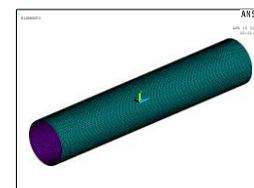
The comparison between MC-PEM analysis and analytic method are illustrated in Figure 9, which shows that the results obtained by the two methods are almost identical. And then the reliability of the Monte Carlo-plate meta-method is established.

#### 4. Example of Single-Layer Cylindrical Shell

A single-layer cylindrical shell can be used as a simplified model of single-hull submarine’s main hull. Cylindrical shell echo is generally considered as the main echo source when the submarine is in the transverse direction. Therefore, the single-layer cylindrical shell model is of great significance to the target strength of the submarine. It can be seen from Figure 10 and Figure 11 that the acoustic structure from the outside to the inside of a single-layer cylindrical shell is water-rubber layer-steel-air, and the thickness of the steel plate is 28mm, the thickness of the anechoic tile is 40mm, the length of the cylinder is 40m, the radius is 3.75m.

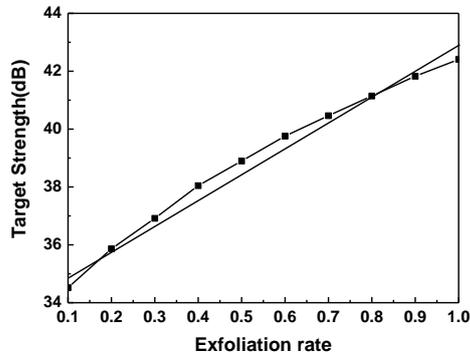


**Figure 10.** Diameter of Single Layer Cylinder

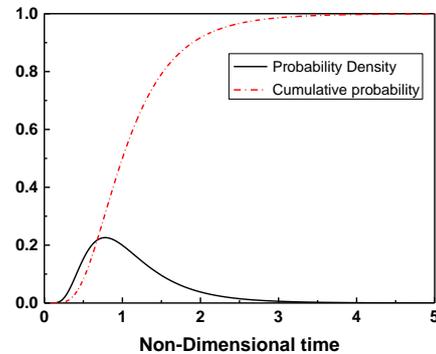


**Figure 11.** Figure of Single Layer Cylinder’s Meshing

When the frequency is 10kHz and the acoustic wave is incident in the horizontal direction, the relationship between the TS and the exfoliation rate is obtained, and the result is linearly fitted, which are shown in Figure 12 and Figure 13.



**Figure 12.** Relationship of Single-layer Cylinder’s TS with the Exfoliation Rate



**Figure 13.** Characteristic of Anechoic Tiles’ Failure Distribution

Assuming that the target intensity is TS, the rate of anechoic tile shedding is  $\eta$  and the linear fit is:  $TS = 34.25 + 8.606\eta$ . It can be drawn that, as anechoic tile shedding rate increased by 10%, the target intensity increased by about 0.86dB.

**5. Time Domain Dynamic Characteristics of TS**

Introducing the theory of reliability, the cumulative failure probability function based on logarithm normal distribution is used to describe the failure state of anechoic tile. And then the dynamic characteristics of the single-layer cylindrical shell with time are analysed.

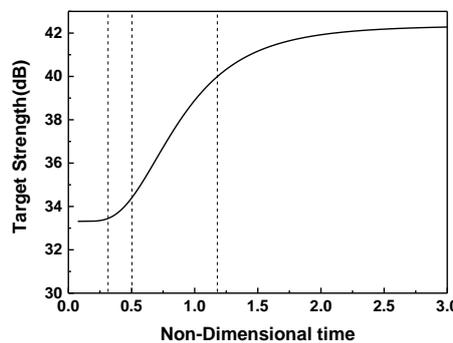
The anechoic tiles’ life expectancy is assumed to be  $\mu$  ( $\mu = 4$ ), their standard deviation of natural logarithm of life is  $\sigma$  ( $\sigma = 0.5$ ). The life of anechoic tiles obeys lognormal distribution  $LN(\ln(4), 0.5^2)$ . In order to facilitate the analysis, anechoic tile life is dimensionless, defined as the ratio of the use of time and average life expectancy  $t^*$ .

$$\hat{t}^* = t / e^\mu \tag{12}$$

According to the inverse function of the cumulative probability distribution of failure, the dimensionless use time  $\hat{t}^*$  corresponding to different shedding rates is obtained.

$$\hat{t}^* = F^{-1}(\eta) / e^\mu = e^{\phi^{-1}(\eta)} / e^\mu \tag{13}$$

Finally, the relationship of TS and  $\hat{t}^*$  is obtained.



**Figure 14.** Relationship of TS and Non-dimensioned Time

It can be seen from the figure.14 that, as the anechoic tiles start to fall off, the target intensity curve of the submarine is approximately S-shaped and can be divided into four regions according to the use time.

(1)  $\hat{t}^* < 0.3$ , initial constant period. As the anechoic tile shedding rate is low, the impact on the TS is not obvious, the target intensity remained almost unchanged.

(2)  $0.3 < \hat{t}^* < 0.5$ , accelerated increasing period. At this stage, the anechoic tile shedding rate continues to increase, the enhancing rate of TS is also increasing, but the increased value is relatively small.

(3)  $0.5 < \hat{t}^* < 1.2$ , rapidly rising period. At this stage, the anechoic tiles have the most considerable shedding rate, and the TS of the submarine increases at a constant rate.

(4)  $1.2 < \hat{t}^* < 3$ , slowly rising period. Muffler tiles have mostly come off, the target intensity slowly increases. When the average life expectancy is more than twice, it increases to the steel boat's target strength level. When the service time exceeds the twice of the average life of anechoic tile, the submarine's TS increases to the high level that lacking anechoic tiles.

Therefore, according to the shedding distribution model of anechoic tiles proposed in this paper, we can draw the following conclusions: In the 0.3 to 0.5 times of the average life of anechoic tiles, detecting and maintaining the anechoic tiles can make the target intensity at a low level.

## 6. Conclusion

Based on the MC-PEM, this paper describes the mechanism of random laying of anechoic tiles on the target strength of cylindrical shell, and quantitatively analyses the relationship between target intensity and shedding rate. This paper draws the following conclusion:

(1) Linear coefficient of the first-order fitting approximates the sensitivity of substructure's target strength to the shedding of anechoic tiles, that is, the target strength of single-shell cylinder increases by 0.86dB for every 10% change of shedding rate.

(2) By introducing the cumulative failure probability function of anechoic tile based on lognormal distribution, the analysis shows that the target strength of cylindrical shell changes with time in four stages: initial constant period, accelerated increasing period, rapidly rising period, slowly rising period. In the 0.3 to 0.5 times of the average life of anechoic tiles, detecting and maintaining the anechoic tiles can make the target intensity at a low level.

The exfoliating mechanism analysis of anechoic tiles can be carried out in the follow-up work. It is generally believed that the probability of shedding of anechoic tiles is closely related to the distribution of alternating pressure on the surface of the submarine. Based on the CFD simulation technology, it helps to establish a more realistic TS prediction model of submarines coating with anechoic tile.

## 7. References

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