

# Construction Method of Telecommunication System for Corrective Information Distribution

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**Abstract.** The paper presents a methodology for optimal construction of telecommunication system for corrective information distribution based on the criterion of minimizing of expendable resources. A calculation method of range of control and correction stations (CCS) action transmitting the corrective information to mobile consumers under various conditions (sea, land areas with various electrical properties) is developed. A construction algorithm of the optimal telecommunication system has been created. The software in the software shell MATHLAB was developed for implementation of the obtained algorithm. The obtained algorithm and the developed program were tested. The results of comparison of calculations with the analytical solution and experimental data for the specific example are given. These results gave a good matching between the computations obtained by the algorithm and the analytical calculations. The developed algorithm and program were applied to optimize the composition of CCS of the local differential subsystem in the rivers basins of Siberia and the Far East. An example of the formation of action zones of CCS in the basin of the Ob River on inland waterways of Russian Federation is given. In conclusion, the inference is made about the effective optimization of the local differential subsystem, as a result of which it became possible to provide all inner waterways in the Ob basin by a continuous field of the differential correction.

## 1. Introduction

The basis of the technical component of e-Navigation concept implanted in the civil fleet accepted by the International Maritime Organization (IMO) is telecommunication systems for various purposes, one of which is the telecommunication system for providing of corrective information for GNSS GLONASS / GPS [1]. The local differential subsystems (LDSS) based on control and correction stations (CCS) working on medium frequency band (283.5-325.0 kHz), the range of which is about 200 ~ 500 km, are widely used in maritime and especially river conditions as telecommunication systems for providing of corrective information [2]. Such a wide spread is determined by the significant influence of underlying surface on the propagation range of radio waves of this band. In this connection, there is a problem of structure optimization of this telecommunication system (the number and location of the CCS which provide a continuous coverage of navigable sections of river basins of the given region).

Synthesis of river LDSS belongs to the class of tasks at which solution the goal is achieved with a minimum of required resources. In our case, the resource, first of all, means the power and the number of CCS transmitters.



## 2. Construction methodology of optimal telecommunication system for corrective information distribution

To optimize the structure of the river LDSS, it is necessary to determine the range of CCS including into it. The criterion for the action range calculation is the correspondence of the error probability of piece-by-piece reception of digital message to its permissible value. The estimation methodology is based on the well-known relationship between the error probability of piece-by-piece reception  $p_{err}$  and the signal power  $h^2$  under conditions of functional noise impact:

$$p_{err.} = 0.5 \exp(-0.5h^2). \quad (1)$$

Where the signal power is determined by the relation:

$$h^2 = (P_r T) / v^2, \quad (2)$$

where  $P_r$  – signal power in a reception point;  $T$  – transmission duration;  $v^2$  – spectral density of white functional noise.

However, it is necessary to consider that the main requirement to the river LDSS is providing of continuous covering of the internal waterways (IW) with the field of the differential correction. In this connection there is a need to overlap the action zones of adjacent CCS, which leads to significant mutual hindrances in the areas of intersection of these zones.

In these areas, the estimation methodology will be based on the coefficient of mutual distinction (CMD) of signal and mutual hindrance [3]:

$$g_{rq}^2 = K_0 \left\{ \left[ \int_0^{\tau} z_r(t) z_{nq}(t) dt \right]^2 + \left[ \int_0^{\tau} z_r(t) \tilde{z}_{nq}(t) dt \right]^2 \right\}, \quad (3)$$

where  $z_r(t)$ ,  $z_{nq}(t)$  – structures of the  $r^{\text{th}}$  version of a signal and the  $q^{\text{th}}$  concentrated noise;  $\tilde{z}_{nq}(t)$  – function adjoint to  $z_{nq}(t)$  by Hilbert;  $K_0$  – normalizing factor.

The normalizing factor  $K_0$  for a digital signal is given by:

$$K_0 = \{ \mu_r \mu_{nq} / (TP_r) \}^2, \quad (4)$$

where  $\mu_r$ ,  $\mu_{nq}$  – amplitude coefficients of signal transmission and concentrated noise transmission, respectively.

Then, at non-coherent reception the error probability of piece-by-piece reception of the digital message will be determined by expression:

$$p_{err} = -0.5 \exp \left[ -\frac{h_{eq}^2}{2} \right] \cdot I_0(R_0 h_{eq}^2) + Q \left[ \sqrt{\frac{h_{eq}^2}{2} (1 - \sqrt{1 - R_0^2})}; \sqrt{\frac{h_{eq}^2}{2} (1 + \sqrt{1 - R_0^2})} \right], \quad (5)$$

where  $Q$  – Markum function;  $I_0(R_0 h_{eq}^2)$  – modified Bessel function of the first kind, zero order;  $h_{eq}^2$  – Equivalent signal power in the presence of several mutual hindrances;  $R_0$  – radio channel parameter.

For opposite non-fading signals:

$$h_{eq}^2 = h^2 \left[ 1 - \sum_{q=1}^N \frac{g_{0q}^2 h_{nq}^2}{1 + h_{nq}^2} \right] = h^2 (1 - \delta), \quad (6)$$

where  $g_{0q}$  – normalized CMD of the signal and the  $q^{\text{th}}$  mutual hindrance;  $h_{nq}$  – power of the  $q^{\text{th}}$  mutual hindrance;  $N$  – number of mutual hindrances.

$$\delta = \sum_{q=1}^N \frac{g_{0q}^2 h_{nq}^2}{1 + h_{nq}^2}, \quad (7)$$

$$R_0 = \delta / (1 - \delta). \quad (8)$$

### 3. Method of calculation of structure of the optimal differential subsystem

Depending on whether the signal receiver is under the influence of only functional noise or it is under the influence of mutual hindrances, either (1) or (5) will be used to determine the error probability of piece-by-piece reception.

In both cases, by (2), it is necessary to determine the signal power, and in the second case it is also necessary to determine the power of each mutual hindrance. In order to determine the signal power or hindrance power presented in (2), it is necessary to find the field intensity of the signal at the receiving point  $E(r)$ , which for emitter in the form of a point dipole is determined by the expression [4]:

$$|E(r)| = -\frac{3\sqrt{10P_{Tr}}}{r} |w(r)|, \quad (9)$$

where  $P_{Tr}$  – radiated power of CCS transmitter;  $r$  – distance from emitter to receiving point;  $w(r)$  – signal attenuation function due to the influence of the underlying surface.

The power at the receiver input  $P_{res}$  is related to the field intensity by the known relation:

$$P_{res} = (A_{eff} E^2) / 240\pi. \quad (10)$$

Where  $A_{eff}$  – effective antenna area, defined by expression:

$$A_{eff} = (D_{res} \lambda^2) / 4\pi, \quad (11)$$

where  $D_{res}$  – Directivity factor of receiving antenna;  $\lambda$  – radiation wavelength.

Substituting (9) and (11) into (10) we obtain:

$$P_{res} = (3D_{Tr} D_{res} \lambda^2 P_{Tr} w(r)^2) / 32\pi r^2. \quad (12)$$

Here  $D_{Tr}$  – directivity factor of the transmitting antenna used in (9) when the dipole is replaced by a real antenna.

Thence

$$r^2 = (3D_{Tr} D_{res} \lambda^2 P_{Tr} w(r)^2) / 32\pi P_{res}. \quad (13)$$

Then the action range of CCS will be determined by the expression:

$$r_{max} = B |w(r)|, \quad (14)$$

where  $B$  – energy potential of the radio channel [5].

$$B = \frac{3\lambda}{4\sqrt{6}\pi} \sqrt{\frac{P_{Tr}}{P_{res,min}} D_{Tr} D_{res}}, \quad (15)$$

The value  $P_{res,min}$ , which determines the minimum permissible sensitivity of the receiver when only the fluctuation noise affects the receiver, is determined from (2) by the specified minimum permissible value of the signal power  $h_{min}^2$ , which is calculated from (1) by the maximum permissible error probability of piece-by-piece reception  $p_{err,val}$ .

Substituting (2) into (1) taking into account (12), we can express  $r_{max}$  in explicit form:

$$r_{max} = \frac{3\lambda}{8\pi} \left[ \frac{D_{Tr} D_{res} P_{Tr} T}{3 |\ln(2 p_{err,val})| v^2} \right]^{1/2} |w(r)|, \quad (16)$$

When the mutual hindrances affect the receiver input in general this algorithm for calculation of the CCS action range is not applicable. Here, the maximum permissible value of error probability of the piece-by-piece reception directly appears as the limiting condition. At the same time the action range of CCS  $r_{\max}$  corresponds to the condition:

$$P_{err.} = P_{errval.} \quad (17)$$

The error probability in this case is calculated according to (5). The power of signal and mutual hindrances included in (5) are determined by (2), taking into account (9) - (11). The equivalent energy is calculated by (6). The normalized CMD for a signal and a mutual hindrance, having the same structure with it, is given by:

$$g_{0q}^2 = \left\{ \frac{\sin[0.5\Omega_{nq}T]}{0.5\Omega_{nq}T} \right\}^2, \quad (18)$$

where  $\Omega_{nq}$  – shift of carrier frequencies of the signal and  $q^{\text{th}}$  hindrance.

In the explicit form dependence of  $r_{\max}$  from  $p_{errval}$  at incoherent reception of phase-shift keyed signals can be obtained only under the influence of one concentrated hindrance for the case: a non-fading signal – Rayleigh fading noise. Then the expression for the error probability of the piece-by-piece reception will take the form:

$$p_{err} = 0.5 \exp \left[ -h^2 \frac{1}{2 + h_n^2 g_0^2} \right], \quad (19)$$

Taking into account (9) - (11), we obtain:

$$r_{\max} = \frac{3\lambda}{8\pi} \left[ -\frac{D_{Tr} D_{res} P_{Tr} T}{|\ln(2p_{err, val})| v^2 (2 + h_n^2 g_0^2)} \right]^{1/2} |w(r)|, \quad (20)$$

The main difficulty at solving of (16) and (20) is the definition of the attenuation function  $w(r)$ . For distances of 200 ~ 300 km the attenuation function can be described by the simplified Hufford's equation for a spherical surface [6]:

$$w(r) = 1 + i\sqrt{\frac{ir}{\lambda}} \int_0^r w(x) \left[ \frac{1}{\sqrt{\varepsilon(x)}} + \sin\left(\frac{r-x}{2a}\right) \right] \frac{1}{\sqrt{x(r-x)}} dx, \quad (21)$$

where  $\lambda$  – wavelength,  $a$  – Earth's radius,  $\varepsilon(x)$  – complex dielectric constant of the underlying surface,  $x$  – distance from the source to the current integration point,  $r$  – distance from the transmitter to the receiving point.

The expression (21) is Volterra integral equation of the second kind:

$$w(r) = f(r) + \rho \int_0^r w(x) K(r, x) dx, \quad (22)$$

In our case, the free term is identically equal to one, and the core contains a complex variable (dielectric constant  $\varepsilon(x)$ ). Therefore, the solution of (21) will be find in the form:

$$w(r) = Re(w(r)) + iIm(w(r)). \quad (23)$$

After simple transformations, we finally obtain:

$$w(r) = 1 - \int_0^r w(x) \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left( \sqrt{|\varepsilon| - \varepsilon''(x)} + \sin\left(\frac{r-x}{2a}\right) |\varepsilon| \right) dx + i \int_0^r w(x) \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left( \sqrt{|\varepsilon| + \varepsilon''(x)} + \sin\left(\frac{R-x}{2a}\right) |\varepsilon| \right) dx, \quad (24)$$

Herein  $|\varepsilon|$  – module of complex dielectric constant, which is defined by expression:

$$\varepsilon = \varepsilon' + i2c\sigma / \lambda, \quad (25)$$

where  $\varepsilon'$  – relative permittivity;  $c$  – light speed;  $\sigma$  – conductivity.

#### 4. Results and discussion

The equation (24) has analytical solution only for a number of special cases. Therefore, its solution was carried out by numerical integration using the quadrature method.

The core of equation (24) has form:

$$K(r, x) = \sqrt{\frac{r}{2\lambda x(r-x)|\varepsilon|^2}} \left[ -\left\{ \sqrt{|\varepsilon| - \varepsilon''(x)} + \sin\left(\frac{r-x}{2a}\right) |\varepsilon| \right\} + i \left\{ \sqrt{|\varepsilon| + \varepsilon''(x)} + \sin\left(\frac{r-x}{2a}\right) |\varepsilon| \right\} \right], \quad (26)$$

Then the numerical solution can be found through the known iterative formula:

$$w_k = \frac{1 + \sum_{j=1}^{k-1} A_{kj} K_{kj} w_j}{1 - A_{kk} K_{kk}}, \quad (27)$$

where  $k = 0, 1, 2, \dots, N$ ;  $N$  – number of knots;  $A_{kj} = p_k h$  – set of integration coefficients;  $h$  – integration step;  $p_k h = B a_k$  – weight coefficients for different degrees of quadrature formulas;  $w = w(r_k)$ ;  $K = K(r_k, x_j)$ .

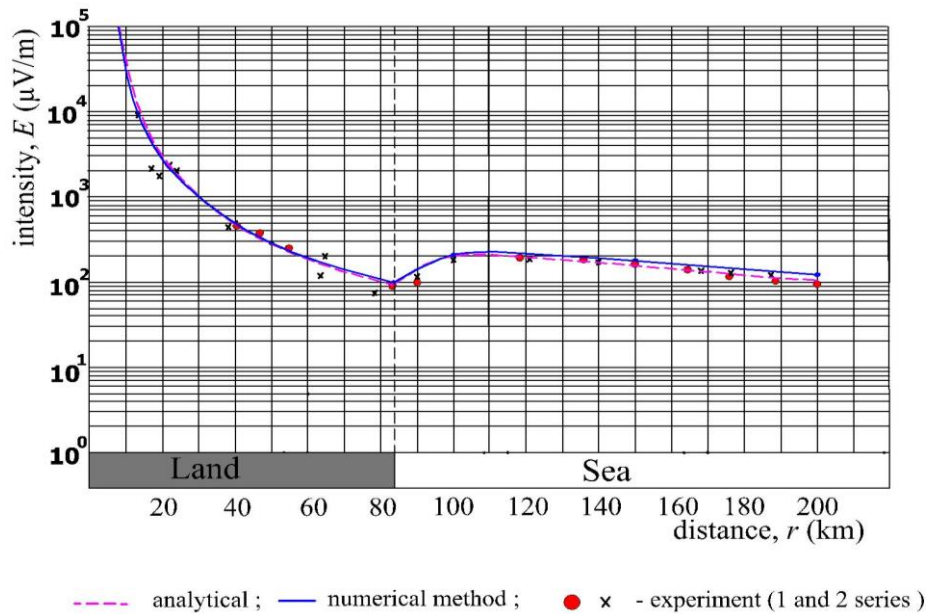
Since the core (26) has singularities at the ends because of the denominator  $x(r-x)$ , direct use of (27) is impossible. To eliminate singularities, each integral in (24) is divided into three parts. In the first integral, the variable is replaced by the condition  $x = y^2$ , and in the third integral by  $x = r - y^2$ . Then we get:

$$\int_0^r K(r, x) w(x) dx = \int_0^{\sqrt{n_1 h}} K_1(r, y^2) w(y^2) dy + \int_{n_1 h}^{r-n_2 h} K(r, x) dx + \int_0^{\sqrt{n_2 h}} K_3(r, r-y^2) w(r-y^2) dy, \quad (28)$$

where  $n_1$  and  $n_2$  – number of points by which the first and the last integrals in (28) are calculated, respectively.

The software for implementation of the obtained algorithm was developed in MATHLAB [7]. The underlying surface is considered as piecewise-homogeneous. The initial data to the program are the Earth's radius, the light speed, the wavelength, the splitting step of track, the number of sections with different permittivity and electrical conductivity and the number of nodal points of each section. In addition, the values arrays of  $\varepsilon'$  and  $\sigma$  are specified at the nodal points of integration.

Task solution for the track consisting of two sections (well conducting soil and sea), an analytical solution and experimental data on which are given in [4] (p. 310) was obtained for testing of obtained algorithm and developed program. The test results are shown in Figure 1.



**Figure 1.** Comparison of theoretical and experimental data with the results of numerical integration.

Initial data for calculation:  $P_{Tr}=10$  kW;  $\lambda = 96$  m; land area – 84 km; sea section – 116 km; land:  $\varepsilon' = 10$ ,  $\sigma = 0.009$  S/m; sea:  $\varepsilon' = 80$ ,  $\sigma = 4$  S/m.

As can be seen from the plot the developed algorithm of the numerical integration of Hufford's equation gave a good matching with the analytical solution for the spherical surface and the experiment results.

The developed algorithm and program were used to optimize the composition of CCS of LDSS in the rivers basins of Siberia and the Far East. The underlying surface was considered as piecewise-homogeneous. To determine the action zones of the CCS, beams were formed, diverging from the station, along which the dimensions of the sections with different electrical properties were set and in each of them the parameters  $\varepsilon'$  and  $\sigma$  were set. The size of the action zones for each directions was defined from (16) or (20) after determination of values of attenuation function by the numerical method. Equation (20) was used if in the given direction there were areas of intersection of the action zones of adjacent CCS with the presence of mutual hindrances.

Since (16) and (20) belong to the class of transcendental equations, their solution, and, consequently, the action range of the CCS in each of the given directions are determined by the successive approximation method. In this case,  $r_{ass}$  is specified in the first step and  $w(r_{ass})$  is determined by (21). Then,  $r_{max}$  is determined by (16) or (20). If  $r_{max} > r_{ass}$ , on the next step  $r_{ass}$  increases, if  $r_{max} < r_{ass}$ ,  $r_{ass}$  decreases and the calculation process is repeated until the condition  $|r_{max} - r_{ass}| < \Delta r$  will be met. Herein the value  $\Delta r$  determines the adjusted accuracy of determination of the CCS action range.

An example of construction of CCS action zones is shown in Fig. 2. The Ob basin of the Russian inland waterways is considered in this figure. The following initial data were used in the calculations: the output power of all transmitters is 400 W;  $\lambda = 1000$  m;  $v^2 = 10E-12$ ;  $T = 0.01$  s;  $p_{err, val} = 10E-3$ ; step of cutting of frequencies of CCS transmitters – 500 Hz;  $\Omega_{nq} = 2 \div 20$  kHz;  $D_{Tr}$ ,  $D_{res} = 0.9$ .

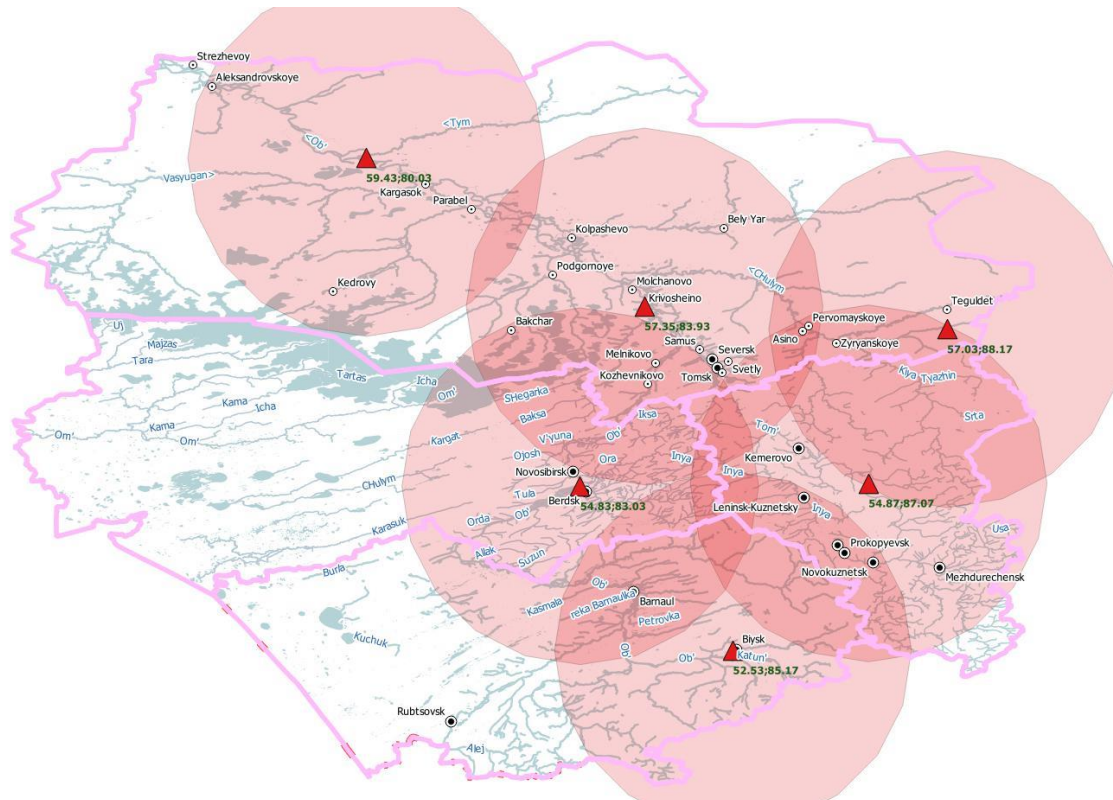
## 5. Conclusion

As can be seen from Fig. 2, the optimization of the LDSS structure made it possible to provide all inland waterways in the Ob basin by continuous field of differential correction. As can be seen from the results of mathematical simulation, only six CCS (triangles in Figure 2) were required to solve this task.



In addition, it should be noted that, with a rational separation of the carrier frequencies of adjacent CCS, in most cases it was possible to achieve a significant reduction in the influence of mutual hindrances in the areas of intersection of the action zones of adjacent CCS, which allows increasing the range of confident reception of corrective information.

The main problem in determination of the CCS action range was the creation of arrays of electrical parameters of the underlying surface  $\varepsilon'$  and  $\sigma$ . Existing Russian soil maps are not detailed enough, therefore errors appear. Taking into account these errors at the step-by-step solution of the transcendental equations (16) and (20), it makes sense to specify the accuracy  $\Delta r$  not less than 3 ~ 5 km.



**Figure 2.** Coverage of the Ob basin the areas of CCS

## 6. References

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