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Negative Linear Compressibility of Generic Tetragonal Beam Structure

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Abstract. The compressibility properties of systems consisting of generic tetragonal beam structures are analyzed and discussed. It is shown that these systems can exhibit negative linear compressibility (i.e. NLC) for certain conformations, that is, the systems with particular geometry will expand along one direction when compressed hydrostatically. It is also shown that through carefully choosing the geometric features, one may control the magnitude and range of NLC.

1. Introduction

When materials are compressed hydrostatically, they usually contract in all directions. However, the theory of elasticity still brings us some surprises as evidenced by occasional reports of materials [1-6] that NLC can actually be exhibited, meaning that some materials can expand along one direction when subjected to hydrostatic stress. Such materials are predicted to have a number of applications ranging from extremely sensitive pressure detectors, telecommunication line systems, to optical materials with high refractive index [1].

Some two-dimensional theoretical models and structures exhibiting negative linear compressibility have been proposed including truss-type systems [2], bi-material strips [3], systems made from rigid units [4], hexagonal honeycombs and wine-rack structures [5]. In view of these models, it is necessary to mention that the wine-rack structure is the most widely used model and the success of this model lies in the fact that the effect of NLC in other models can be strengthened when the geometry of such models become similar to wine-rack structure, for example, in hexagonal honeycombs when $h<<l$ [5] and in rotating rigid triangles when r becomes closer to 0 [4]. Here it should be noted that wine-rack structure is the special case of the tetragonal beam structures proposed in this paper and the compressibility properties of this more generic models have not been discussed. In view of this, in this paper, we aim to extend the early work by discussing how the generic tetragonal beam structures can be made to exhibit negative linear compressibility. And the expressions of mechanical properties including Young's moduli, Poisson's ratios and on-axis linear compressibilities are derived through an energy approach. It is shown that this generic structure can exhibit negative linear compressibility for particular conformations and it is also shown that the wine-rack structure is not always the best choice when one wants to get a NLC met material.

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2. Analytical Model

The tetragonal beam structure taken into consideration in this paper is consisting of beams of two different lengths as shown in Fig. 1. It should be noted that the structure relies on the beams being rigid in both axial and transverse directions. And these beams are connected through simple flexure hinges which enable the structure can deform when it is subjected to a stress and restore the original shape when the stress is removed. In particular, the structure modelled is one having dimensions l_1, l_2, θ_1, θ_2 defined as in Fig. 1.

The orientation of the structure is such that the unit cell vectors are aligned along the OX_1 and OX_2 directions such that the projections of the unit cell in these directions are given by:

$$X_1 = 2l_1 \cos\theta_1 = 2l_2 \cos\theta_2 \tag{1}$$

$$X_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \tag{2}$$

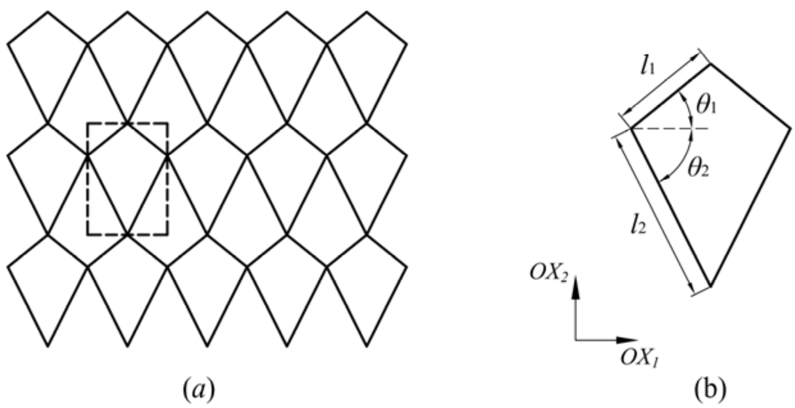

Figure 1. (a) Systems consisting of generic tetragonal beam structures, (b) the unit cell.

Note that in this model, apart from the normal conditions that $l_1>0$, $l_1>0$, $0^\circ<\theta_1<90^\circ$, and $0^\circ<\theta_2<90^\circ$, the following condition must be satisfied synchronously:

$$l_1 \cos\theta_1 = l_2 \cos\theta_2 \tag{3}$$

And from Eq. (1), we can assume that when this unit cell is subjected to a stress, the strain in OX_1 direction can be given by:

$$d\varepsilon_1 = \frac{1}{X_1}\frac{dX_1}{d\theta_1}d\theta_1 = \frac{1}{X_1}\frac{dX_1}{d\theta_2}d\theta_2 \tag{4}$$

Then the relationship among $d\theta_1$ and $d\theta_2$ can be derived:

$$l_1 \sin\theta_1 d\theta_1 = l_2 \sin\theta_2 d\theta_2 \tag{5}$$

Then we can use an energy approach to derive the expressions of the Young's moduli. The work done by each unit cell due to changes in angles for loading along OX_i (i=1, 2) axis is given by:

$$W = 4\cdot\left[\frac{1}{2}k_h\cdot\left(d\theta_1^{[i]}\right)^2\right] + 4\cdot\left[\frac{1}{2}k_h\cdot\left(d\theta_2^{[i]}\right)^2\right] \tag{6}$$

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And the work done per unit volume due to a strain $d\varepsilon_i^{[i]}$ for loading in OX_i directions is given by:

$$U = \frac{1}{2}E_i\left(d\varepsilon_i^{[i]}\right)^2 = \frac{1}{2}E_i \cdot \left(\frac{1}{X_i}\right)^2 \cdot \left(\frac{\partial X_i}{\partial \theta_1}d\theta_1^{[i]} + \frac{\partial X_i}{\partial \theta_2}d\theta_1^{[i]}\right)^2 \tag{7}$$

From the principle of conservation of energy, the expressions of the Young's moduli can be given by:

$$E_1 = \frac{2k_h}{b} \cdot \frac{l_1^2\sin^2\theta_1 + l_2^2\sin^2\theta_2}{l_2^2\sin^2\theta_2} \cdot \frac{\cos\theta_1}{l_1\sin^2\theta_1 \cdot \left(l_1\sin\theta_1 + l_2\sin\theta_2\right)} \tag{8}$$

$$E_2 = \frac{2k_h}{b} \cdot \frac{l_1^2\sin^2\theta_1 + l_2^2\sin^2\theta_2}{l_2^2\sin^2\left(\theta_1+\theta_2\right)} \cdot \frac{l_1\sin\theta_1 + l_2\sin\theta_2}{l_1^3\cos\theta_1} \tag{9}$$

Where b is the thickness of this model. Also for this model, the Poisson's ratio may be defined by (i, j=1, 2):

$$\nu_{ij} = -\frac{d\varepsilon_j^{[i]}}{d\varepsilon_i^{[i]}} \tag{10}$$

Where $d\varepsilon_j^{[i]}$ is an infinitesimally small change in OX_j direction due to the loading in OX_i direction. Hence the Poisson's ratio can be given by:

$$\nu_{12} = \nu_{21}^{-1} = \frac{l_1\sin\left(\theta_1+\theta_2\right)}{l_1\sin\theta_1 + l_2\sin\theta_2} \cdot \frac{\cos\theta_1}{\sin\theta_1\sin\theta_2} \tag{11}$$

Having determined Young's moduli and Poisson's ratios, one may obtain the expressions for β_L (OX_i):

$$\beta_L\left(OX_1\right) = \frac{1}{E_1} - \frac{\nu_{21}}{E_2} = \frac{b}{2k_h} \cdot \frac{l_1^2 l_2^2\sin\theta_1\sin\theta_2}{l_1^2\sin^2\theta_1 + l_2^2\sin^2\theta_2} \cdot \left(\frac{l_1\sin\theta_1 + l_2\sin\theta_2}{l_1\cos\theta_1}\sin\theta_1\sin\theta_2 - \sin\left(\theta_1+\theta_2\right)\right) \tag{12}$$

$$\beta_L\left(OX_2\right) = \frac{1}{E_2} - \frac{\nu_{12}}{E_1} = \frac{b}{2k_h} \cdot \frac{l_1^2 l_2^2\sin\left(\theta_1+\theta_2\right)}{l_1^2\sin^2\theta_1 + l_2^2\sin^2\theta_2} \cdot \left(\frac{l_1\cos\theta_1}{l_1\sin\theta_1 + l_2\sin\theta_2}\sin\left(\theta_1+\theta_2\right) - \sin\theta_1\sin\theta_2\right) \tag{13}$$

And then the area compressibility can be found from the linear compressibility:

$$\beta_A\left(OX_i - OX_j\right) = \beta_L\left(OX_i\right) + \beta_L\left(OX_j\right) \tag{14}$$

3. Discussion

In this model, the values of the linear and area compressibility are dependent on the geometry of the model including the magnitudes of θ_1, θ_2, l_1 and l_2. And particularly, according to Eq. (3), the expressions of βL (OX_1), β_L (OX_2) and β_A can be viewed as functions of θ_1 and l_1/l_2. In this structure, the linear compressibility is clearly illustrated by plots of θ_1 and various combinations of l1 and l_2 as shown in Fig. 2, which shows that β_L can be negative for some values of θ_1 and the effect of NLC is also influenced by the values of l_1/l_2.

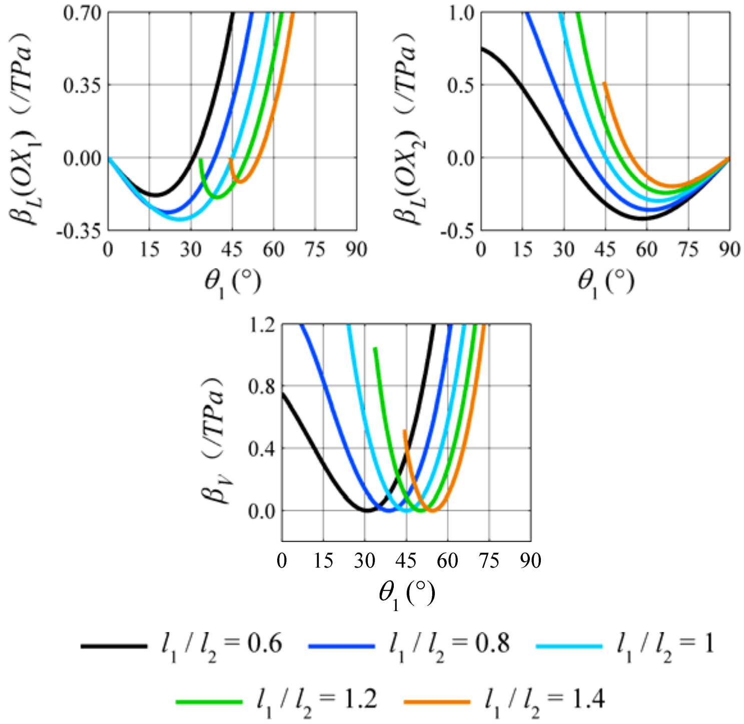

Figure 2. The linear and area compressibilities of this model across various angles of θ_1 with k_h= 1 KJ/rad^{-2}, l_1= 1mm, b=2mm.

In fact, Eqs. (12 - 13) suggest that NLC can be exhibited in this structure when the following conditions are satisfied:

(1) For negative β_L (OX_1): $(l_1/l_2 \bullet \sin\theta_1 + \sin\theta_2)\sin\theta_1\sin\theta_2 < l_1/l_2 \bullet \cos\theta_1 \sin(\theta_1+\theta_2)$

(2) For negative β_L (OX_2): $l_1/l_2 \bullet \cos\theta_1 \sin(\theta_1+\theta_2) < (l_1/l_2 \bullet \sin\theta_1 + \sin\theta_2)\sin\theta_1\sin\theta_2$

Which clearly suggest that NLC in OX_1 direction will arise at the exclusion of NLC in OX_2 direction. And not only that, from Fig. 2 we can know that NLC is always exhibited in this model. To be more specific, the existence of NLC in OX_1 direction and OX_2 direction can cover the whole range of variable θ_1. Here we should highlight the fact that the range of variable θ_1 is from 0° to 90° when $l_1/l_2 \leq 1$, however, once the ratio of l_1 to l_2 becoming larger than 1, the range of variable θ_1 will shrink to *arcos* $(l_2/l_1) \leq \theta_1 \leq 90°$ because of the relationship revealed in Eq. (3). Moreover, the effect of NLC in OX_1 direction can be maximized (i.e. widening the region of NLC and increasing the magnitude of the most negative value) to a greater degree by decreasing the magnitude of $|l_1/l_2-1|$ and particularly the maximum effect of NLC in OX_1 direction will be achieved when $l_1/l_2=1$, which corresponds to the wine-rack geometry. From this aspect, wine-rack geometry seems still to be the best choice to obtain NLC in 2D structure. However, when we turn our attention to the linear compressibility in OX_2 direction, the superiority of the generic tetragonal beam structure proposed in this paper will be visible. The effect of NLC in OX_2 direction can be strengthened by decreasing the magnitude of l_1/l_2 and the effect of NLC in this direction is always greater than that in OX_1 direction except when $l_1=l_2$, and once $l_2/l_1<1$, the effect of NLC in OX_2 direction will be more evident than that in wine-rack structure, which may suggest that the effect of NLC in wine-rack structure is not always the best.

Before we conclude, it is important to highlight that the work presented here indicates that the extent of NLC of this model can be tuned to specific values by choosing the magnitude of θ_1, l_1 and l_2, which would be useful when one needs to use this structure to attain a met material for particular

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application. And it should be noted that wine-rack geometry is not always the best choice to achieve apparent effect of NLC. Note also that for the mechanism responsible for NLC working, requires that the pressure should be exerted on the structure from outside just like the case in methanol monohydrate system studied by Fortes ET. al. [6]. Also it is important to highlight that the compressibility properties discussed above are only valid for small change of pressure. Finally, it should be noted that although two-dimensional models have their limitations when compared with three-dimensional models, their superiorities lie in the fact that they are simpler to analyze and also they are adequate enough to predict the behavior of particular projections of a complex 3D model where the linear compressibility is measured.

4. Conclusion

In summary, it has been shown through analytical modelling that this generic tetragonal beam structure can exhibit the special property of negative linear compressibility in certain directions. And in some cases, the effect of NLC in this structure can even greater than that in wine-rack geometry, which may be very useful when one needs to achieve evident NLC effect. Given the importance of this work, it is hoped that the findings obtained here will serve as a blueprint that can be used to either design new met materials or widen the search for materials which exhibit NLC.

Acknowledgments

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