

Analysis of Aeolian Vibration on Transmission Line Based on Analytical Model

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Abstract. To reveal the mechanism of aeolian vibration on transmission line, the steady amplitude of aeolian vibration is calculated based on the principle of energy balance, and the correction coefficient of amplitude is proposed under considering the non-uniformity coefficient of wind turbulence, environmental conditions and other factors; then the balance equation of aeolian vibration on transmission line is established, and the nonlinear motion balance equation is derived based on the D'Alembert's principle. Finally, the above two methods are compared and discussed. The result indicates that: The calculation result calculated by the traditional energy balance method is more conservative due to it amplifies the input of wind energy. The correction coefficient of amplitude is provided under considering influences included the turbulence of wind field, environmental conditions, wind speed and wind direction; The calculation result calculated by the improved energy balance method is more reasonable, which is close to the result of dynamics analytical method.

1. Introduction

So far, computing methods of aeolian vibration on transmission line could be divided into two major types: energy balance method and dynamical method. The energy balance method was based on the principle which the input energy of wind was equal to the consumed energy of transmission - antivibrator system to calculate the stable amplitude of transmission line, then to obtain the aeolian vibration strength and the analysis parameter of transmission line [1]. Whereas the dynamical method established the system equation of the transmission - antivibrator system to simulate the coupling effect between the wind, the transmission line and the antivibrator. Then the system response was obtained by using the dynamic equation [2]. The former had a clear relation of energy transfer and a simple concept. It could compute the stable amplitude of aeolian vibration by a preliminary calculation, which was the most extensive adopted method nowadays. The latter had a clear theoretical framework and a reliable calculation result. It considered complicated phenomena such as the installation site of antivibrator and traveling wave effect. As a supplement to the energy balance method, it had developed rapidly in recent years.



2. Energy balance method

When the input energy reaches a balance with the consumed energy, the amplitude of transmission line will keep up with the maximum level A_0 , the formula is as follows:

$$P_w = P_c \quad (2.1)$$

Where: P_w is the input power of wind; P_c is the self-damping power of transmission line.

2.1. The input power of wind

The paper regards transmission lines as cylinders, the vortex shedding of leeward side will generate the periodic lift force F_y on cylinders, its expression is:

$$F_y \sin \omega t = \left(C_L \frac{1}{2} \rho U^2 D \right) \sin \omega t \quad (2.2)$$

Where: C_L is the lift coefficient; ρ is the air density; U is the wind speed; D is the external diameter of transmission line; ω is the lift circular frequency.

Vertical displacement of vibration on transmission line y is set to:

$$y = -A_0 \cos(\omega t - \varphi) \quad (2.3)$$

Therefore, in half a cycle, the input power mean of wind is:

$$P_w = \frac{2}{T} \int_0^{T/2} \left\{ F_y \sin \omega t \frac{d}{dt} [-A_0 \cos(\omega t - \varphi)] \right\} dt \quad (2.4)$$

Plugging $A_0 = \eta D \rho = 1.3 \frac{S_t U}{D}$, $S = 0.2$ into equation (2.4) to get:

$$\frac{P_w}{f^3 D^4} = 51.05 C_L \eta \quad (2.5)$$

According to the literature [3], and considering the influence of regional conditions on the wind input power, the fitting formula of wind power is as follows:

$$\begin{cases} P_w = \beta_w \left(\frac{y}{D} \right)^{2a_1} \left(\frac{y}{D} \right)^{a_2} e^{a_3} f^3 D^4 & y < 0.6D \\ P_w = 27.51 \beta_w f^3 D^4 & 0.6D \leq y < D \\ P_w = 0 & y > D \end{cases} \quad (2.6)$$

Where: $a_1 = 0.0526$, $a_2 = 1.4074$, $a_3 = 4.0324$, y is the single amplitude; D is the diameter of transmission line; β_w is the non-uniformity correction coefficient of wind input power [4], its expression is:

$$\beta_w = \frac{1}{\sqrt{1 + (I_V / I_L)^2}} \quad (2.7)$$

Where: I_v is the wind turbulence intensity at the suspension point of transmission line; I_L is the locking constant 0.09.

2.2. Self-damping power of transmission line

Because the self-damping of transmission line has a large scatter and is influenced by many factors, the theoretical calculation is very difficult. It is usually measured through tests. In the paper, the theoretical expression of transmission line self-damping is [5]:

$$P_c = 1.07 \times 10^6 E_{eq} \sqrt{k_D} k_s K_0 L \frac{D^{4.5} f^{6+\beta} y^{2.5}}{V_c^5} \quad (2.8)$$

Where: E_{eq} is the equivalent bending stiffness; K_D is the factor of experience, for the aluminium cable steel reinforced, it can be taken to 0.54; for the steel conductor, it can be taken to 0.65; y is the single amplitude; K_s is the reduction coefficient of maximum bending stiffness, it can be taken to 0.5.

V_c is the wave velocity of transmission line $V_c = L \sqrt{\frac{g}{8sag}}$, g is the acceleration of gravity; s is the sag;

for the aluminium cable steel reinforced, $K_0 = 0.0042$, $\beta = -0.4256$.

2.3. Amplitude correction coefficient of aeolian vibration on transmission line

The energy balance method assumes that the steady mean wind speed which has a right angle with transmission line remains constant along all the span to calculate the aeolian vibration response of transmission line. Obviously the assumption is not correspond with the actual situation. In the actual circuit, the speed and the direction of wind will change with space and time, which can have a direct influence on the amplitude and the duration of transmission line.

Therefore, the paper put forward a correction to the amplitude of aeolian vibration on transmission line which is calculated by using energy method. The expression of correction coefficient is [6]:

$$F_T = \frac{y}{y_0} \quad (2.9)$$

Where: y_0 is the maximum theoretical amplitude which is calculated by using energy balance method; y is the maximum amplitude which is obtained through measuring a long period vibration in the site. The correction coefficient can be taken according to table 1.1[7].

3. Dynamics analytic method

3.1. Vortex-induced force

When the stable wind speed of x - z plane is about 0.5-10m/s, the vortices that alternates up and down will be generated on the leeward side of transmission line, which applies the force alternating up and down on the y direction of transmission line to make transmission line vibrate in plane. Based on the *Scanlan* empirical nonlinear model [8], the vortex-induced force expression of transmission line is obtained as follows in the paper.

$$f_y = \frac{1}{2} \rho U^2 D [Y_1(K) (1 - \varepsilon \frac{v^2}{D^2}) \frac{\dot{v}}{U} + Y_2(K) \frac{v}{D} + \frac{1}{2} C_L(K) \sin(\omega_{st} t + \phi)] \quad (3.1)$$

Where: U is the mean wind speed; Y_1 is linear self-excited damping; ε is nonlinear self-excited damping; Y_2 is the linear aerodynamic stiffness; C_L is the lift coefficient of resonance; v is the displacement of the direction which is perpendicular the direction of wind.

Researches show that $Y_2(K)$ and $C_L(K)$ can be negligible in the locking area, equation (3.1) can be simplified to:

$$F_y = \frac{1}{2} \rho U^2 D \left(Y_1 \frac{\dot{v}}{U} - Y_2 \frac{v^2}{D^2} \frac{\dot{v}}{U} \right) \quad (3.2)$$

3.2. Dynamic equilibrium equation

By means of the D'Alembert principle, the dynamic equilibrium equation of cable element is built:

$$\frac{\partial}{\partial s} \{ (T + \tau) \delta \} = m v_{tt} + c_y v_t + f_y + G_y \quad (3.3)$$

Where: T , τ respectively are the axial tension and the additional axial tension after vibrating; S is the arc length coordinate of transmission line; m is the unit mass; c_y is the damping coefficient of y direction; G_y is the self-weight of transmission lines; f_y is the vortex-induced force of per unit length.

According to the deformation compatibility equations and static equilibrium condition of cable element [9], equation (3.3) can be simplified to:

$$H v_{xx} + h \frac{\partial}{\partial s} (y_x + v_x) = m u_{tt} + c_y u_t + f_y \quad (3.4)$$

To conduct the dynamic analysis, equation (3.4) is converted into ordinary differential equation. The motion equation of cable in y direction is assumed to [10] [11]:

$$v(x, t) = \sum_n \varphi_{2n}(x) \cdot q_{2n}(t) \quad (3.5)$$

Where: φ_{2n} is the mode function of y direction; q_{2n} is the vibration function of y direction.

For the cable whose sag-span ratio is small, the vibration mode of cable is generally taken as the vibration mode of standard chord [12], and conduct the truncation of first order mode by means of the *Galerkin* method [13]. The partial differential equation of (3.4) can be converted into the nonlinear dynamic equilibrium equation of y direction which is expressed by the ordinary differential equation:

$$b_1 \ddot{q}_2 + b_2 \dot{q}_2 + b_3 q_2 + b_4 q_2^2 + b_5 q_2^3 + b_6 q_2 + b_7 q_2^2 = F_y(t) \quad (3.6)$$

Where:

$$b_1 = -m \int_0^l (\varphi_2)^2 dx, \quad b_2 = -c_y \int_0^l (\varphi_2)^2 dx, \quad b_3 = H \int_0^l \varphi_2'' \varphi_2 dx + \gamma \int_0^l y_x \varphi_2' dx \cdot \int_0^l y_{xx} \varphi_2 dx$$

$$b_4 = \gamma \int_0^l y_x \varphi_2' dx \cdot \int_0^l \varphi_2'' \varphi_2 dx + \frac{\gamma}{2} \int_0^l y_{xx} \varphi_2 dx \cdot \int_0^l (\varphi_2')^2 dx, \quad b_5 = \frac{\gamma}{2} \int_0^l (\varphi_2')^2 dx \cdot \int_0^l \varphi_2'' \varphi_2 dx$$

$$b_6 = \gamma \int_0^l y_x \varphi_2' dx \cdot \int_0^l y_{xx} \varphi_2 dx, \quad b_7 = \frac{\gamma}{2} \int_0^l (\varphi_2')^2 dx \cdot \int_0^l y_{xx} \varphi_2 dx$$

The vortex-induced force f_y is the aerodynamic negative damping of system; equation (3.6) can be simplified into:

$$\ddot{q}_2 + a_1 q_2 + a_2 q_2^3 + a_3 \dot{q}_2 + a_4 \dot{q}_2 q_2^2 = 0 \quad (3.7)$$

Plugging $\varphi_{2n} = \sin\left(\frac{n\pi x}{l}\right)$ into equation (3.5), the steady-state amplitude $v(x, t)$ of aeolian vibration on transmission line is obtained.

4. Comparative analysis of example

The paper takes ground wires of a section 1000KV transmission line in the southeast of Jin-Nanyang–Jingmen as an example to analyze the aeolian vibration. The span is 522m, the conductor type is JLB20B-240, the external diameter is 0.02m, the mass per unit length is 1.5955kg/m, the everyday tension is 40677.9N. The sectional areas A of aluminium (alloy) and steel are respectively 59.69 mm² and 179.07 mm². The combined elastic coefficient E is 1.472×10^{11} Pa, the calculated comprehensive breaking strength T is 3.152×10^5 N, the horizontal component of tension H is 4.068×10^4 N, the diameter of conductor d is 2×10^{-2} m, the horizontal distance of two adjacent tower L_x is 522m, the mass per unit length μ is 1.5955 kg.m⁻¹. According to the improved energy balance method and the dynamics analytical method, the vibration response of transmission line is calculated.

When the wind speed is 0.5m/s~5m/s, the locking frequency is 5HZ~50HZ, the transmission line is easy to generate the aeolian vibration and the double amplitude generally does not exceed the diameter of transmission line. In the figure 4.1, the amplitudes calculated by *Claren R* method and *Diana* method have exceeded the diameter of transmission line, which has a big difference with the other two results.

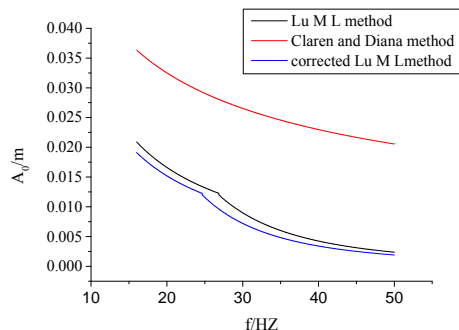


Figure 4.1. Comparison between corrected Lu M L with traditional Lu M L method, Claren and Diana method

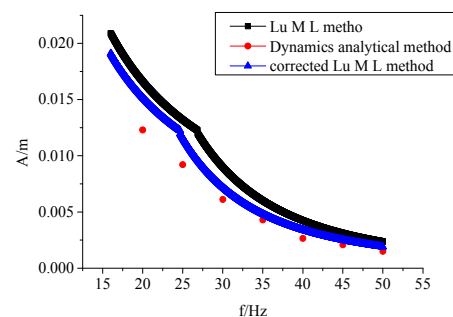
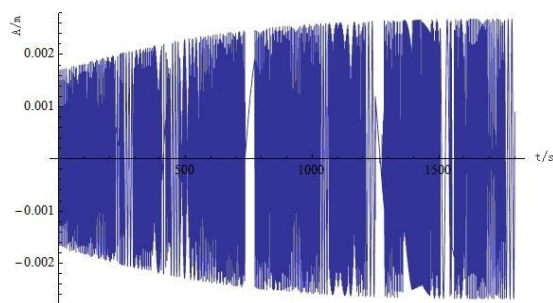
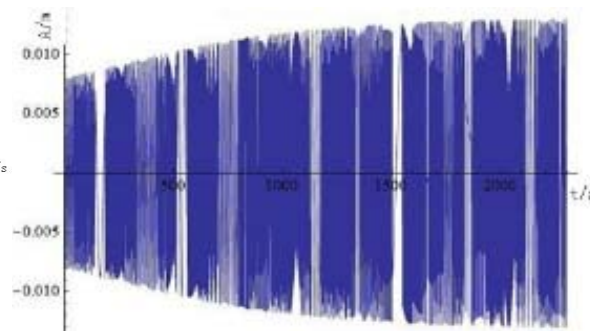


Figure 4.2. Comparison between Dynamics to VIV based on analytical model with Lu M L method

In addition, to ensure the security of structure, when calculating the aeolian vibration of transmission line by means of the energy balance method, the input power model of wind is relatively conservative. The value of input power is generally larger, moreover, the self-damping power model of transmission line has a great difference. The paper comprehensively considers the influences of the non-uniform coefficient of wind input power, environmental conditions, wind speed and wind direction to correct the *Lu M L* method. Figure 4.1 shows that the calculation result is smaller than that of *Lu M L* method.

Because the paper has limited space, two typical wind speeds (3m/s~4m/s) are used to analyze, response frequencies of aeolian vibration are respectively 30Hz and 40Hz. The calculation result are shown in figure 4.2. The continuous curve in the figure is the calculation result according to literature [12]. The calculation result is larger than that of the example in the paper. However considering influences of the terrain and other conditions, after correcting the amplitude, the calculation result is relatively close to that of the dynamic analytic method.

**Figure 4.3.1.** VIV of 3m/s wind velocity**Figure 4.3.2.** VIV of 4m/s wind velocity

Figures 4.3.1-2 are the calculation results by means of numerical algorithm (wind speeds are respectively 3m/s, 4m/s), the amplitude of transmission line can reach steady state ultimately.

5. Conclusion

- (1) Considering influences of the non-uniformity coefficient of wind turbulence, environment conditions, wind speed and wind direction, the traditional energy balance method is corrected, the calculation result of which is more reasonable.
- (2) Making a comparison between the calculation results of correction energy balance method with that of dynamic analytic method, which indicates that: because of the big wind power input and the symmetrical mode truncation of cable in the finite element model, which leads to some error into calculation results. However, the calculation result of correction energy balance method is relatively close to that of the dynamic analytic method, which is more reasonable.

References

- [1] Hagedorn P. On the computation of damped wind-excited vibrations of overhead transmission line. Journal of Sound and Vibration.1982, 83 (2): 253-271.
- [2] Claren R, Diana G. Mathematical analysis of transmission line vibration.IEEE Transactions on Power Apparatus and Systems.1969, PAS—88 (12): 1741-1771.
- [3] Diana G, Faleo M. On the forces transmitted to a vibrating cylinder by a blowing fluid. Mechanica.1971, 6 (1): 9-22.
- [4] Ervik M., Berg A, Boelle A, et al. Report on aeolian vibration. Electra. 1989, (124): 40-77.
- [5] Lu M L, Chan J K. An efficient algorithm for Aeolian vibration of single conductor with multiple dampers IEEE Transactions on power Delivery.200722 (3): 1822-1529.
- [6] Report of CIGRE, Aeolian vibration on overhead transmission line. CIGRE, 1970, No.22-11.
- [7] Shao Tianxiao, Mechanics calculation of electrical wires of overhead transmission line. Beijing: China Electric Power Press, 2003.
- [8] INDRANIL GOSWAMI, R H SCANLAN. N P JONES. Vortex-induced vibration of circular cylinders II :new model [J]. Journal of Engineering Mechanics. 1993 , 119 (11): 2271-2287.
- [9] Yang Zhenhua. Analysis of the aeolian vibration on large cross transmission line [D]. Chongqing University, 2010.
- [10] P. Warnitchai. Nonlinear vibration and active control of cable-stayed bridges. Ph. D. dissertation. Eng, The University of Tokyo, 1990.
- [11] P Wamitchai, Y Fujino, T. Susumpow. A non-linear dynamic model for cables and its application to a cable-system. Journal of Sound and Vibration, 187 (4) (1995): 695-712.
- [12] H. M. Irvine. Cable Structures. The M.L.T. Press, Cambridge, Massachusetts, 1981.
- [13] Wang Lianhua. Nonlinear dynamic analysis of stay cable [D]. Master's thesis of Hunan University, 2001.