

# Sustainability Index Metric System Based on AHP and the GRA-Entropy Method

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**Abstract.** When reconciling balance between economic development and ecosystem health, a sustainable future is imperative but hard to distinguish. We focus on setting up an index assessment system for evaluating sustainable development. Using the data from 1995 to 2016, we utilize the AHP serving for data selection. Besides, the Gray Relational Analysis and the Entropy Method are employed to identify the index value. Two methods' calculation results lead to different weighting structures so the combination is adopted. We draw on six sample countries to inspect the impact of national behavior on sustainability. The result shows that the countries that are sustainable are the minority. The index of a country varies from year to year.

## 1. Introduction

Since the Industrial Revolution, human accelerate the plundering of natural resources and cause such as tremendous earthquakes, tsunamis, volcanic eruption to the environment. In order to gain the harmony between man and nature, the *Sustainable Development* was proposed [1] and the scholars and governments made great efforts to complete it.

In 1992, the United Nations convened its General Assembly on Environment and Development, adopted and signed the important documents such as Rio de Janeiro Declaration on Environment and Development, *The Declaration of Principles on Forests and Agenda 21*. This conference firstly put the sustainability theory into practice. In 2002, The United Nations Summit on Sustainable Development held in Johannesburg, South Africa adopted the plan of implementation of the World Summit on Sustainable Development, which means the sustainable development theory has entered a new stage [2]. From then on, the sustainable development system is becoming rich and perfect. The United Nations Commission on Sustainable Development (UNCSD) put forward the "driving force - state - response" (DSR) indicator system. Later, the World Bank proposed a new one which combines four elements of natural resources, productive capital, human resources and social capital [3]. The index proposed by Eurostat in 2000 contains 63 indicators and these are distributed in economic, society,



environment and mechanism. In addition, different countries and regions have also constructed their own national sustainable development index systems and there exists no one standardized.

We build a sustainability index system based on the indicators; they are economy, society and environment. Firstly, based on several selection principles and the Analytical Hierarchy Process (AHP), we establish the sustainability index system. Then, we combine the Gray Relational Analysis (GRA) and the entropy method to determine the weights of these indexes. Finally, we calculate countries' sustainability index value and take six countries as examples in our paper.

## 2. Preparation for Sustainability Index Metric System

### 2.1. Selection of Indexes

To measure the sustainability efficiently, we identified several principles to select the indexes for our Sustainability Index Metric System (SIMS) [4]. They are shown as follows:

The data of indexes is easy to get and be calculated;

Indexes can be affected by the policy;

Indexes have good representativeness of sustainability;

It is easy to distinguish different indexes.

Based on these principles, we construct an index system with three aspects including Environment, Economy and Society, which contains 17 secondary indexes shown in Table 1.

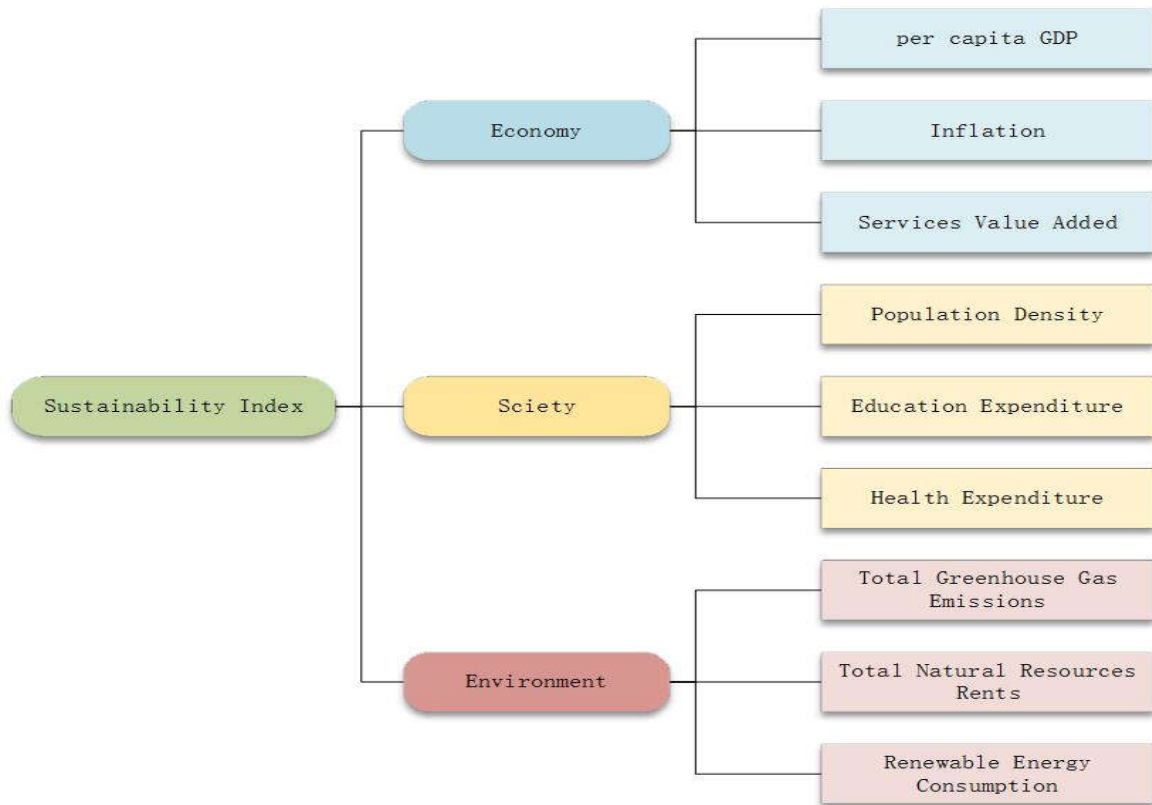
### 2.2. AHP-Based System Simplification

To simplify our index system, we introduce the Analytic Hierarchy Process (AHP) to screen those indexes mentioned above [5, 6]. The results are shown as follows:

**Table 1.** AHP Result of Secondary Indexes

First-Level Index-A	Second-Level Index-B	Weight
Environment Indexes-A1	Arable land B1	0.1024
	Renewable energy consumption-B2 (% of total final energy consumption)	0.3075
	Total natural resources rents-B3 (% of GDP)	0.1128
	Total greenhouse gas emissions-B4 (kt of CO <sub>2</sub> equivalent)	0.3075
	Forest area-B5 (sq. km)	0.1035
	Energy use per \$1,000 GDP-B6	0.0663
Economic Indexes-A2	Inflation-B7	0.2612
	GDP per capita-B8	0.5190
	Exports of goods and services-B9 (% of GDP)	0.0580
	Broad money-B10 (% of GDP)	0.0580
	Services value added-B11 (% of GDP)	0.1038
Society indexes-A3	Health expenditure, total-B12	0.1913
	Unemployment, total-B13 (% of total labor force)	0.0779
	Education expenditure-B14	0.4247
	Land under cereal production-B15	0.1913
	Mortality rate, under-5-B16 (per 1,000 live births)	0.0368
	Population density-B17	0.0779

To simplify our index system, we select the top 3 second-level indexes in the rank of weight. Finally, we get the index system shown as Figure 1.



**Figure 1.** Sustainability Index Metric System

Besides, the data used in our model comes from the World Bank Database ([data.worldbank.org.cn](http://data.worldbank.org.cn)), which covers years from 1961 to 2014. Unfortunately, the lack of data still cannot be avoided. So we decided to eliminate some countries losing too much important information. And finally we get 104 countries including countries and areas from 1995 to 2014. We complete the missing data by averaging and then standardize the value between 0 and 1.

### 3. Establishment of SIMS Based on GRA-Entropy Method

#### 3.1. Algorithm Introduction of the Gray Relational Analysis & Entropy Method

We introduce the Gray Relational Analysis (GRA) and Entropy Method to calculate the weight of indexes. Here, we introduce the algorithm principle of GRA and Entropy Method [7, 8].

##### (1) Gray Relational Analysis

**Step 1** Determine comparison matrix. Define the evaluation matrix  $H_{m \times n}$ , entry  $h_j(i)$  represents  $i(i = 1, 2, \dots, m)$  bottom index of  $j(j = 1, 2, \dots, n)$  country. Thus 104 comparison sequences are  $H_j = (h_j(1), h_j(2), \dots, h_j(m))$

**Step 2** Arrange parameter sequence. Here,  $H_0 = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$

**Step 3** Calculate correlation of  $H_j$  and  $H_0$ . Here,

$$r(H_0(i), H_j(i)) = \frac{\min_j \min_i \Delta_j(i) + \alpha \max_j \max_i \Delta_j(i)}{\Delta_j(i) + \alpha \max_j \max_i \Delta_j(i)}$$

Where,  $\alpha = 0.5$ .

*Step 4* Calculate correlations. Then we measure the gray correlation of  $H_j$  and  $H_0$ . The expression is

$$r_i = \frac{1}{n} \sum_{j=1}^n r(H_0(i), H_j(i)) (i = 1, 2, \dots, m)$$

*Step 5* Calculate weights. As for a certain year, the weight  $w_i = r_i / \sum_{k=1}^m r_k$   $i = 1, 2, \dots, m$ ,

We average weights of 11 years and get the final weight of an index.

(2) Entropy Method

*Step 1* Calculate weights and entropy value. Here,  $e_z = -k \sum_{j=1}^n p'_{zj} \ln p'_{zj}$   $z = 1, 2, \dots, p$ .

*Step 2* Calculate different coefficients. We define different coefficient is  $g_z = \frac{1 - e_z}{9 - E_e}$

Among them,  $E_e = \sum_{z=1}^p e_z$ .  $0 \leq g_z \leq 1$ , and it satisfies  $\sum_{z=1}^p g_z = 1$ .

*Step 3* Calculate weights. For index  $z$ ,  $w_{z-e} = \frac{g_z}{\sum_{z=1}^p g_z} = g_z$ ,  $z = 1, 2, \dots, p$

### 3.2. Method Combination: GRA-Entropy Model

To establish our SIMS accurately and scientifically, we decide to combine GRA and entropy methods [8]. The combined model is shown as follows:

$$w_{zi} = 0.618w_{zi-g} + 0.382w_{zi-e}$$

Where,  $w_{zi}$  is the weight calculated by GRA-Entropy Method,  $w_{zi-g}$  is the weight by GRA,  $w_{zi-e}$  is the weight by entropy method.

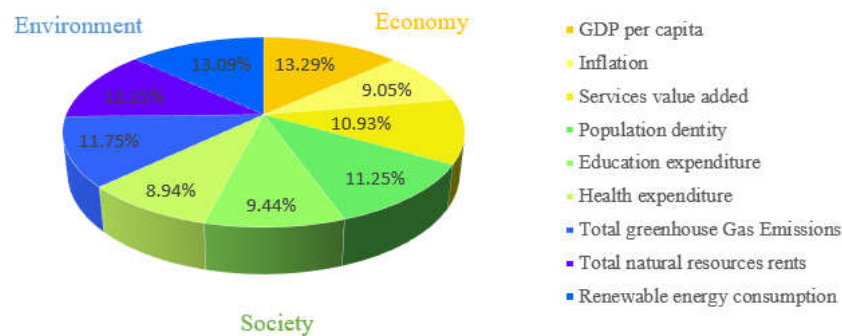
Then, sustainability index can be calculated as follows:

$$SI_{zj} = 0.618SI_{zj-g} + 0.382SI_{zj-e}$$

Where,  $SI_{zj}$  is the weight calculated by GRA-Entropy Method,  $SI_{zj-g}$  is the weight by GRA,  $SI_{zj-e}$  is the weight by entropy method.

### 3.3. Result Analysis

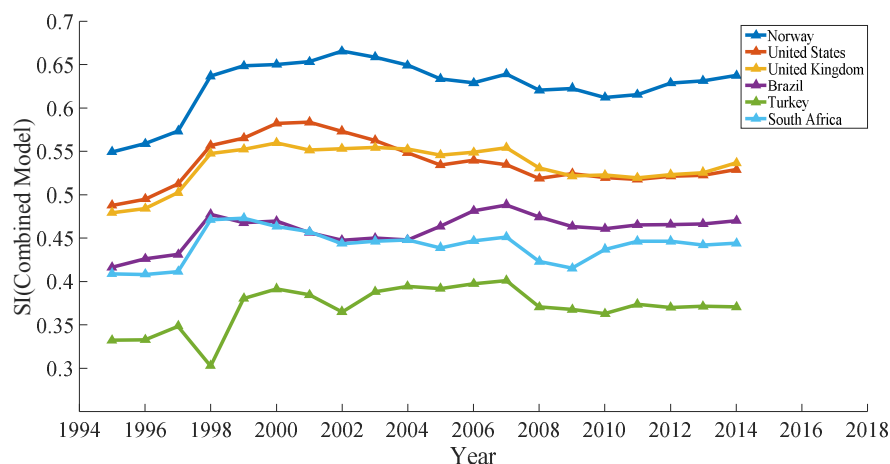
The weight of each secondary index in the combined model is calculated as shown in Figure 2:



**Figure 2.** Weights of GRA-Entropy Model

According to figure 2, the secondary weights of the 9 indexes calculated by the GRA-Entropy model are relatively balanced. Among them, the environmental comprehensive index contributes most to SI. This proves that the environment is still a very important factor that can not be ignored in the process of sustainable development. This weighting structure is an improvement on the weights obtained by both methods, which is more reasonable and realistic.

Due to the limit of our paper, we use six countries as examples to show the result of GRA-Entropy model and calculate the sustainability indexes value and draw the SI curves in Figure 3.



**Figure 3.** Six countries in the SI value (Combined model)

Also, the ranking of the six countries is shown in Table 2.

**Table 2.** Sustainability Index Ranking of Six Countries (GRA-Entropy model)

Country	1995	2000	2005	2010	2014
Norway	2	1	1	1	1
United States	11	6	7	8	3
United Kingdom	14	8	5	6	9
Brazil	40	32	28	21	8
Turkey	91	89	91	92	49
South Africa	46	35	42	42	26

As we can see in Figure 3, the sustainability indexes of six countries can be clearly divided into 4 levels. Norway belongs to the first level and its SI is always above 0.5 which is still growing slowly; the second level includes the United States and the United Kingdom. Brazil and the South Africa belong to the third level; Turkey is in the fourth level.

As we can see in Table 2, we can clearly find that in our GRA-Entropy model, Norway always ranks first among the countries around the world, which is consistent with the current reality worldwide. Besides, we can find that the rank of UK and US are always closed. As the representatives of developed countries, Brazil and South Africa also have closed ranking. Due to the fact that there are lots of wars and disaster in Middle East, Turkey is the worst country among the selected countries.

#### 4. Conclusion

In our time, encouraging economic development, and maintaining ecosystem health are becoming more and more noteworthy, so the problem comes how to define the sustainability. This paper builds an index evaluation system based on indicators of sustainable development coverage. The consequence shows that the sustainability of most countries is not decent so much as optimistic, and their scores need to be further improved. The sustainability index will change caused by national policies in different years. Effective actions can improve the sustainability index to some extent so it is urgent to formulate improvement policies.

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