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Stability Analysis for Subgrade Settlement Prediction by Curve Fitting Methods

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Abstract. The curve fitting methods are widely used in predicting the settlement of subgrade because of few and easily determined parameters, especially for hyperbola method, Xingye method and Asaoka method. In applying this kind of prediction methods to predict the subgrade settlement, the afterward settlement can be obtained by the regression analysis of advance monitoring data, and the stability analysis for subgrade settlement prediction by curve fitting methods has become a noteworthy and valuable issue in engineering. In this paper, the evaluation indicators of fitting parameters by hyperbola method, Xingye method and Asaoka method are discussed, and the stability of predicted results by these three methods is analyzed through some engineering examples. The results show that there is a closely relationship between the stability of predicted results by curve fitting methods and fitting parameters in the regression formula. In predicting the subgrade settlement based on hyperbola method and Xingye method, the error amplification coefficient of fitting parameters is small and the predicted results show good stability when setting different initial time. However, as for Asaoka method, the error amplification coefficient of fitting parameters is large and the predicted results show instability when setting different initial time and time spacing. In addition, the predicted results obtained from Asaoka method are greatly affected by randomness of sample points.

1. Introduction

In the practical engineering, it is difficult to make accurate calculations for soft soil subgrade settlement, and there often exists some differences between the calculated results and the measured results. The reasons for that are as follows: (1) the selection of calculation parameters is different, (2) it is really hard to determine settlement coefficients on settlement calculation due to a wide range of settlement coefficient, (3) it is also hard to estimate the lateral extrusion, especially for high filling on soft soil subgrade [1]. In view of the above-mentioned facts, a new type of prediction method is put forward on basis of curve fitting, which is used for predicting the afterward settlement by the regression analysis of the advance monitoring data, and the curve fitting is made by actual subgrade settlement data and time.

In order to establish corresponding curve model, the settlement of curve fitting methods is regard as the process under some change rules, and fitting process is conducted on measured settlements. In addition, the parameters can be back-calculated with some optimization methods. Nowadays, the curve fitting methods are widely used in predicting the settlement of subgrade because of few and easily determined parameters, and the stability analysis for subgrade settlement prediction by curve fitting methods has become a noteworthy and valuable issue in engineering.

In this paper, the evaluation indicators of fitting parameters by hyperbola method, Xingye method and Asaoka method are dicussed, and the stability of predicted results by these three methods is analyzed through some engineering examples.

2. Prediction models of curve fitting methods

There are several common prediction models of curve fitting methods are mainly analyzed in this paper, such as hyperbola method, Xingye method and Asaoka method [2,3].

2.1. Hyperbola method

It is assumed that the settlement rate of ground decreases with time in hyperbolic form, the settlement calculation formula of hyperbola method is as follows (1):

$$S_t = S_0 + \frac{t - t_0}{\alpha + \beta(t - t_0)} \tag{1}$$

In the formula (1), t_0 and S_0 respectively indicate the observation time and the settlement of the starting point in the fitting calculation; t and S_t respectively indicate any time point of fitting curve and the corresponding settlement; α and β respectively indicate the intercept and slope of the line.

When the parameter t tends to be infinite, using next formula (2) to calculate final settlement S_∞ :

$$S_\infty = S_0 + 1/\beta \tag{2}$$

After a period of loading, residue settlement ΔS can be calculated by formula (3):

$$\Delta S = S_\infty - S_t \tag{3}$$

2.2. Xingye method

The settlement calculation formula of Xingye method is as follows (4):

$$S_t = S_0 + \frac{AK\sqrt{(t - t_0)}}{\sqrt{[1 + K^2(t - t_0)]}} \tag{4}$$

In the formula (4), S_0 indicates the immediate settlement produced by instantaneous loading; A indicates the coefficient of final settlement when the parameter t tends to be infinite. According to formula (4), it is found that the relationship between $(t - t_0)/(S_t - S_0)^2$ and $(t - t_0)$ amounts to a straight line with the intercept is $1/(A^2K^2)$ and slope is $1/A^2$. When the parameter t tends to be infinite, final settlement S_∞ can be calculated by formula (5):

$$S_\infty = S_0 + A \tag{5}$$

2.3. Asaoka method

Asaoka method is based on Terzaghi's unidirectional consolidation equation, its linear fitting equation is as follows (6):

$$S_{tj} = \beta_0 + \beta_1 S_{tj-1} \tag{6}$$

In the formula (6), S_j indicates scattered settlement, β_0 and β_1 indicate undetermined constants.

According to formula (6), when S_{tj} equals $S_{tj\ 1}$, final settlement S_∞ can be calculated by formula (7):

$$S_\infty = \beta_0 /(1-\beta_1) \tag{7}$$

3. Evaluation indicators of stability by curve fitting methods

According to the prediction models of hyperbola method, Xingye method and Asaoka method, it is proved that the key problem is how to determine parameter α and β when using hyperbola method to predict settlement, and the identification of parameter A and K is the key problem for Xingye method. As for Asaoka method, the key problem is to determine parameter β_0 and β_1 [4]. This illustrates that there is a closely relationship between the stability for subgrade settlement prediction based on curve fitting methods and fitting parameters in the regression formula.

3.1. Hyperbola method

For hyperbola method, if settlement s_0 responding to start point t_0 is known, the differential expression of final settlement can be obtained by taking the derivative with respect to the final settlement in the formula (2), which can be seen in formula (8):

$$ds_\infty = -\frac{d\beta}{\beta^2} = -\frac{1}{\beta}\cdot\frac{d\beta}{\beta} \tag{8}$$

According to formula (8), furthermore the simplified differential expression of final settlement can be seen in formula (9):

$$\frac{ds_\infty}{s_\infty} = -\frac{1}{\beta s_\infty}\cdot\frac{d\beta}{\beta} \tag{9}$$

From formula (9), it can be known that it has definite multiple relationship between the ratio of final settlement s_∞ to its micro deformation ds_∞ and the ratio of fitting parameter β to its micro deformation $d\beta$. In other words, if some certain errors are existing in fitting parameter β, it would lead to the prediction settlement s_∞ also exists some certain errors, and it is also found that there exists a relevant relation between them. On the other hand, if micro deformation of final settlement ds_∞ is viewed as the error caused by the regression analysis method, it can be known that the relative error of final settlement is ds_∞ / s_∞ and the relative error of fitting parameter is $d\beta / \beta$. There also exists a definite multiple relationship between these relative errors, and the error amplification coefficient of fitting parameters $k_{\text{Hyperbola}\infty}$ can be defined in following formula (10):

$$k_{\text{Hyperbola}\infty} = 1/(\beta s_\infty) \tag{10}$$

Similarly, with organization, the relative error of the residual settlement after an extended period of time away can be calculated by formula (11):

$$\frac{ds_{\infty}}{s_{\infty}-s_{t_0}}=-\frac{1}{\beta(s_{\infty}-s_{t_0})}\cdot\frac{d\beta}{\beta} \tag{11}$$

As for residual settlement, the error amplification coefficient of fitting parameters $k_{Hyperbola\ Surplus}$ can be calculated by formula (12):

$$k_{Hyperbola\ Surplus}=\frac{1}{\beta(s_{\infty}-s_{t_0})} \tag{12}$$

3.2. Xingye method

As for Xingye method, when immediate settlement s_0 caused by instantaneous loading is known, the differential expression of final settlement can be obtained by taking the derivative with respect to the final settlement in the formula (5), which can be seen in formula (13):

$$ds_{\infty}=dA \tag{13}$$

According to formula (13), furthermore the simplified differential expressions of final settlement can be seen in formula (14) and formula (15):

$$\frac{ds_{\infty}}{s_{\infty}}=\frac{A}{s_{\infty}}\cdot\frac{dA}{A} \tag{14}$$

$$\frac{ds_{\infty}}{s_{\infty}-s_{t_0}}=\frac{A}{s_{\infty}-s_{t_0}}\cdot\frac{dA}{A} \tag{15}$$

From formula (14) and formula (15), it clearly knows that the error amplification coefficient of fitting parameters for final settlement $k_{\mathrm{Xingye}\infty}$ can calculated by formula (16). And as for residual settlement, the error amplification coefficient of fitting parameters $k_{\mathrm{Xingye\ Surplus}}$ can be calculated by formula (17):

$$k_{Xingye\infty}=A/s_{\infty} \tag{16}$$

$$k_{\mathrm{Xingye\ Surplus}}=A/(s_{\infty}-s_{t_0}) \tag{17}$$

3.3. Asaoka method

As for Asaoka method, the differential expression of final settlement can be obtained by taking the derivative with respect to the final settlement in the formula (7), which can be seen in formula (18):

$$dS_{\infty}=\frac{d\beta_0}{1-\beta_1}+\frac{\beta_0}{1-\beta_1}\cdot\frac{d\beta_1}{1-\beta_1} \tag{18}$$

According to formula (7) and formula (18), furthermore the simplified differential expressions of final settlement can be seen as following formulas (19) ~ (21):

$$dS_\infty = S_\infty \cdot (\frac{d\beta_0}{\beta_0} + \frac{\beta_1}{1-\beta_1} \cdot \frac{d\beta_1}{\beta_1}) \tag{19}$$

$$\frac{dS_\infty}{S_\infty} = \frac{d\beta_0}{\beta_0} + \frac{\beta_1}{1-\beta_1} \cdot \frac{d\beta_1}{\beta_1} \tag{20}$$

$$\frac{dS_\infty}{S_\infty - S_{t_0}} = \frac{S_\infty}{S_\infty - S_{t_0}} \cdot (\frac{d\beta_0}{\beta_0} + \frac{\beta_1}{1-\beta_1} \cdot \frac{d\beta_1}{\beta_1}) \tag{21}$$

It can be known in formula (20), the error amplification coefficient of fitting parameters for final settlement $k_{\mathrm{Asaoka}\infty}$ can be calculated by formula (22):

$$k_{\mathrm{Asaoka}\infty} = \frac{\beta_1}{1-\beta_1} \tag{22}$$

Similarly, according to formula (21), there exists a definite multiple relationship between the relative error of residual settlement and fitting parameters β_0 and β_1, and the error amplification coefficient of fitting parameters $k_{\mathrm{Asaoka}\ Surplus\ \beta_0}$ and $k_{\mathrm{Asaoka}\ Surplus\ \beta_1}$ can be defined in following formulas (23)~(24):

$$k_{\mathrm{Asaoka}\ Surplus\ \beta_0} = \frac{s_\infty}{s_\infty - s_{t_0}} \tag{23}$$

$$k_{\mathrm{Asaoka}\ Surplus\ \beta_1} = \frac{s_\infty}{s_\infty - s_{t_0}} \cdot \frac{\beta_1}{1-\beta_1} \tag{24}$$

Finally, through the above analysis and discussion, in applying hyperbola method, Xingye method and Asaoka method to predict the settlement of subgrade, it is found that that stability of predicted results is influenced by the error amplification coefficient of fitting parameters in regression formulas.

4. Stability analysis of curve fitting prediction methods

4.1. Fitting parameters of curve fitting prediction methods

As an example, the monitoring data in the midpoint section of K33+240 of the Wubu-zizhou highway was analyzed. Considering the change of subgrade settlement during construction period, which is significantly interfered by the external world and the settlement variation law is not obvious, so subgrade settlement monitoring data after the construction period is chosen as the sample data to be analyzed[5,6]. When the hyperbola method and Xingye method are used to curve fitting, initial time is set different respectively; but for Asaoka method, curve-fitting through setting up different initial time and time spacing. The prediction of subgrade settlement and the error amplification coefficient of fitting parameters can be seen in Table1~Table3.

Table 1. The subgrade settlement predictions of hyperbola method

Days after the end of the embankment construction (d)	Fitting parameter and correlation coefficient			The settlement prediction		The error amplification coefficient of fitting parameters	
	α	β	R^2	Final settlement (mm)	Residual settlement (mm)	$k_{\mathrm{Hyperbola}\infty}$	$k_{Hyperbola\ Surplus}$
3	0.00164	0.07824	0.994	236.8	75.8	2.577	0.169
6	0.00205	0.13731	0.993	229.6	63.6	2.125	0.115
9	0.00213	0.15183	0.993	226.5	59.5	2.075	0.111
15	0.00220	0.16519	0.991	224.7	54.7	2.024	0.111
30	0.00228	0.17336	0.991	223.6	52.6	1.962	0.110

Table 2. The subgrade settlement predictions of Xingye method

Days after the end of the embankment construction (d)	Fitting parameter and correlation coefficient		The settlement prediction		The error amplification coefficient of fitting parameters	
	A	R^2	Final settlement (mm)	Residual settlement (mm)	$k_{\mathrm{Xingye}\infty}$	$k_{\mathrm{Xingye\ Surplus}}$
3	161.26	0.996	241.6	80.3	0.667	2.008
6	166.54	0.993	240.3	74.3	0.693	2.241
9	167.20	0.994	236.9	69.9	0.706	2.392
15	170.43	0.898	237.4	67.4	0.718	2.529
30	171.33	0.992	233.6	62.6	0.733	2.737

Table 3. The subgrade settlement predictions of Asaoka method

Days after the end of the embankment construction (d)	Time spacing(d)	Fitting parameter and correlation coefficient			The settlement prediction		The error amplification coefficient of fitting parameters		
		β_0	β_1	R^2	Final settlement (mm)	Residual settlement (mm)	$k_{\mathrm{Asaoka}\infty}$	$k_{\mathrm{Asaoka}\ Surplus\ \beta_0}$	$k_{\mathrm{Asaoka}\ Surplus\ \beta_1}$
3	3	126.8	0.9346	0.998	226.2	65.2	15.291	3.469	53.004
6	3	106.9	0.9687	0.996	206.8	40.8	31.949	5.069	161.949
6	6	269.7	0.8547	0.992	238.9	71.9	6.882	3.323	22.869
15	3	280.9	0.8136	0.995	231.6	61.6	5.345	3.760	20.097
15	6	156.7	0.9036	0.993	200.9	29.9	10.373	6.719	69.696

Based on the data in Table 1~Table 3, the following conclusions can be drawn.

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The range of the error amplification coefficient of fitting parameters based on hyperbola method $k_{\text{Hyperbola}\infty}$ is 5.345~31.949 and $k_{Hyperbola\ Surplus}$ is 0.110~0.169; as for Xingye method, its range of the error amplification coefficient of fitting parameters $k_{\text{Xingye}\infty}$ is 0.667~0.733 and $k_{\text{Xingye Surplus}}$ is 2.008~2.737. The results of this study illustrate that predicted results by hyperbola method and Xingye method both show a good stability. When initial time are set different based on the hyperbola method and Xingye method, the error amplification coefficient of fitting parameters is small and the predicted final settlement is very close to the predicted residual settlement, which meant that the error amplification coefficient of fitting parameters has a relatively small impact on predict results for hyperbola method and Xingye method.

In applying Asaoka method to predict the subgrade settlement, it is found that the range of the error amplification coefficient of fitting parameters $k_{\text{Asaoka}\infty}$ is 5.345~31.949, $k_{\text{Asaoka}\ Surplus\ \beta_0}$ is 3.323~6.719 and $k_{\text{Asaoka}\ Surplus\ \beta_1}$ is 20.097~161.949, depending upon the chosen initial time and time spacing. Obviously knowing that the error amplification coefficient of fitting parameters is large and the predicted final settlement has relatively large difference to the predicted residual settlement, which meant the error amplification coefficient of fitting parameters has a relatively large impact on predict results and the predicted result is also unstable for Asaoka method. Therefore, it is necessary to adopt other prediction methods to further proof the results when Asaoka method is used for predicting subgrade settlement.

4.2. Analysis of stability influence factors in predicting settlement based on Asaoka method

From the calculation steps of Asaoka method, it can be seen that the key of this method is the proper time spacing Δt selection [7]. Because different time spacing Δt would lead to different calculated settlement, if Δt is smaller, the volatility of fitting points must be much large and the related coefficient of straight line must be small; on the contrary, it Δt is much large, it would lead to S_i points are too little to easily determine that whether the subgrade enters secondary consolidation period. In general, the reasonable value of the time spacing Δt is 30~100d [8]. In order to select subgrade settlement with a good related coefficient as the final settlement, it is better to take some different time spacing Δt to calculate the corresponding final settlement in the actual process for inference.

In this paper, the settlement monitoring data in the midpoint section of K143+530 of Hebei coastal highway on soft soil ground is taken as an example to illustrate the calculate process. In this engineering, 25d,50d and 75d are selected respectively as the time spacing, and the corresponding settlements are selected based on the monitoring data. According to calculation steps of Asaoka method, $S_{tj}, S_{tj\ 1}$ relation graphs can be drawn and the corresponding fitting lines as shown in Figure 1~Figure 3.

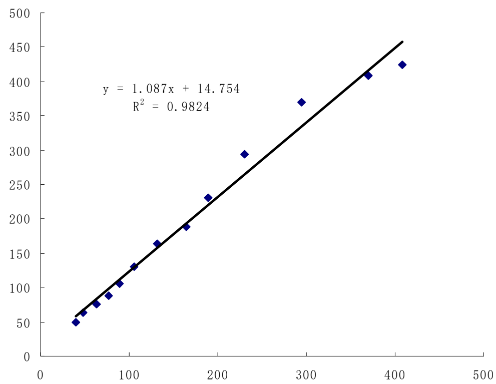

Figure 1. $S_{tj}, S_{tj\ 1}$ relation graph (Δt=25)

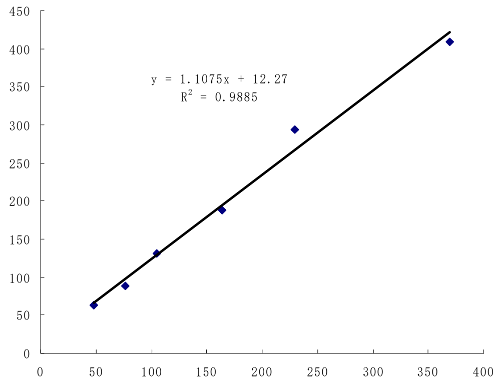

Figure 2. $S_{tj}, S_{tj\ 1}$ relation graph (Δt =50)

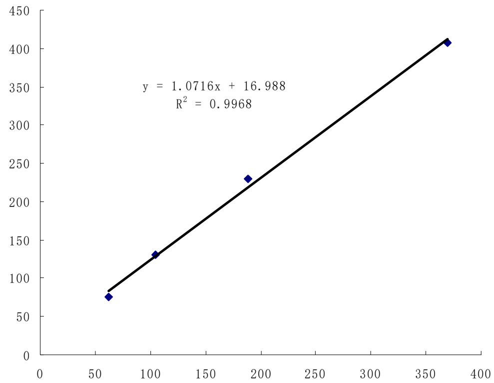

Figure 3. $S_{tj}, S_{tj\ 1}$ relation graph (Δt =75)

According to fitting results, the predicted results of the final settlement can be shown in Table 4, and calculation results of maximum related coefficient are chosen as the predicted final settlement in the midpoint scection of K143+530 on soft soil ground. As seen from Table 4, the predicted results obtained from Asaoka method are greatly affected by randomness of sample points, and the predicted settlement varies in the results when taking different values of time spacing Δt, which suggests that predicted results by Asaoka method show instability.

Table 4. The predicted results of final settlement with different time spacing

Calculation times	Time spacing(d)	Calculation total settlement(mm)	The correlation coefficient	Final settlement(cm)
3	25	169.59	0.9824	237.26
	50	114.14	0.9885	
	75	237.26	0.9968	

5. Conclusion

At present, curve fitting methods are widely used in predicting soft ground settlement, especially for hyperbola method, Xingye method and Asaoka method, which apply more widely than other methods. In this paper, the stability of hyperbola method, Xingye method and Asaoka method is discussed, the main conclusions are as follows:

1 The stability of predicted results by curve fitting methods is closely related to fitting parameters in the regression formula, and the stability of forecasting methods is dependent on the error amplification coefficient of fitting parameters.

2 In predicting the subgrade settlement based on hyperbola method and Xingye method, the error amplification coefficient of fitting parameters is small and the predicted results show good stability when setting different initial time.

3 In predicting the subgrade settlement based on Asaoka method, the error amplification coefficient of fitting parameters is large and the predicted results also show instability when setting different initial time and time spacing. In addition, the predicted results obtained from Asaoka method are greatly affected by randomness of sample points.

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