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Bayes Decision Model for Maintenance Resource Demand

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Abstract. In this paper, combining with the actual situation of equipment damage detection information, a Bayesian decision model for maintenance resource demand is proposed. First, the decision loss is divided into two parts by analysis: decision error loss and delay decision loss, and then use the Bayesian risk function to express the total decision loss. Finally, the decision-making loss of the maintenance resource allocation is minimized to obtain the best decision time and the optimal maintenance resource demand.

Keywords: Maintenance resource; Bayes Decision; Bayesian risk function.

1. Introduction

When making decisions on the supply of warfare equipment maintenance resources, it is necessary not only to consider the equipment maintenance resource requirements determined by equipment damage analysis, but also to take into account the errors existing in battlefield equipment damage detection. The situation on the battlefield is complex. When the equipment is tested, data errors occur due to human error or unstable performance of the testing equipment. Therefore, it is necessary to establish a predictive model of maintenance resource demand with high accuracy. This chapter considers the above issues, analyzes the influencing factors for the determination of maintenance resource demand, establishes a Bayesian [1] decision model for the consumption of maintenance resources, and simulates and analyzes the sensitivity of each parameter in the model to ensure that the model reflects real wartime. The process of determining the supply of equipment maintenance resources lays the foundation for the subsequent allocation of maintenance resources.

2. Bayesian sequential decision model

2.1. Model description

According to the above analysis, a Bayesian decision model is established. The structure of the Bayesian decision model is shown in Figure 1.

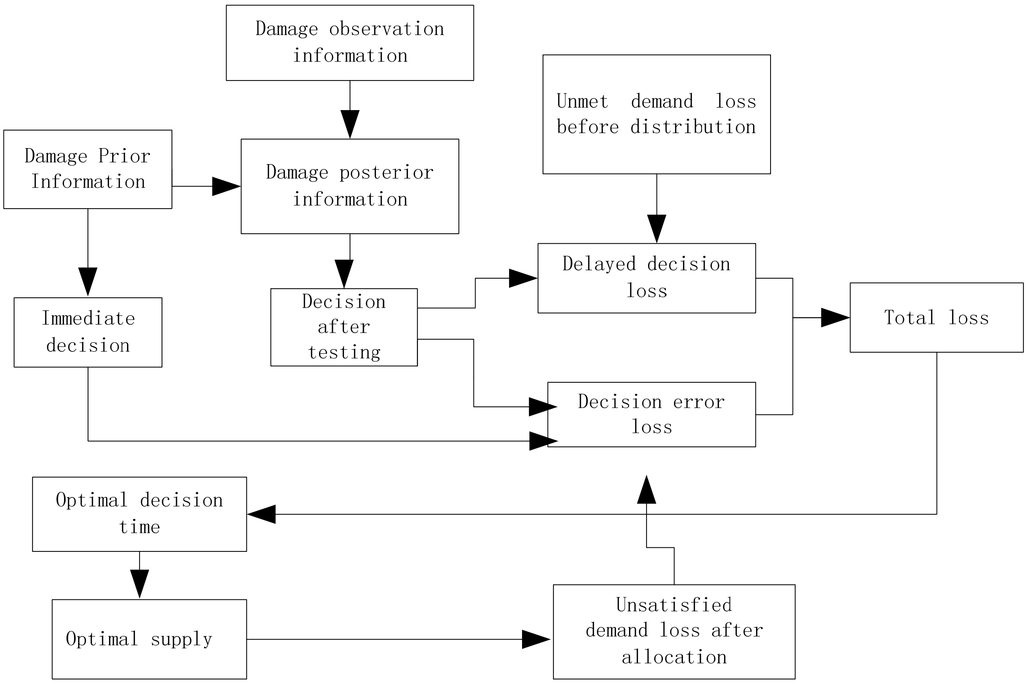

Figure 1. Bayesian decision procedure

Equipment damage prior information reflects the damage of equipment components in the initial stage of damage. At this time, the equipment command management department needs to decide whether to make repair decisions for the damage, that is, maintenance resource allocation. At this time, due to the timely decision, the allocation does not produce delayed decision loss, however, at this point, the decision may be due to the detection error and other issues caused by decision errors. As time goes on, equipment status information is constantly updated and the damage data is getting more and more accurate. Decisions are made at this time to reduce the decision error, but a delay error of maintenance resource allocation decision is generated. In the case of continuous updating of information, the last inspection Information is a priori information for the next inspection. In the process of maintenance decision-making, both the decision error loss and the delay decision loss have the opposite effect, and the objective function is the smallest total loss. The optimum maintenance resource demand can be obtained through calculation.

2.2. Model assumption

(1) Assume that the prior distribution of equipment battlefield damage is known

Assume that the equipment has been tested for live ammunition, and data on the actual damage of the equipment has been obtained through experiments to form a database. And when equipment battle damage occurs after combat, the equipment command department will receive a rough inspection of the equipment to obtain a general damage situation, and the combat department can estimate the equipment damage through the combat situation, and thus get the equipment damage prior distribution. That is, the known damage ratio of the equipment γ is assumed and its prior density $\pi(\gamma)$ is assumed to follow the normal distribution $N(\mu,\tau^2)$.

(2) Damage information can be continuously revised after equipment damage

Modern weapons and equipment testing equipment can collect information on equipment component damage (due to the impact of the operational environment, equipment damage detection may occur errors), through the maintenance and protection information transmission system uploaded to the equipment management department, to provide the basis for equipment maintenance decision, thus making Equipment damage information is constantly updated and revised. Suppose that

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$X^q = (X_1, X_2, \cdots, X_q)$ is a sequential sample of the proportion of damaged equipment detected by the testing equipment. The condition density of the sample is $f(x|\gamma)$, and the distribution of compliance is $N(\gamma, \sigma^2)$, $x \in \Omega$. Assume that the distribution of the equipment damage ratio after detection of the sample value x is $\pi(\gamma, x)$.

(3) Demand for damage resources by repair equipment can be expressed as a function of damage ratio

Assume that the number of equipment in a combat squad is E , and the damage ratio of equipment is γ : the number of damaged equipment in a combat squad is $E\gamma$, and the demand of certain damaged equipment for a type of maintenance resource is e , so it can be concluded that the combat squad maintains certain types of maintenance is $y = E\gamma e$. the prior distribution of maintenance resource demand is subject to a normal distribution $N(E\gamma e, \gamma^2 E^2 \sigma^2)$.

4) Equipment maintenance resource allocation decision total loss including delay decision loss and decision error loss

For equipment, each damage detection and evaluation take time. When a maintenance decision is made after the new damage information is detected, delays in the allocation of maintenance resources will occur due to the time constraints, resulting in delayed allocation of such maintenance resources. The loss of equipment is called delayed decision loss L_d . Delayed decision loss is the loss due to time delay before the decision on the allocation of equipment maintenance resources, assuming that the delay decision loss is expressed by the following formula:

$$L_d = \alpha(E\gamma e - 0)^2 = \alpha(E\gamma e)^2 \tag{1}$$

Where α is the delay decision loss adjustment factor and is positive.

When the equipment repair rate and the maintenance resource demand information are indefinite, the maintenance resource allocation quantity cannot meet the equipment maintenance requirement and it will result in a loss. Assuming that the loss is the decision error loss, denoted by L_f , the loss represents the assigned maintenance resources and demand. Poor maintenance resources. Similarly, assume that the decision error is the square of the unsatisfied supply, assuming that the decision error is expressed by the following formula:

$$L_f(\gamma, H) = \beta(E\gamma e - H)^2 \tag{2}$$

Where β denotes the decision error loss adjustment factor and is positive.

For equipment testing, assuming that the detection interval is the same, the maximum number of detections is Q . After the equipment damage detection is performed q times, the total decision loss is the sum of the decision error loss and the delay decision loss. The total decision loss can be expressed by the following formula:

$$L(\gamma, H, q) = L_f(\gamma, H) + \frac{q}{Q} L_d \tag{3}$$

In the formula, $\frac{q}{Q}L_d$ denotes the delay decision loss caused by time accumulation before the decision, which accumulates with time. $L_f(\gamma,H)$ represents the decision error loss and does not accumulate over time.

2.3. Symbol definition

γ : The proportion of equipment damage in the combat unit;
E : the number of equipment in the combat unit;
μ : A priori mean of damage ratio;
τ^2 : The prior variance of the damage ratio;
σ^2 : The variance of the test sample of the damage ratio;
e : Demand for certain types of resources by unit damage equipment;
q : The number of equipment damage detections;
Q : The maximum number of detections of equipment damage;
$\overline{T}^*$: optimal decision time;
T : Actual best decision time;
q^* : The optimal number of decision-making on the allocation of maintenance resources;
H : The amount of maintenance resources supplied to the unit where the damaged equipment is located;
y : Demand for damaged equipment for maintenance resources;
L_f : decision error loss;
L_d : Delayed decision loss;
L_d : Bayesian decision risk;
ρ : postponed expected loss;

2.4. Symbol definition

The Bayesian risk function can express the total decision loss expectation well. Through the Bayesian risk definition, the Bayesian risk of the decision loss can be:

$$\begin{aligned} r^q(\pi) &= E^\pi E_\gamma^{X^q}[L(\gamma,\delta_q^\pi(X^q),q)] \\ &= E^\pi E_\gamma^{X^q}[L(\gamma,\delta_q^\pi(X^q))]+\frac{q}{Q}L_d \end{aligned} \tag{4}$$

In the formula, $\delta_q^\pi(X^q)$ is the Bayesian decision rule for delayed decision loss $L(\gamma,H,q)$, which indicates the maintenance resource requirement after giving sequential detection sample X^p of equipment damage ratio. In this regard, assuming $r(\pi,\delta_q^\pi)=E^\pi E_\gamma^{X^q}[L(\gamma,\delta_q^\pi(X^q))]$, this represents the expectation of the decision error loss; $DL=\frac{q}{Q}L_d$ represents the expectation of the delay decision loss, so formula (4) can be expressed by the following formula:

$$\rho(\pi(\gamma|x),H,q)=\int_\Theta L(\gamma,H,q)\pi(\gamma|x)\,\mathrm{d}\gamma \tag{5}$$

From the formula, it can be seen that the assumed loss contains two parts of the decision error loss expectation and the delay decision loss expectation loss.

So, the expectation for posterior loss can be defined as:

$$\rho(\pi(\gamma|x),H,q)=\int_{\Theta}L(\gamma,H,q)\pi(\gamma|x)\,\mathrm{d}\gamma \tag{6}$$

2.5. Model solving

The Bayesian decision model established for the above problems needs to determine the optimal maintenance resource allocation decision time and the optimal allocation of maintenance resources. Even if the Bayesian risk function $r^q(\pi)$ obtains the minimum value, the model solution objective is:

$$\min_{q,H} r^q(\pi) \tag{7}$$

The solution to the above problem [2] is to use the Bayesian formula to use the damage ratio test sample to correct the prior distribution, and then obtain the posterior distribution of the equipment damage ratio and the number of maintenance resource requirements and obtain the total loss through decision. The Bayesian decision rule is derived, and the Bayesian decision rule is used to derive the Bayesian risk expression formula. The Bayesian risk expression is derived to obtain the best decision time and the optimal maintenance resource demand.

The solution steps are as follows:

Step 1: First, find the Bayesian posterior distribution $\pi(\gamma|x)$ of the equipment damage ratio γ and the Bayesian posterior distribution $\pi(H|x)$ of the maintenance resource demand. The theorem is given in [77]:

Theorem 3.1 : Assume that the Bayesian prior distribution of the random variable γ is such that $\pi(\gamma|x)$ obeys the normal distribution $\pi(H|x)$, and that μ and τ are both known quantities. For the equipment damage detection sample X , the conditional density is $f(x|\gamma)$ obeys the normal distribution $N(\gamma,\sigma^2)$, and γ Unknown, σ^2 is known. According to the Bayesian formula, after the given sequential sample X^q , the Bayesian posterior distribution γ of $\pi(\gamma|x)$ obeys the normal distribution $N(\mu_q(\bar{x}_q),\phi_q)$, and in the normal distribution, $\mu_q(\bar{x}_q)=\frac{\sigma^2}{\sigma^2+q\tau^2}\mu+\frac{q\tau^2}{\sigma^2+q\tau^2}\bar{x}_q$, $\phi_q=\frac{\sigma^2\tau^2}{\sigma^2+q\tau^2}$, where $\bar{x}_q$ is the sequential sample. The average value of the detected value of X^q .

Step 2: Calculate Bayesian Decision Rule by Calculating Bayesian Risk Function

Theorem 3.2: There are two methods for calculating Bayesian decision rule, which are Bayesian risk function minimization and Bayesian posterior expectation loss minimization. Since the Bayesian posterior expectation loss solution method is relatively simple, the posterior expectation is used to solve the Bayesian decision rule. Substituting $L(\gamma,H,q)$ into equation (6) gives the formula:

$$\rho(\pi(\gamma|x),H,q)=\int_{\Theta}[(\alpha E\gamma e-H)^2+\frac{q}{Q}\beta(Ee\mu)^2]\pi(\gamma|x)\,\mathrm{d}\gamma$$

$$=\alpha E^2e^2\int_{\Theta}\gamma^2\pi(\gamma|x)\,\mathrm{d}\gamma-2\alpha eEH\int_{\Theta}\gamma\pi(\gamma|x)\,\mathrm{d}\gamma+\alpha H^2+\beta\frac{q}{Q}(eE\mu)^2$$

Deriving $\rho(\pi(\gamma|x),H,q)$ from H and letting $\frac{\mathrm{d}\rho}{\mathrm{d}H}=0$ can push:

$-2\gamma E\int_{\Theta}\gamma\pi(\gamma|x)\,\mathrm{d}\gamma+2H=0$

So, you can find the Bayesian decision rule:

$$\delta_q^{\pi}(X^q)=E^{\pi(\gamma|x)}(\gamma)=eE\mu_q(\bar{x}_q) \tag{8}$$

Therefore, the Bayesian decision rule is the posterior average of maintenance resource requirements.

Step 3: Bring the Bayesian decision rule into the Bayesian risk function. By minimizing the Bayesian risk function, the best decision time and the best number of maintenance resource allocations can be obtained.

Substituting $\delta_q^{\pi}(X^q)=eE\mu_q(\bar{x}_q)$ obtained in the previous step into equation (4), the Bayesian risk function for maintenance resource allocation is:

$$\begin{aligned} r^q(\pi)&=\int_{\Theta}\int_{\Omega}\alpha[eE\gamma-eE\mu_q(\bar{x}_q)]^2 f(x|\gamma)\,\mathrm{d}x\pi(\gamma)\,\mathrm{d}\gamma+\frac{q}{Q}\beta(eE\mu)^2\\ &=\alpha e^2E^2\frac{\sigma^2\tau^2}{\sigma^2+q\tau^2}+\frac{q}{Q}\beta(eE\mu)^2\\ &=\alpha e^2E^2\int_{\Theta}\int_{\Omega}[\gamma-\mu_q(\bar{x}_q)]^2\pi(\gamma\mid x)\,\mathrm{d}\gamma m(x)\,\mathrm{d}x+\frac{q}{Q}\beta(eE\mu)^2 \end{aligned} \tag{9}$$

The next step is to find the best decision time. Since q is an integer in the problem, for the sake of calculation, assuming that q is a continuous variable and $r^q(\pi)$ is a derivative of q, we can get the formula:

$$\frac{\mathrm{d}r^q(\pi)}{\mathrm{d}q}=\frac{-\alpha e^2E^2\sigma^2\tau^4}{(\sigma^2+q\tau^2)^2}+\frac{\beta}{Q}(eE\mu)^2 \tag{10}$$

Let the formula $\frac{\mathrm{d}r^q(\pi)}{\mathrm{d}q}=0$, because the parameters in the formula are all positive, so we can get the best detection times:

$$q^*=\frac{\sqrt{\frac{\alpha\sigma^2\tau^4Q}{\beta\mu^2}}-\sigma^2}{\tau^2}=\frac{\sigma\tau^2\sqrt{\alpha\beta Q}-\mu\beta\sigma^2}{\mu\beta\tau^2} \tag{11}$$

The ultimate goal of this problem is to obtain the best decision time, so we need to convert the optimal number of decision times to the actual best decision time, that is $\overline{T}^*$, given the latest decision times, that is the latest decision time, and the detection time is assumed to be The average allocation, so the best decision time needed can be obtained by the product of the optimal number of detections q_{int}^* and the interval of each detection. So, the best decision time is:

$$T^* = T \cdot q_{\text{int}}^* \cdot \frac{1}{Q} = [T \cdot \frac{\sigma\tau^2\sqrt{\alpha\beta Q} - \mu\beta\sigma^2}{\mu\beta Q\tau^2}] \tag{12}$$

When $0 < q^* < Q$, the optimal number of detections q^* is substituted into equation (3-8), that is, the formula for the decision rule, the optimal maintenance resource requirements can be obtained, that is:

$$H^*(q_{\text{int}}^*, X^q) = eE\mu_q(\bar{x}_q) = \frac{eE\mu\sigma^2 + eE\bar{x}_{q^*}(\sqrt{\frac{\alpha\sigma^2\tau^2 Q}{\mu\beta}} - \sigma^2)}{\sqrt{\frac{\alpha\sigma^2\tau^2 Q}{\mu\beta}}}$$
$$= \frac{eE\sigma\mu^2\sqrt{\alpha\beta Q} + \alpha eE\bar{x}_{q^*}\tau^2 - \mu eE\sigma\bar{x}_{q^*}\sqrt{\alpha\beta Q}}{\alpha Q\tau^2} \tag{13}$$

3. Example analyses

Assume that prior to combat, according to expert predictions and equipment damage simulations, a priori distribution $N(0.3, 0.3^2)$ of equipment damage ratio γ is obtained, namely $\mu = 0.3$, $\tau = 0.3$. Assume that equipment damage detection is performed by means of personnel inspection and detection equipment detection so as to obtain detection data of equipment damage ratio. Assume that the test sample obeys the normal distribution $N(\gamma, \sigma^2)$, γ is unknown, and $\sigma = 0.25$. Due to equipment working status and physical fitness, the detected damaged information is updated every 0.3 hours, and uploaded to the equipment command department through the data information system. The equipment command department makes maintenance resource allocation decisions, and due to the emergency, the decision must be 4 Made within hours. The purpose is to minimize the total decision loss expectation of maintenance resource allocation, that is, to minimize the sum of delay decision loss and decision error loss. A certain combat equipment group has 60 vehicles, that is, $E = 60$, and the unit of damaged equipment requires 5 units of a certain type of maintenance resources, that is, $e = 5$. In order to reflect the importance of delay decision loss and decision error loss, set $\alpha = 4$, $\beta = 1$.

According to formula (11), find:

$$q^* = \frac{\sigma\tau^2\sqrt{\alpha\beta Q} - \mu\beta\sigma^2}{\mu\beta\tau^2} = \frac{0.25\times0.3^2\times\sqrt{4\times1\times12} - 0.3\times1\times0.25^2}{0.3\times1\times0.3^2} = 5.0791$$

The integer number of optimal decision times is 5, which can be calculated according to formula (12):

$$T^* = q_{\text{int}}^* \times \frac{1}{3} \times 60 = \frac{\sigma\tau^2\sqrt{\alpha\beta Q} - \mu\beta\sigma^2}{\mu\beta\tau^2} \times 20 = \frac{0.25\times0.3^2\times\sqrt{4\times1\times12} - 0.3\times1\times0.25^2}{0.3\times1\times0.3^2\times3}$$
$$= 101.5812$$

Through rounding, and converted into actual time is about 100 minutes, that is, the 5th information update must be made to make maintenance decisions. At this time, the maintenance resource allocation decision is optimal, and the resulting maintenance resource allocation decision loss is minimal.

Maintenance resource demand solution:

First, we need to generate a random number that obeys a normal distribution with a mean of 0.3 and a variance of 0.25. Use the normrnd function in MATLABR2014a and select the first 12 samples

X^{12} = (0.4344, 0.7585, 0.5155, 0.3797, 0.1916, 0.3857, 0.9924, 1.0587, 0.4814, 0.2842, 0.4787, 0.2488). It can be obtained that when the optimal detection number is 5, the average value of the equipment damage detection sample is $\overline{x}_5 = 0.4559$, and $\mu_5(\overline{x}_5) = 0.4146$ can be obtained. At this time, according to formula (13), namely:

$$H^*(5, X^5) = eE\frac{\mu\sigma^2 + q_{\text{int}}^* \overline{x}_{q_{\text{int}}^*}\tau^2}{\sigma^2 + q_{\text{int}}^*\tau^2} = \frac{5\times 60\times(0.3\times 0.25^2 + 3\times 0.4559\times 0.3^2)}{0.25^2 + 3\times 0.3^2} = 127.9$$

4. Relationship between parameters simulation

① Use MATLABR2014a to simulate and obtain the relationship between decision error loss expectation $r(\pi, \delta_q^\pi)$, delay decision loss DL, total decision loss expectation $r^q(\pi)$, and equipment damage detection times q for maintenance resource allocation, as shown in Figure 2.

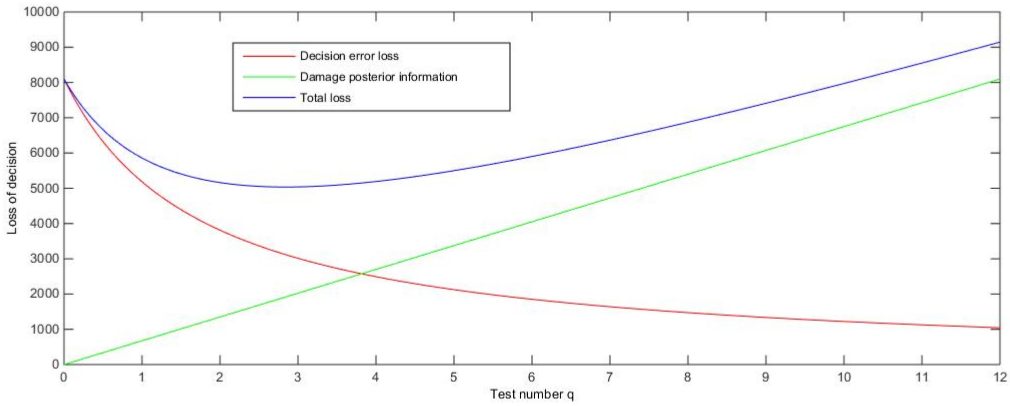

Figure 2. Simulation of the relationship between maintenance resource allocation decision loss and detection time

It can be seen that the decision error loss expectation decreases as the number of detections increases, but the curve tends to be flat, indicating that the marginal cost of the damage detection decreases; the linear relationship between the delay decision loss and the number of damage detections, that is, the damage detection the marginal cost is constant; the total decision loss expectation decreases first and then increases again. When the number of damage detections reaches about X times, the total decision loss is expected to be minimal, so the optimal number of equipment damage detections is 5 times. The actual best decision time is 100 minutes after equipment damage.

②Maintenance resource demand simulation

Let $\overline{x}_{q^*}$ be the range of [0,1]. The settings of other parameters are the same as the previous parameters. Use MATLABR2014a to simulate and get the relationship between the prior average of the damage ratio, the mean value of the damage detection sample, and the expected total loss, as shown in Figure 3 shows. As can be seen from the figure, the maintenance resource demand value based on the posterior mean is based on the prior mean and the detection mean, indicating that the repair resource demand value based on the posterior is corrected by prior information and test samples, and the data is more accurate. Make the decision more reasonable.

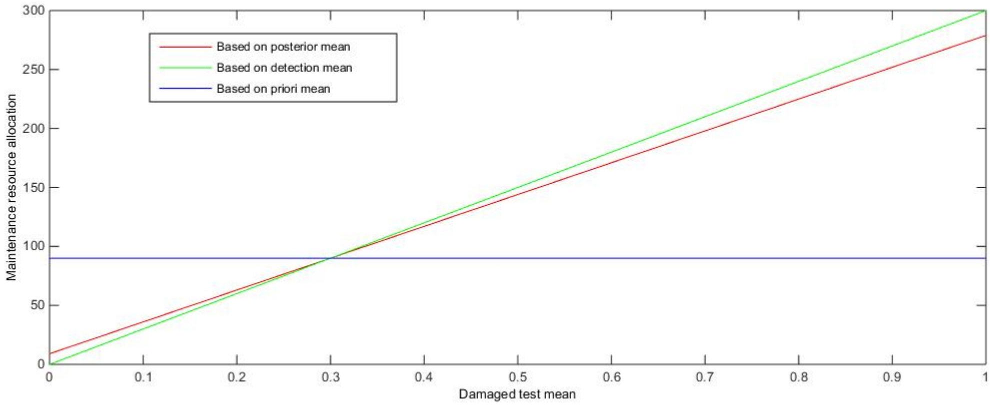

Figure 3. Simulation of maintenance resource demand under different information types

5. Conclusion

This article assumes that the equipment damage information is continuously updated and considers two types of losses, namely, delay decision loss and decision error loss, establish a Bayesian decision model for maintenance resource demand, and determine the demand for maintenance resources. The numerical solution and simulation analysis using MATLAB software verified the a priori average value of the equipment damage ratio, the prior standard deviation of the damage ratio, the standard deviation of the damage ratio detection, the maximum number of detections and other parameters and the optimal number of damage detections. The relationship between total decision loss expectations. The conclusions are as follows:

(1) Delay decision loss and decision error loss constitute total losses. However, there is a contradiction between the two. When the marginal return of detection and delay decision is equal to the marginal cost, the total decision loss is the least, and the best detection times can be obtained.

(2) The greater the maximum number of detections, that is, the higher the detection frequency, the less the loss will be. Therefore, as far as conditions permit, the detection frequency of the damage should be increased as much as possible, and even continuous detection can be achieved.

(3) The smaller the standard deviation of the detection of the damage ratio, the smaller the actual optimal number of detections and the less the total loss. Therefore, equipment damage detection should improve the accuracy of testing equipment.

(4) The greater the a priori mean value of the damage ratio, the greater the a priori standard deviation and the greater the total loss expectation. Therefore, in order to cope with this situation, it is necessary to increase the number of actual equipment tests and obtain more damage data and establish an equipment damage database, which is more conducive to the future equipment wartime maintenance work.

(5) The demand for maintenance resources worthy of the equipment's damage ratio posterior test is based on the maintenance resource demand obtained based on the a priori mean and the mean value of the repair resource obtained from the average damage detection. After the correction of the posterior mean value, the obtained data is more accurate, thus making the decision more reasonable.

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