

# An Active Detection Method for Buried Target Based on LFMCW Waveform

Lei Yue \*

Kunming Shipbuilding Equipment Research and Test Center, Kunming 650051,  
Yunnan, China

\*Corresponding author e-mail: yuelei750@foxmail.com

**Abstract.** Underwater clutter is the main interference for active sonar to detect buried target. Anti-clutter waveform designing and processing method play a significant role in clutter suppression. This paper proposes a processing framework for detecting buried target. The linear-frequency-modulated continuous wave (LFMCW) signal is selected as the detecting waveform, and the properties of time-frequency distribution, spectrum, periodic fractional Fourier transform (PFRFT) and wideband ambiguity function (WAF) for LFMCW are analysed, then the waveform parameters is designed to well detect buried target. Furthermore, on the basis of PFRFT and WAF, this paper proposes a processing method for LFMCW. Both the theoretical analysis and simulation results validate the proposed method. Sine-square weighted LFMCW outperforms the rectangular weighted one.

## 1. Introduction

Underwater clutter is the main interference for active sonar to detect buried target. The main methods for clutter suppression are anti-clutter waveform designing and signal processing. There are two ways to design anti-clutter waveform: one is deriving the optimum anti-clutter waveform by combining the prior knowledge of the target, the scattering distribution of the scatters and the autocorrelation function of the signal; another one is designing anti-clutter waveform according to the application environment and the ambiguity function matching principle. However, the former method is very difficult in mathematical derivation, whose results are not much, but the latter one is convenient to calculate. As a low probability of intercept signal, linear-frequency-modulated continuous wave(LFMCW) has reflected some advantages in detecting target under interference environment. Therefore, LFMCW signal can be selected in detecting buried target, and its waveform parameters can be designed according to the requirements.

In addition, the detection of buried target requires high spatial resolution. Therefore, the hydrophone arrays and transmitted signal should have high angular resolution and high range resolution, respectively. Some effective processing methods should be used to suppress clutter and reduce the effect of multipath propagation, which caused by propagation medium's fluctuation and uniformity. The frequency of the transmitted signal can't be too high, because it is necessary to consider that the sound wave should have some capability of penetrating sediment [1].

This paper is organized as follows. In section 2, preliminary work including the wideband ambiguity function (WAF), clutter model and periodic fractional Fourier transform (PFRFT) algorithm will be



introduced briefly. In section 3, the processing method for detecting buried target is discussed in detail, LFM CW signal is analyzed, and experimental results are provided to validate the performance of the proposed method. Finally, the conclusion will be presented.

## 2. Methodology

This section introduces the theories of wideband ambiguity function, clutter model and periodic fractional Fourier transform briefly.

### 2.1. Wideband Ambiguity Function(WAF)

The ambiguity function is a concept proposed in the study of radar resolution, which is the ability to distinguish two adjacent targets. Ambiguity function is an effective tool for signal analysis and waveform design. At the same time, the mathematical model of the moving point target echo and the least mean square error criterion are used to derive the ambiguity function [2].

Considering broadband model of moving point target echo [3]

$$g(t) = \sqrt{\eta} s[\eta(t - \tau)] \quad (1)$$

where  $\eta = (c - v)/(c + v)$  represents doppler scale parameter.

For a wideband signal, the effect of target velocity must also consider the compression of the signal. The wideband ambiguity function of time delay and doppler scale factor is thus defined as [4]:

$$\chi(\eta, \tau) = \sqrt{\eta} \int_{-\infty}^{+\infty} s(t) s^*[\eta(t - \tau)] dt \quad (2)$$

The quantity of interest is usually the square of the envelope of the matched filter response, and the ambiguity function is thus often defined as

$$\Psi(\eta, \tau) = |\chi(\eta, \tau)|^2 = \left| \sqrt{\eta} \int_{-\infty}^{+\infty} s(t) s^*[\eta(t - \tau)] dt \right|^2 \quad (3)$$

Let  $\eta = 1$ , then (3) becomes

$$\Psi(1, \tau) = \left| \int_{-\infty}^{+\infty} s(t) s^*(t - \tau) dt \right|^2 \quad (4)$$

which is called time delay ambiguity function.

Let  $\tau = 0$ , then (3) becomes

$$\Psi(\eta, 0) = \left| \sqrt{\eta} \int_{-\infty}^{+\infty} s(t) s^*(\eta t) dt \right|^2 \quad (5)$$

which is called velocity ambiguity function.

WAF can be realized by FFT algorithm, shown in (6)

$$\chi(\eta, \tau) \Leftrightarrow U(f) U^*(f/\eta) / \sqrt{\eta} \quad (6)$$

Different waveform's ability to distinguish two targets, including the  $\tau_e$  (distance resolution) and  $\eta_e$  (velocity resolution), is an important indicator for sonar system, which can be obtained by making  $\Psi(1, \tau) = 1/2$  and  $\Psi(\eta, 0) = 1/2$ , respectively.

## 2.2. Clutter Model

Making the following assumptions in order to simplify the calculation and complete the reverberation simulation:

The scatters are randomly distributed at the bottom of the sea.

Considering only the primary scattering of scatters, and it is thought that the sound wave travels along a straight line without considering the multipath effect.

In a sufficiently short pulse width, the angular variation of the scattered and the receiving array caused by the motion of the sonar platform is ignored.

Based on the above assumptions, the clutter model is given by [5], and actually its basis lies in [6].

The geometric description of the backscattering model is shown in Fig. 1. At a given time instant, the platform is assumed to be traveling along the x-axis at speed  $v$ . Scatters is distributed randomly and evenly in the bottom of the sea. The emission signal reaches scatters then backscattering to the receive source superposition forming clutter. Hence, the clutter signal can be expressed as:

$$y(t) = \sum_{n=1}^N A_n s[\eta_n(t - \tau_n)] \quad (7)$$

where  $N$  indicates the number of scatters,  $A_n$ ,  $\eta_n$  and  $\tau_n$  indicate the amplitude, the Doppler scale parameter and time delay of echo from the  $n$ th scatters, respectively.

The scatters' echo amplitude is related to the strength of emission signal, scattering strength, the area of scatters, the beam directivity of transmit and receive and transmission loss, etc. In this study we calculate scattered' echo amplitude ignoring the beam directivity of transmit and receive and transmission loss. The distance from platform to scatters of point  $A$  and  $B$  are  $r_{min}$  and  $r_{max}$ , respectively.  $N \cdot dr / (r_{max} - r_{min})$  scatters are distributed on quite small space  $dr$ , whose shape resemble as semicircles on seabed. The whole area of scatters is

$$S_a = \pi \sqrt{r^2 - H^2} dr / \cos \varphi \quad (8)$$

Hence, the area from platform to scattered of point  $A$  is

$$S_n = \pi \sqrt{r^2 - H^2} (r_{max} - r_{min}) / N \cos \varphi \quad (9)$$

For  $\sqrt{r^2 - H^2} = r \cos \theta$ , the expression turns to

$$S_n = \pi r (r_{max} - r_{min}) / N \quad (10)$$

The scattering strength can be calculated by Lambert theorem

$$S_b = -27 + 10 \lg(\sin^2 \varphi) \quad (11)$$

So the echo's amplitude is

$$A_n = 10^{(S_b/10) \times S_n} \quad (12)$$

The Doppler scale parameter of the echo is

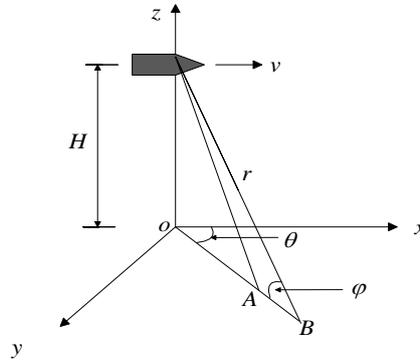
$$\eta_n = (c - v_n \cos \theta_n \cos \varphi_n) / (c + v_n \cos \theta_n \cos \varphi_n) \quad (13)$$

Here  $\theta$  stands for azimuth,  $c$  stands for sound velocity in water.

The time delay of the echo is

$$\tau_n = 2r_n/c \tag{14}$$

Submitting (10), (11), (12) into (7), the clutter signal can be obtained.



**Fig. 1.** Geometric description of clutter model

2.3. Periodic Fractional Fourier Transform(PFRFT)

The Fractional Fourier Transform (FRFT) of continuous function  $x(t)$  is defined as [7]:

$$F_\alpha(u) = \int_{-\infty}^{+\infty} x(t)K_\alpha(t, u)dt \tag{15}$$

The kernel function of FRFT is

$$K_\alpha(t, u) = \begin{cases} A_\alpha(u)e^{-j2\pi ut \csc \alpha + j\pi t^2} & \alpha \neq n\pi \\ \delta(t - u) & \alpha = 2n\pi \\ \delta(t + u) & \alpha = (2n + 1)\pi \end{cases} \tag{16}$$

where  $A_\alpha(u) = \sqrt{1 - j \cot \alpha} e^{j\pi u^2 \cot \alpha}$ ,  $\alpha = p\pi/2$  is rotation angle,  $p$  is fractional order of FRFT.

FRFT can accumulate the energy optimally for LFM signal but not for LFM CW. For LFM CW, FRFT can only process it as multi-component LFM, which against detecting buried target.

To solve the problems above, periodic fractional Fourier transform (PFRFT) is proposed, which defined as:

$$P_{\alpha,\tau,T}(u) = \int_{-\infty}^{+\infty} x(t)K_{\alpha,\tau,T}(t, u)dt \tag{17}$$

The kernel function of PFRFT is

$$K_{\alpha,\tau,T}(t, u) = \begin{cases} A_\alpha(u)e^{-j\pi[2ut \csc \alpha - \text{mod}(t+\tau)^2 \cot \alpha]} & \alpha \neq n\pi \\ \delta(t - u) & \alpha = 2n\pi \\ \delta(t + u) & \alpha = (2n + 1)\pi \end{cases} \tag{18}$$

The relationship between FRFT and PFRFT is

$$\lim_{\tau \rightarrow 0, T \rightarrow \infty} P_{\alpha,\tau,T}[x(t)] = F_\alpha[x(t)] \tag{19}$$

The PFRFT of  $x(t)$  can be realized by calculating FRFT of sub-pulse and compensating phase, see detailed in ref [8], which can be expressed as

$$P_{\alpha,\tau,T}[x(t)] = F_{\alpha} \left[ \sum_n e^{-j\pi u(nT-\tau) \csc \alpha} g_T(t-\tau+nT)x(t) \right] \\ = \sum_n e^{-j\pi u(nT-\tau) \csc \alpha} F_{\alpha}[g_T(t-\tau+nT)x(t)] \quad (20)$$

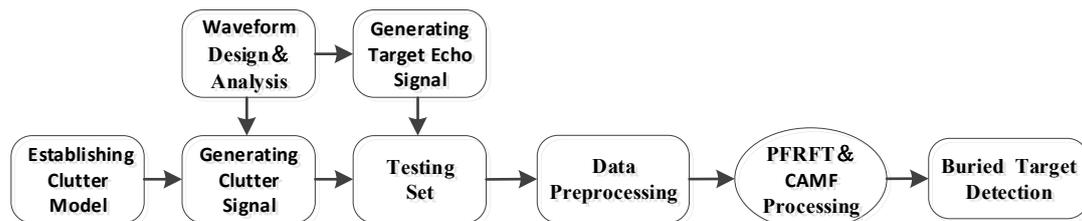
PFRFT can accumulate the energy optimally for LFM CW, which can be used for filtering some clutter out.

### 3. Proposed processing framework for detecting buried target and simulation results analysis

This section proposes the processing framework for detecting buried target based on LFM CW signal, and then the properties of time-frequency distribution, spectrum, PFRFT and WAF for LFM CW are analyzed by simulation. In addition, the ROC for LFM CW signal in clutter is given.

#### 3.1. Proposed processing framework for detecting buried target

To realize the goal of detecting buried target and verify the effect of processing method, the PFRFT and WAF algorithm is combined to obtain a processing method. The detailed process is shown in Fig.2.



**Fig. 2.** Proposed framework for detecting buried target based on LFM CW waveform

#### Step 1: Waveform design, analysis and target echo signal generation

Linear-frequency-modulated continuous wave (LFMCW) signal is selected to detect buried target, and LFM CW signal can be written as

$$s(t) = A(t)e^{j(\varphi+2\pi f_0 t+\pi k \text{mod}(t+\tau,T)^2)} \quad (21)$$

where  $A(t)$  is the amplitude,  $f_0$  (Hz) is the initial frequency,  $k = B/T$  (Hz/sec) is the chirp-rate,  $B$  (Hz) is the bandwidth,  $\tau$  (sec) is an initial time-offset, and  $T$  (sec) is the modulation period. The symbol  $\varphi$  represents the initial phase. We make  $\varphi = 0$  in this study to simplify some procedures but not affect validation effect.

**Table 1.** Waveform designing parameters of LFM CW

Label	Frequency(kHz)	Duration(Ms)	No. of sub-pulses	Amplitude type
parameters	10-20	50	5	rec sin

The waveform designing parameters of LFM CW is shown in Table I. The waveform's frequency is designed range from 10 kHz to 20 kHz to obtain high range resolution and some ability to penetrate seabed. The waveform's duration and number of sub-pulses are designed 50ms and 5 respectively to accumulate sufficiently high echo energies. Generally, the amplitude type of LFM CW is rectangular

window function. The sine-square window function<sup>[9]</sup> is proposed in weighting LFM CW's sub-pulse to achieve better effect of detection.

The properties of time-frequency distribution, spectrum, PFRFT and WAF for LFM CW are analyzed. The time-frequency distribution of LFM CW is realized by spectrogram with hamming window, which is the squared magnitude of the short-time transform.

As shown in (20), LFM CW signal is formed by periodic extension of LFM signal, which can cause comb-like spectrum. This phenomenon can also cause side-lobes emerging in both FRFT and AMF domain, which is of disadvantage for detecting buried target. Nevertheless, it is great beneficial for detecting buried target if accumulating the signal's energy by its time diversity. PFRFT (see details in section 2) can solve this problem well and filter some clutter out. It remains sub-pulse (LFM signal) to be processed after PFRFT processing, and CAMF is used for processing sub-pulse. Thus, the LFM signal's ambiguity function (especially for time delay ambiguity function) should be analyzed. Compared with rectangular window function, the sine-square window function has better energy focusing property.

Considering broadband model of static point target echo as formula (1), the target echo signal can be expressed as

$$g(t) = s(t - \tau_0) \quad (22)$$

The target echo is obtained by setting  $\tau_0$ .

Step 2: Establishing clutter model and generating clutter signal

The clutter model is described in the second section of methodology, and the clutter signal is generated by setting clutter model's parameters.

Step 3: Testing set

The clutter and echo signal are described in previous section, then the target echo under clutter background is obtained by mixing clutter signal and echo signal, and the clutter interference intensity is adjusted by signal to clutter ratio (SCR). White Gaussian noise is added to obtain data according with the actual situation, whose interference intensity is adjusted by clutter to noise ratio (CNR).

Step 4: Data preprocessing

Data preprocessing include two aspects: filtering and smoothing. A band-pass fir filter is used to filter out noise outside the system band for obtaining high processing gain. Time-varying gain control (TVGC) is used to smooth the data for obtaining stationary processing results.

Step 5: PFRFT and CAMF processing

PFRFT is used for transforming time domain of LFM CW into  $u$  domain, some clutter is filtered out in  $u$  domain and then inverse transform into time domain, finally the data is processed by CAMF.

The PFRFT property of rectangular weighted LFM CW in  $u$  domain is calculated as follows:

Let  $cot\alpha = -k$ , the FRFT of sub-pulse signal is

$$\begin{aligned} F_p[g_T(t - \tau + nT)x(t)] &= A_\alpha \exp(-j\pi u^2 k) \int_0^T \exp[j2\pi t(f_0 - ucsc\alpha)] dt \\ &= \left\{ \frac{jA_\alpha \exp(-j\pi u^2 k)}{2\pi(ucsc\alpha - f_0)} \exp[j2\pi T(f_0 - ucsc\alpha)] - 1 \right\} \end{aligned} \quad (23)$$

When  $u = u_0 = f_0 \sin\alpha$ , according to l hospital law the above formula becomes

$$F_p[g_T(t - \tau + nT)x(t)]_{u=u_0=f_0 \sin\alpha} = A_\alpha T \exp(-j\pi u_0^2 k) \quad (24)$$

In general,  $F_p[g_T(t - \tau + nT)x(t)] = \frac{A_\alpha |\exp[j2\pi T(f_0 - ucsc\alpha)] - 1|}{2\pi(ucsc\alpha - f_0)}$ . It's symmetric around the peak point  $u = u_0 = f_0 \sin\alpha$ , the first zero point is  $u = u_0 \pm \sin\alpha/T$ , and the main-lobe width is  $2\sin\alpha/T$ .

The expression of sine-square weighted LFMCW's PFRFT in  $u$  domain is not calculated but qualitatively analyzed here. The main-lobe width of sine-square weighted LFMCW is narrower than  $2\sin\alpha/T$  on account of the sine-square window function has better energy focusing property than rectangular window function.

Then the spectrum and time delay AMF properties of LFM signal are analyzed as follows.

If the duration of LFM signal is infinite, its spectral shape is rectangular when  $BT > 20$  (time-bandwidth product is more than 20). Actually, the spectrum broadening and energy leakage are affected by truncation effect according to the weighted function, which is more serious for rectangular than sine-square weighted.

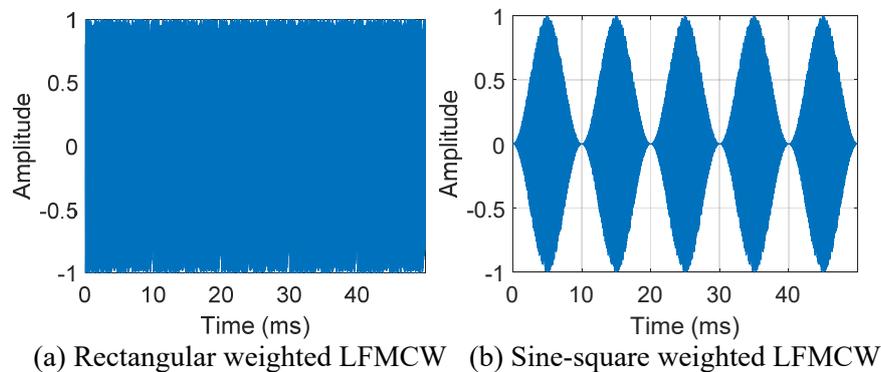
Let  $\eta = 1$ , (6) becomes

$$\chi(1, \tau) \Leftrightarrow U(f)U^*(f) \quad (25)$$

The time delay AMF can be obtained by FFT algorithm. Since the spectrum broadening and energy leakage for rectangular weighted signal are serious than sine-square weighted signal, the distance resolution  $\tau_e$  for former are better than latter whereas the side-lobe for former are lower than later according to the uncertainty principle, which is beneficial for detecting buried target.

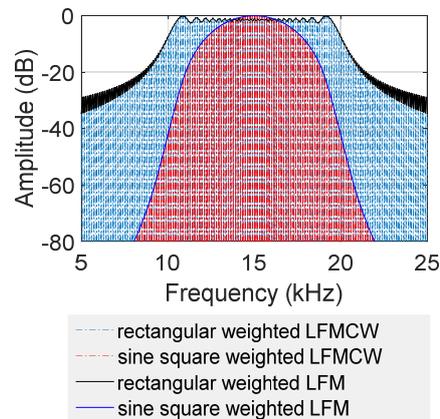
### 3.2. LFMCW signal analysis

In the above section, the properties of time-frequency distribution, spectrum, PFRFT and WAF for LFMCW are analyzed from the theory point of view. The properties are analyzed by simulation in this section.



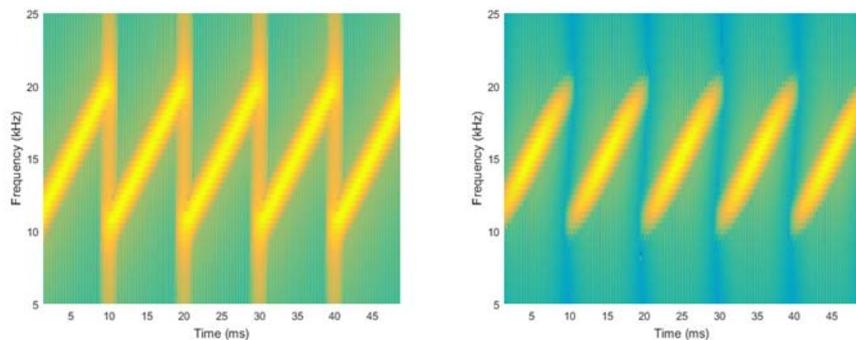
**Fig. 3.** LFMCW waveform in time domain

The LFMCW waveform in time domain shows in Fig.3. The parameters of LFMCW are described in Table I.



**Fig. 4.** LFM CW waveform in frequency domain

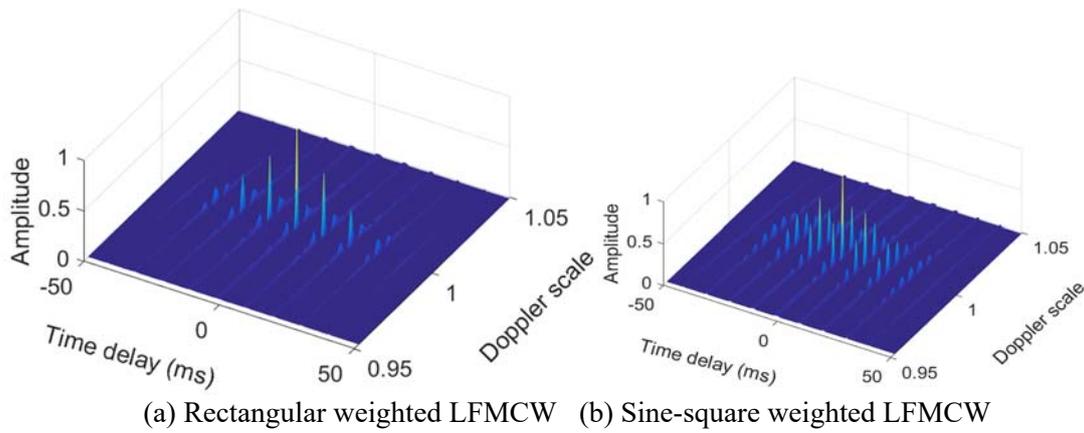
The LFM CW waveform in frequency domain shows in Fig.4. The envelope of rectangular and sine weighted LFM CW's comb spectrum are rectangular and sine weighted LFM's spectrum respectively. The sine-square weighted LFM have a narrower band and lower side-lobe than rectangular weighted LFM, showing that sine-square weighted LFM has better energy focusing property than rectangular weighted LFM, which in accordance with the theoretical analysis above.



(a) Rectangular weighted LFM CW (b) Sine-square weighted LFM CW

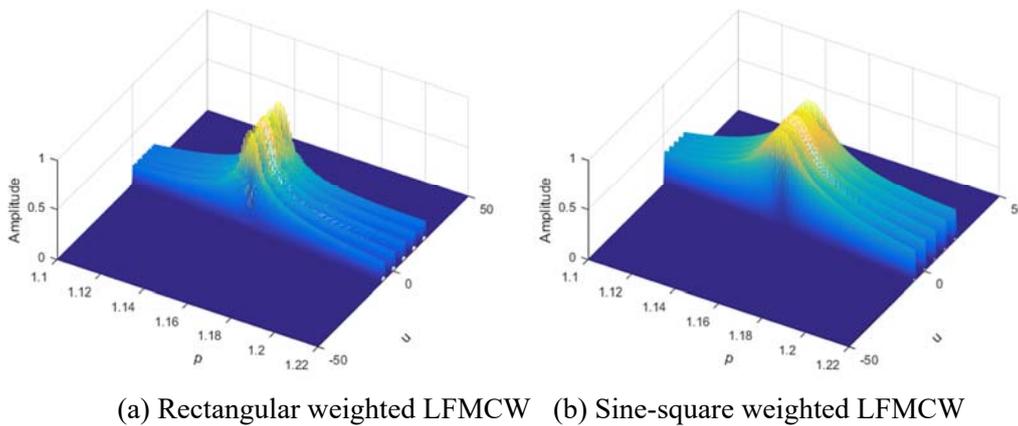
**Fig. 5.** Short-time Fourier transform of signal

The Short-time Fourier transform time-frequency diagrams of signal shows in Fig.5. The intersecting of time-frequency energy distribution is obvious for rectangular weighted LFM CW, while sine-square weighted LFM CW signal has no this phenomenon, which verified the theoretical analysis above.



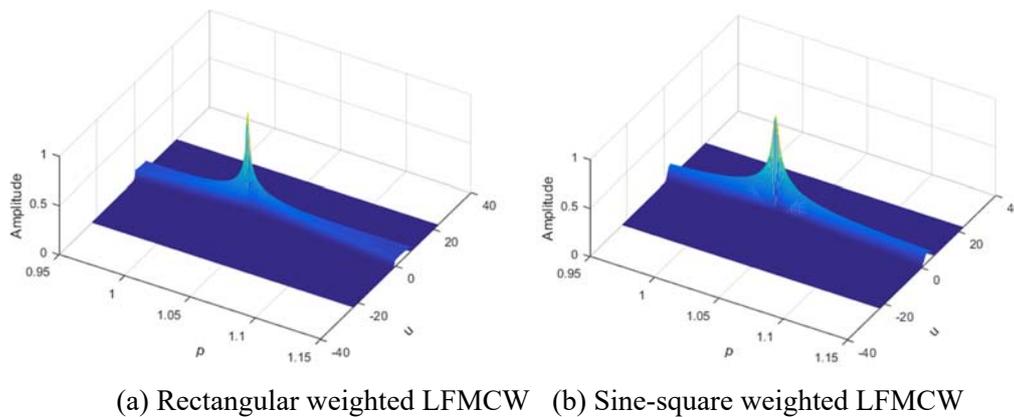
**Fig. 6.** AMF of signal

The AMF of signal shows in Fig.6. Both rectangular weighted and sine-square weighted LFM CW have many side-lobes.

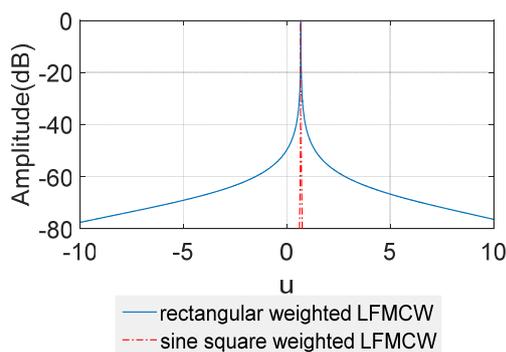
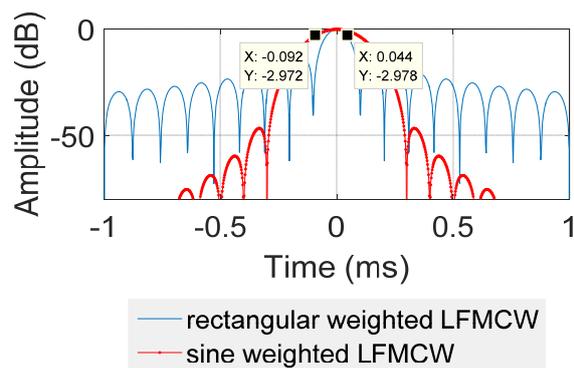


**Fig. 7.** FRFT of signal

The FRFT of signal shows in Fig.7. Both rectangular weighted and sine-square weighted LFM CW have many side-lobes.

**Fig. 8.** PFRFT of signal

The PFRFT of signal shows in Fig.8. Both rectangular weighted and sine-square weighted LFM CW have only one main-lobe.

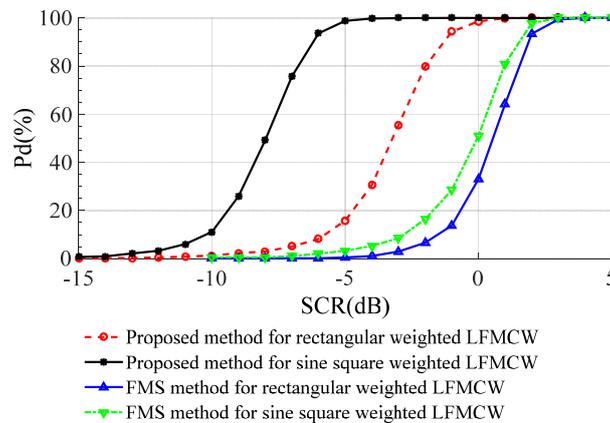
**Fig. 9.** PFRFT of signal in  $u$  domain**Fig. 10.** The time delay AMF of LFM signal

The PFRFT of signal in  $u$  domain shows in Fig.9. The sine-square weighted LFM CW have a sharper peak and lower side-lobe than rectangular weighted LFM CW, indicating that sine-square weighted LFM CW have better energy focusing and anti-clutter properties than rectangular weighted LFM CW, which in accordance with the theoretical analysis above.

The time delay AMF of LFM signal shows in Fig.10. The sine-square weighted LFM have a weaker distance resolution but lower side-lobe than rectangular weighted LFM, showing that sine-square weighted LFM has better ability of reducing the multipath propagation effect caused by propagation medium's fluctuation and uniformity than rectangular weighted LFM, which in accordance with the theoretical analysis above.

### 3.3. Experiential results and analysis

Furthermore, the proposed processing method is validated and the result is analyzed.



**Fig. 11.** ROC for LFM CW signal in clutter

The ROC for LFM CW signal in clutter shows in Fig.11. To validate the proposed method, we compare the frequency match search (FMS) method in the ref [10] with the proposed method. Monte-Carlo method is used to complete the test validation. Some test parameters settings as follows: The Monte-Carlo test times is 1000, probability of false alarm is 0.001, gaussian white noise is added and CNR is 5dB, and SCR ranges from -15 dB to 5 dB with interval of 1 db. For the proposed method, the probability of detection for rectangular weighted LFM CW is more than 90% when SCR is equal to 0dB, whereas the probability of detection for sine-square weighted LFM CW is more than 90% when SCR is equal to -5dB. For the FMS method, the probability of detection for rectangular and sine square weighted LFM CW is more than 90% when SCR is equal to 2dB.

#### 4. Conclusion

The test results are analyzed in section 3.3. By analyzing the results in above section, we can see that:

The LFM CW signal and its processing method can be used for detecting buried target underwater.

The sine-square weighted LFM CW has better anti-clutter property than the rectangular weighted LFM CW.

#### Acknowledgments

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