

# A framework for region based quantitative mapping using hybrid constrained PSO based approach

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**Abstract.** In hyperspectral imaging, spectral unmixing and classification of the pixels are some of the major post-processing operations. The spectral unmixing operation is used to map the pixels quantitatively. In general, it is noticed that the algorithm computes abundance fractions of some endmembers computationally, but does not exist in such part of a real scene. These endmembers may be available in other parts of the real scene. To address this issue, a framework is proposed to do quantitative mapping of the data. First, divide the data into the regions of equal pixels size. Subsequently, hybrid constrained PSO based approach is applied for mapping pixels quantitatively. Combination of Spectral Angle Mapper (SAM) and PSO based approach are used for quantitative mapping respectively. For mapping, fully constrained supervised linear mixing model is considered to estimate the abundance fractions. In this work, hybridization of SAM and PSO is done in order to perform the mapping of pixels quantitatively. The proposed framework is tested over synthetic data and has been performing well.

## 1. Introduction

Hyperspectral imaging is one of the popular and most widely used spectral imaging techniques. It has been using in a number of applications which includes minerals identification and mapping [1], biomedical applications [2], vegetation mapping [3], water resources management [4], and agricultural applications [5] and so on. The aim of spectral unmixing is to decompose mixed pixels into a set of constituent materials called “endmembers” and their corresponding proportions called “abundances” [11]. In general, endmembers extraction and abundance mapping are the sub-parts of spectral unmixing operation. Majorly, there are two types of spectral mixing models i.e., Linear Mixing Model (LMM) and the Nonlinear Mixing Model (NMM). Omran et al. [12] proposed Particle Swarm Optimization (PSO) based approach for endmembers selection to do unmix of multispectral satellite images. Similarly, other authors have also been used PSO based approaches [13], [14], [15] for spectral unmixing operation.

In the real scene, it is generally observed that the one part of the scene contains some endmembers while another part contains different endmembers. In general, optimization algorithms compute abundances of the other endmembers, where such endmembers are not available. Along with, it is also observed that the scene consist a combination of pure pixels and mixed pixels. Hence, in a scene under observation pure pixel needs mapping at pixel level while mixed pixel needs at sub-pixel level mapping i.e., quantitatively. In order to address such issue, data is divided into sub-regions and estimated the abundance fractions of each sub-region individually. Thereby, PSO based approach follows global best of individuals region and along with keep track on their individual best. Further, Spectral Angle Mapper (SAM) [6] is used in combination with Particle Swarm Optimization (PSO) [7] based



approach output to do pixel and sub-pixel mapping (quantitatively) to increase the accuracy. In this way, this hybrid approach is used to increase the efficiency of quantitative mapping. In this work, SAM is used to compute the angle difference between the test pixel spectra and endmembers. If the angle between the spectrums is less than some threshold, it indicates that the pixel spectrum is closed to the reference spectrum.

The remainder of the paper is organized in the following manner: Section 2, 3 and 4 discussed the spectral angle mapper, particle swarm optimization and linear mixing model as a common ground. Section 4 presents the methodology for the proposed framework. Section 5 discussed the experiment and analysis of the proposed approach over synthetic data. Section 6 ends with the conclusion along with future scopes.

## 2. Spectral Angle Mapper (SAM)

Spectral Angle Mapper (SAM) [6] is based on the assumption that the single pixel represents a single material and on matching assigns reference spectrum to such single pixel. This technique is used to measure the spectral similarity between the reference spectra and test spectra by calculating the angle in between them, treating them as vectors in a space with dimensionality equal to the number of bands. Equation (1) represents the SAM expression for determining the spectral angle difference:

$$\alpha = \cos^{-1} \left( \frac{\sum_{i=1}^n t_i r_i}{\sqrt{\sum_{i=1}^n t_i^2 \sum_{i=1}^n r_i^2}} \right) \quad (1)$$

In Equation (1),  $n$  represents the number of bands,  $t$  and  $r$  indicate the test pixel spectrum and reference spectrum, and  $\alpha$  indicates the spectral angle between the test and reference spectra. Lesser value of  $\alpha$  represents the more similarity between the spectras.

## 3. Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) [7] is proposed by Eberhart and Kennedy in 1995. PSO is inspired by the food searching behavior of birds and fishes. It is an evolutionary computation technique based on swarm intelligence. It is very simple as compared to other evolutionary computation algorithms and requires few parameters for optimization purpose. PSO has stronger global convergence and very popular in the multi-objective optimization problem, non-linear programming, and many other areas. It may be applied to most of the optimization tasks and the tasks which can be converted into the optimized ones. In our case, PSO based approach is applied to estimate the abundance fractions following the constraints of LMM. Equation (2) represents the velocity and position of a particle which is given below, respectively.

$$v_{i+1} = w_i v_i + c_1 * rand() * (p_i - s_i) + c_2 * rand() * (p_g - s_i) \quad (2)$$

$$s_{i+1} = s_i + v_{i+1}$$

In above equations,  $v_{i+1}$  and  $s_{i+1}$  represent the velocity and position at  $i+1$  step.  $w_i$  denotes inertia weight,  $c_1$  and  $c_2$  are the acceleration coefficients whose values lie in the range  $[0, 1]$ , and  $rand()$  is a uniform independent random variable.  $p_i$  And  $p_g$  represent personal and global best of the particle.

## 4. Linear Mixing Model (LMM)

Pixel-wise classification detects the object class that most closely matches with the pixel spectrum. However, it does not provide any information that might be present within the boundaries of the

pixel. It may be possible that mixed pixels may be present at such place. A mixed pixel contains a mixture of more than one material which occurs due to two reasons. First, due to the low spatial resolution of the imaging sensor, so that adjacent endmembers jointly occupy a single pixel. Consequently, resulting spectra will be the combination of individual spectrums. Second, when mixed pixels appear when different materials are combined into a homogeneous mixture. Hence, in such case, spectral unmixing [10] is one of the vital post-processing tasks of hyperspectral data. It is a case of a blind separation problem. Mixing of the spectral signatures may be linear or non-linear in nature. However, Linear Mixing Model (LMM) [8] is mathematically simple and easy to solve. Here, fully constrained based LMM is utilized for optimization to estimate the abundance fractions. It follows sum-to-one and non-negative constraints, respectively.

In LMM, a linear relationship exists in between the fractional abundance of the substances comprising the area being imaged and the spectrum of the reflected radiation. An expression for the LMM has been given below in Equation (3):

$$y = a_1s_1 + a_2s_2 + \dots + a_Ms_M + w$$

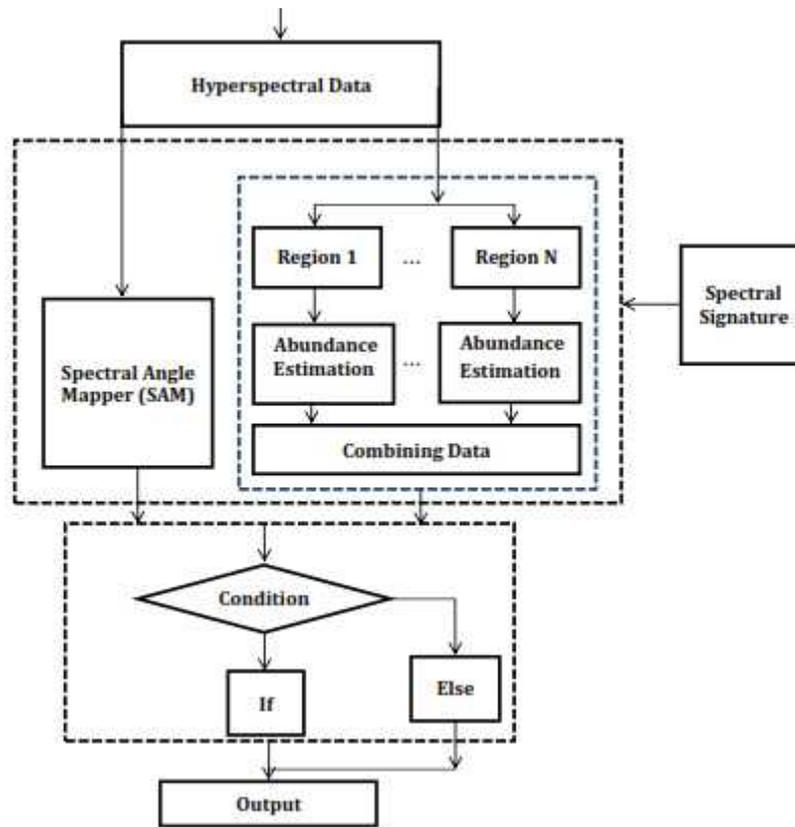
$$y = \sum_{i=1}^M a_i s_i + w \quad (3)$$

In above equation,  $s_i$  represents  $i$ th endmember and  $a_i$  indicates the abundance fraction of  $i$ th endmember,  $M$  is the number of endmembers and  $w$  is the error term.

## 5. Methodology

Methodology for the proposed work has been shown in Figure 1. Initially, SAM is used to find the spectral angle difference between the endmembers and test pixels' spectrum. In parallel, data is divided into equal regions and PSO based approach is applied to each region individually in order to estimate the abundance fractions. In this methodology, conventional PSO based approach is not used for estimation. A PSO based approach is used in which each pixel is considered as a particle and each particle is our solution i.e., estimating the number of abundance fractions per pixel. Each pixel keeps track over their personal best and follows global best of that region. Hence, a swarm is equal to the number of the pixel in a region, and each swarm finds its own solution i.e., abundance fraction estimation. Hence, this PSO based approach is computationally less complex as compared to the conventional PSO. There is no swarm size per solution.

In general, it is observed that one endmember is presented in one part of the scene while other endmembers are not presented in that considered part of the scene. However, optimization algorithm does not know which endmember is available or not in that part of the scene. In such a case, estimated output of optimization algorithm which is computationally right but does not exist like a real scene. Due to this, data is divided into sub-regions before finding abundance estimation. In such a case, PSO based approach follows global and personal best of such region for estimation. If-else conditions are applied, after finding spectral angle difference and estimated output. In this block, the output of SAM and PSO based approach is used for quantitatively mapping in terms of pixels and sub-pixels operation quantitatively. As shown in the pseudo code, a loop is applied for pixel basis. If the minimum value of spectral angle difference is less than '0.001' by any endmember per pixel, then, the value of '1' is allotted for that endmember and others endmembers contribution for such pixel will be zero. Else, abundance estimated value by PSO based approach will be allotted to such pixel. In such a way, a quantitative mapping is achieved. Pseudo code for the if-else block of methodology has been given below.



**Figure 1.** Methodology for the Proposed Framework

#### Pseudo Code : if-else Block of Methodology

*Given:* Calculated Spectral Angle Difference (SAD) between endmembers and pixel spectrum of data and Estimated abundance fractions output (K\_ab) by PSO based approach.

**Dimensions:** SAD( Number of abundance fractions, Number of pixels), and  
K\_ab( Number of abundance fractions, Number of pixels)

*Step 1:* **for** j = 1 → Number of Pixels

*Step 2:* [error, location] = min(SAD(:, j))      % Number of abundance fractions per pixel

*Step 3:*     **if** error <= 0.001

*Step 4:*         Abundance ( location, j ) = 1

*Step 5:*         **Else**

*Step 6:*         Abundance ( :, j ) = K\_ab(:, j)

*Step 7:*         **End**

*Step 8:*         **end**

## 6. Experiment and Analysis

Synthetic data is used for testing the proposed framework. It consists of 20 bands and 28 pixels. In this framework, assuming endmembers for abundance estimation are available. Synthetically generated endmembers have been used to do unmixing operation. For abundance estimation, supervised linear mixing model is considered. Data is divided into four regions of 7 pixels. Each consists of 20 bands. The objective function for the minimization is given below:

$$f_j = \sum (y_j - Ea_j)^2 \quad (4)$$

In above equation,  $y_j$  is the  $j$ th mixed pixel,  $E$  represents the endmembers and  $a_j$  indicates the abundance fractions for the  $j$ th mixed pixel. Synthetically generated endmembers for abundance estimation has been given in Figure 2. Herein, four endmembers are used which are labeled as E1, E2, E3, and E4, respectively. Root Mean Square Error (RMSE) is used as a statistical measure to check the performance of the proposed framework. An expression for the RMSE is given below in Equation (5):

$$RMSE = \sqrt{\frac{1}{N} \sum_{j=1}^N (actual_j - estimated_j)^2} \quad (5)$$

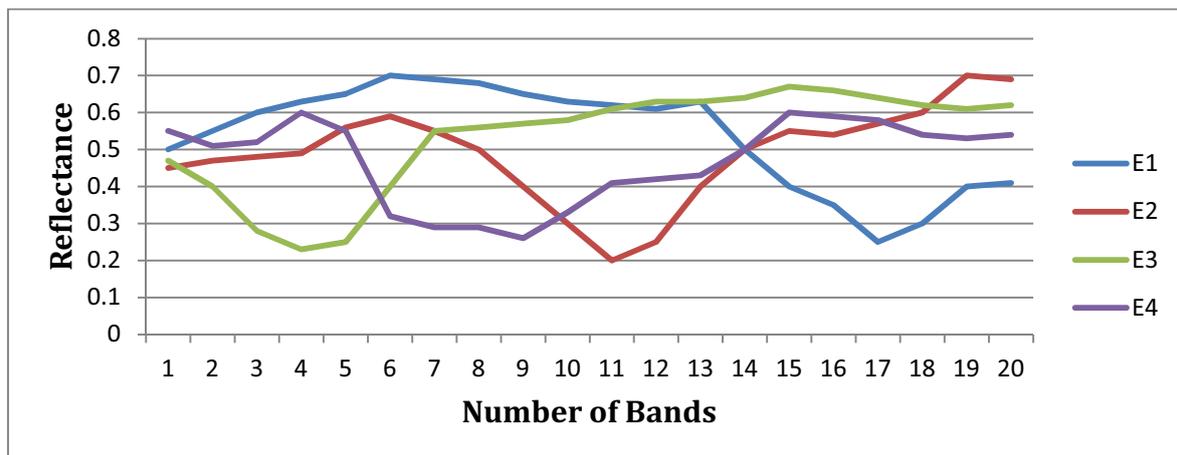
In above equation,  $N$  is the number of pixels, and  $actual_j$  and  $estimated_j$  represent the abundance fractions of a  $j$ th pixel. Hence, RMSE value is calculated for each endmember. Parameters for the PSO based approach in order to estimate the abundance fractions are given in Table 1. Stopping criteria for the PSO based approach is the number of iterations i.e., 1000. Value of  $C_1$  and  $C_2$  are taken as 1.1 and 1.05. Sugeno function [9] is considered for the inertia weight strategy and the expression is given below in Equation (6):

$$w = \frac{1 - \beta}{1 - s\beta} \quad (6)$$

In above equation,  $\beta$  is (current iteration/maximum iterations) and  $s$  is constant larger than -1. For this experiment, a value of  $s$  is taken as -1.5.

**Table 1.** Parameters consideration for the PSO based approach

Parameters	Values
Number of Iterations (Stopping Criteria)	1000
Inertia Weight Strategy	Sugeno function
$C_1$ (Social Component)	1.1
$C_2$ (Cognitive Component)	1.05

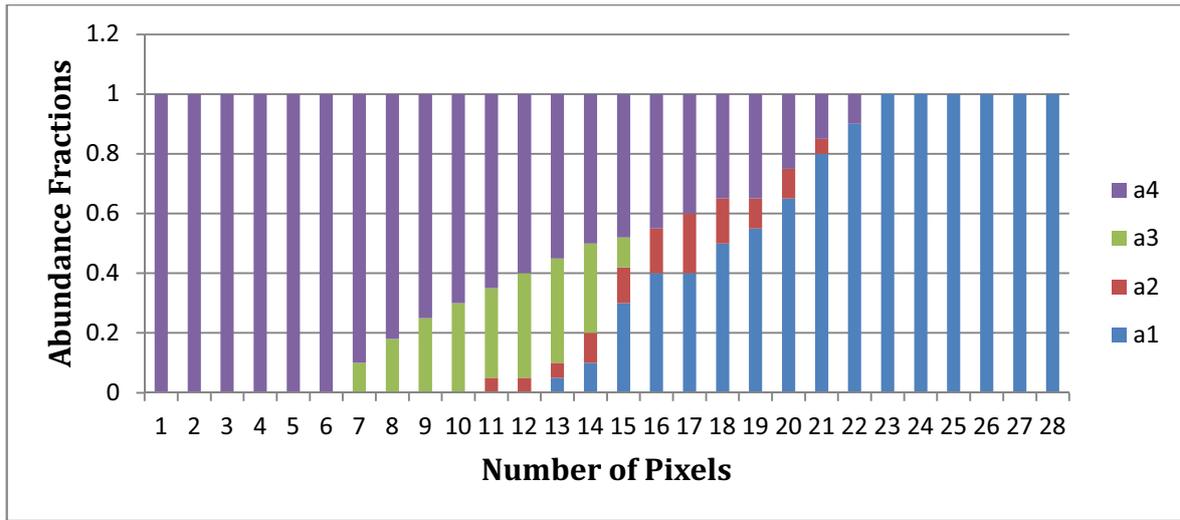


**Figure 2.** Synthetic generated endmembers for abundance estimation

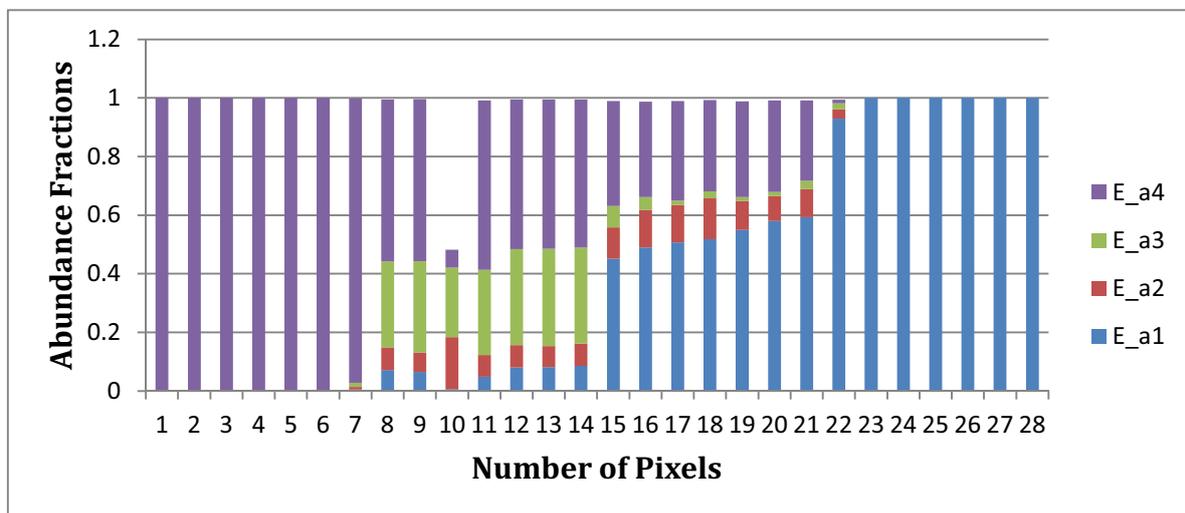
Distribution of synthetically generated abundance fractions is shown in Figure 3. In the given figure,  $a_1$ ,  $a_2$ ,  $a_3$  and  $a_4$  represent abundance fractions of endmembers E1, E2, E3, and E4, respectively. No noise is added to the mixture data. As shown in the figure, abundance fractions distribution fractions follow sum-to-one and non-negative constraints, respectively. In some pixels, some pixels are pure and others are mixed. A framework is proposed in order to do mapping of the

spectral data quantitatively. Estimated abundance fractions labeled as E\_a1, E\_a2, E\_a3, and E\_a4 using hybrid constrained PSO based approach by endmembers E1, E2, E3, and E4 are shown in Figure 4. The result indicates that the proposed approach has been performing well. On visualization Figure 3 and 4, estimated abundance framework has been performing well.

It is noticed that the RMSE value should lie in the range of [0, 1]. In our case, RMSE values between the actual and estimated abundance fractions for each endmember are 0.0629, 0.0439, 0.0351 and 0.1465 which have been given in Table 1. Obtained RMSE values indicate that the proposed framework has been performing well.



**Figure 3.** Distribution of synthetically generated abundance fractions



**Figure 4.** Distribution of estimated abundance fractions using proposed framework

**Table 2.** RMSE values between actual and estimated abundance fractions for each endmember

Abundance Fractions	RMSE
Abundance Fraction for Endmember 1	0.0629
Abundance Fraction for Endmember 2	0.0439
Abundance Fraction for Endmember 3	0.0351
Abundance Fraction for Endmember 4	0.1465

## 7. Conclusion

In this paper, a novel framework is proposed to do quantitative mapping using hybrid constrained PSO based approach. In this work, hybridization of SAM and PSO is done in order to perform the task. The aim of the paper is to present a framework for quantitatively mapping. It is tested over synthetic data and performing well. The suitability of this framework may be tested over real data set and noisy data. Furthermore, the potential of a framework for abundance estimation may be tested over non-linear mixing model.

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