

Investigation of friction power consumption and the performance of a water turbine seal based on the imbalanced rotation of magnetic nanofluids

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Abstract. The calculation of friction power consumption and the performance of magnetic nanofluids in sealed water turbines have proved to be hurdles in studies on magnetofluidic device designs and applications. In this paper, we investigate the imbalanced rotation $\Omega \neq 1/2$ (rot v) of a magnetic nanofluid suspended in a paramagnetic carrier liquid. Through Langevin and Navier–Stokes equations, the formula for the theoretical friction power consumption can be calculated for the magnetic particles in the magnetofluid. The calculated value fits the test result. Further simulation indicates that in a sealed magnetic nanofluid device of a water turbine, the magnetic field gradient ΔB_{sum} between the tooth space (wave trough) and the pole tooth (wave crest) has the most influence on the imbalanced rotation. Specifically, the larger the ΔB_{sum} is, the more obviously imbalanced the magnetofluid rotation will be at that location; and the imbalanced torque will be larger, so the seal differential pressure will be obvious as well. As a result, the sealing capacity will be better and the frictional power consumption will be larger; and the reverse is also true. This study result can serve as a reference for designs for sealed magnetic nanofluidic devices for water turbines, which is significant, especially given the heat from friction power consumption in the water turbine and its cooling equipment.

1. Introduction

Accounting for 80% of renewable clean energy, hydropower is the most stable, efficient, and transportable energy form. Worldwide, there are about 1.05 billion kilowatts of installed capacity yet to be developed^[1-3]. On the other hand, the main shaft sealing technology in water turbines has always been the technical bottleneck limiting hydroelectric devices. Classic sealing structures with regular materials^[4-5] often lead to “nonprogrammed halts” and serious environmental pollution. The Nanjingdu Hydropower Station experienced main shaft leaking 28 times from 1991 to 2005; the #1–3 unit at the OzaAkko Hydropower Station has had serious leaks on multiple occasions; similar problems have appeared in Tugur, Russia, the Gulf of Cambay in India, Three Gorges in China, Liujiaxia Gorges, Bapanxia Gorges, and the Ertan Hydropower Station many times, as well; the water leakage at hydropower stations including Baishiyao, Feilaixia, Hekou, and Honghua, etc., have caused a mix of oil and water which extensively contaminated water resources.

Solving the problem of leaking seals in the main shaft of a water turbine using magnetic nanofluids has become a hot topic currently in the international hydropower industry. Magnetic nanofluids, a



liquid material with important value in industrial applications, have been applied widely in areas such as astronavigation, computers, vacuum seals, printing, dampers, speakers, and so on [6]. Researchers have pursued many studies on magnetic nanofluids [7-8]. In particular, they have achieved success in the manufacture and production of magnetic nanofluids. Gou Maling [9] and Hu Yu [10], et al. applied chemical precipitation methods to produce magnetofluids of a 50 nm particle size for Fe₃O₄. Kanno et al. [11] produced a vacuum-sealed magnetofluid with a saturation magnetization of 35.0 emu/g. The research shows that the performance and stability of magnetofluids are greatly affected by the size and distribution of the micro-magnetic particles [12-13]. In recent years, many studies on the characteristics of microfluids [14-16] have greatly benefited the manufacture and production of magnetofluids.

As magnetofluid seals have been increasingly and widely applied, there have been more in-depth studies on the laws and dynamic characteristics affecting operations with magnetofluids. Hayat et al. considered the Soret and Dufour effects when studying the hydrodynamics of magnetic nanofluids. Through structure-level numerical solutions for homotopy, they found that under different physical parameters, the increase in the Casson number β and Hartman number Ha will cause a decrease in the speed $f(\eta)$ [17]. Hayat et al. analyzed the influence of radiation on the flow of micro-nano magnetohydrodynamics induced by an extension-type surface [18]. Arhad and Asghar analyzed how when the surface stretches out and draws back at any speed, the boundary layer of the second grade fluid flows [19]. Hassani et al. studied the analytical solutions for the boundary layer flow when a nanofluid flows past an extension-type surface [20]. Zou [21] studied the centrifugal force of magnetic nanofluids. Chi [22] studied the non-Newtonian flow characteristics of ferrofluids, and achieved some breakthroughs on the bearing capacity of magnetofluids. Hu [23] studied the stability and fluidity of magnetofluid particles. Nemala [24] studied the impact of temperature on magnetofluidic dynamics. These achievements have laid a good foundation for a theoretical system for magnetofluidics.

The application of magnetic nanofluids in a water turbine seal is a brand-new technology, in which the tribological properties greatly affect the sealing stability. The friction torque caused by the imbalanced rotation of the magnetic nanofluid is the issue of most concern in this area. Wang et al. [25] studied the tribological properties when adding a Mn-Zn ferrite magnetic nano-particle fluid. Based on the fact that the fluid viscosity increases as the magnetic field strength is enhanced, in the study, the comprehensive wear value of Mn-Zn ferrite magnetic-particle lubricating oil was increased, with the maximum being 1.43 times of the base liquid [25]. Yang [26] studied the friction power consumption and kinetic characteristics of magnetofluids. Both Chinese and international studies have shown good results. However, there is a bottleneck that currently exists in the theoretical calculation and experiment methods for friction power consumption, which is caused by the imbalanced rotation in the magnetofluidic seals and has hindered the development of magnetic nanofluidic seal technology for water turbines.

2. Mathematical derivation of friction power consumption caused by the imbalanced rotation of magnetic particles

2.1. Laws of motion for imbalanced rotations

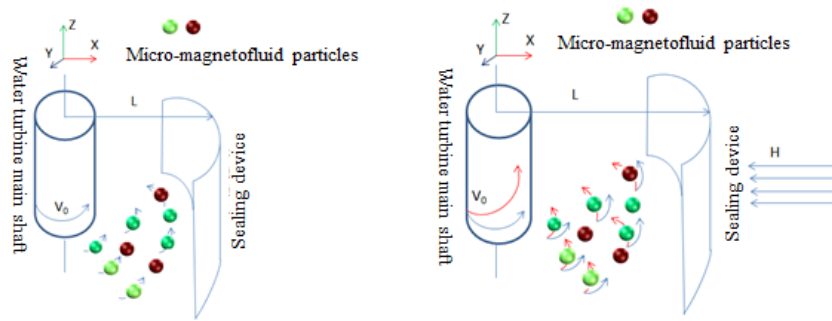
Magnetic nanofluid particles have magnetic dipole moments when rotating around an axis. Without an external magnetic field, the suspended particles are distributed randomly. In the sealing device for the main shaft in a water turbine, the motion of the magnetofluid can be indicated by figure 1(a). When an external magnetic field is exerted, the angles for the dipole moments of these micro-nano magnetic particles will change, as indicated in figure 1(b). The nanoparticles show paramagnetism, and the magnetization intensity can be calculated by the Langevin equation:

$$M(H) = M_s L(a) = M_s \left(\coth a - \frac{1}{a} \right) \quad (1)$$

Nano magnetofluidic superparamagnetism fits the basic theories of paramagnetic materials. The Langevin function $L(\xi)$ can be used to describe its magnetization characteristics:

$$M(H) = nm(\coth \xi - \frac{1}{\xi})$$

where a is Langevin's parameter, (magnetic-thermo ratio); n is the number of magnetic particles within the unit volume m^3 ; m is the magnetic moment of the particles, Am^2 , $\xi = \mu_0 m H / (k_B T)$; μ_0 is the magnetic conductivity in a vacuum; k is the Boltzmann constant; T is the absolute temperature, $^{\circ}C$; and, H is the magnetic field intensity, T .



(a) Non-magnetic carrier liquid microfluid flow. (b) External magnetic field magnetofluid flow.

Figure 1. Magnetic nanofluid shear flow.

According to Newton's internal friction model, a nano magnetofluid has a viscosity-temperature characteristic, just like an ordinary liquid. The imbalanced rotation of a nano magnetofluid $\Omega \neq 1/2$ (rot v) will cause the friction torque. The magnetofluid motion of the rotational axial magnetofluidic sealing device can be described in figure 1a. This flow field can be indicated as:

$$v(r) = v_0 \frac{z}{L} e^x \quad (2)$$

where v_0 is the rotational speed of the main shaft, m/s ; z is the coordinate of the z -axis; L is the distance between the rotational axis surface and the magnetic pole, m ; and, e^x is basic vector of x direction.

For a magnetic nanofluid $\Omega \neq 1/2$ (rot v), the rotation is considered to be:

$$\Omega(r) = \frac{1}{2} \text{rot} v = \frac{V_0}{2L} e_y \quad (3)$$

2.2. Mathematical formula derivation for the friction power consumption of magnetic particles

When the magnetic field $H = H_0 e_z$ (Figure 1b), through rotation Ω , the magnetization intensity of the nano magnetofluid has an offset smaller than e^x . This feature is shown as:

$$\text{div} v = 0 \quad (4)$$

$$d \frac{dv}{dt} = -\nabla p + Z \nabla v + \frac{1}{2} Z_r \text{rot} \left(\Omega - \frac{1}{2} \text{rot} v \right) - \mu_0 (M \nabla) H \quad (5)$$

$$\frac{dM}{dt} = -\frac{1}{f} \left(M - \frac{M}{H} e_H \right) + \Omega \times M \quad (6)$$

$$\theta \frac{d\Omega}{dt} = -Z_r \left(\Omega - \frac{1}{2} \text{rot} v \right) + \mu_0 \Omega \times M \quad (7)$$

where d is the magnetofluid density in Kg/m^3 ; Z is the shear coefficient of viscosity; Z_r is the rotation coefficient of viscosity; f is the relaxation time of magnetization intensity; and θ is the average value of rotational inertia.

Influenced by the magnetic field, and considering the static Maxwell equation and Navier–Stokes

equation, the total viscosity of the nano magnetofluid in the rotational axis of the water turbine Z^{eff} is:

$$Z^{\text{eff}} = Z + \Delta Z \quad (8)$$

$$\Delta Z = \frac{\frac{1}{2} f M(H) \mu_0 H}{Z_r + f M(H) \mu_0 H} \quad (9)$$

Thus, compared with Z_r , ΔZ can express rotational viscosity more conveniently.

When considering the influence of rotation viscosity, considering equations (4)–(8), the formula can be expressed as:

$$\theta \frac{d\Omega}{dt} = -Z_r \omega + \mu_0 M \times H + (T + U) \Delta \text{div} \Omega + V \Omega \quad (10)$$

where T , U , and V are the non-single uncertain rotation diffusion coefficients caused by rotation, $e(H) = -\mu_0 H_0 M(H)$.

H_0 is the magnetic field intensity asserted on the magnetofluid. H is the magnetic field intensity inside the fluid. Here, considering the demagnetization coefficient of the container, $M(H)$ can be derived from equation (1).

The friction torque caused by the external magnetic field asserted on the magnetofluid is

$$\int dV \mu_0 M \times H = \int dV Z_r \omega \quad (11)$$

where V is the total volume of the magnetofluid, m^3 . After exerting the external magnetic field, a magnetic particle friction torque was added; when a new stable state was reached, the output torque of the dragging generator increased by ΔT . To calculate this better, a function $K(H)$ has been introduced:

$$K(H) = V e(H) K_0 \Delta T \quad (12)$$

then:

$$K(H) = \frac{1}{f} + \frac{1}{Z_0} e(H) \quad (13)$$

Take $e(H)$ in the equation and get the increment for the friction torque

$$\Delta T = \frac{V_0 H_0 M(H) K_0}{\frac{1}{f} + \frac{1}{Z_0} H_0 M(H)} \quad (14)$$

From this equation, we can further calculate the friction power consumption.

3. Simulation calculation of the micro magnetofluid sealing performance

Affected by the external magnetic field, and considering the imbalanced rotation $\Omega \neq 1/2$ (rot v), the total viscosity of the micro magnetofluid rotation and the friction torque asserted on the magnetofluid are two main factors influencing the sealing capacity of the water-turbine sealing device. However, the relationship between the magnetic field and the micro-magnetofluid friction power consumption is complicated and non-linear, so we cannot draw accurate, quantitative conclusions from the mathematical models. Ansys software is an efficient way to find the non-linear performance of the micro magnetofluid seal.

3.1. Micro-magnetofluid sealing structure of the water turbine

The object of study is a typical low head water turbine on the Yellow River. It has a water-sealing diameter of 0.052m for the main shaft in the water turbine model. The wheel diameter is 0.34m. The water head is 5m. The rotational speed is 68.18rpm. According to $\rho g H = (1/2(\rho v^2) + D/Y)$, the sealing pressure difference $\Delta p < 0.5 \text{ MPa}$. The structural design is indicated in Figure 2. The rotational units include the wheel, the water turbine main shaft, and the rotor. The static units are composed of the inner wall of the flow pass. The water flows between the rotating units and the static units. The microfluid sealing device is installed between the main shaft and the wheels so as to create a seal

between the dynamic and static units.

Design the sealing device for the water turbine main shaft into one model with both ends face seal and radial seal. The magnetic pole is in tooth form and designed into a counter gear structure so as to increase the magnetic concentration. The magnetic pole is 2mm in width and the distance between the tooth spaces is 1.5mm. The magnetic pole is 2mm in height and the sealing clearance is 0.3mm.

Ferroferric oxide is chosen for the magnetic nanofluid, with a saturation magnetization of 0.04T. The relative permeability of the magnetic nanofluid is MURX of 1. The base liquid is oil. The permanent magnetic material is neodymium iron boron (NdFeB). An SA1010 model (#10 grade carbon steel) is selected for the permeable magnetic material. The material for the main shaft is #45 grade steel.

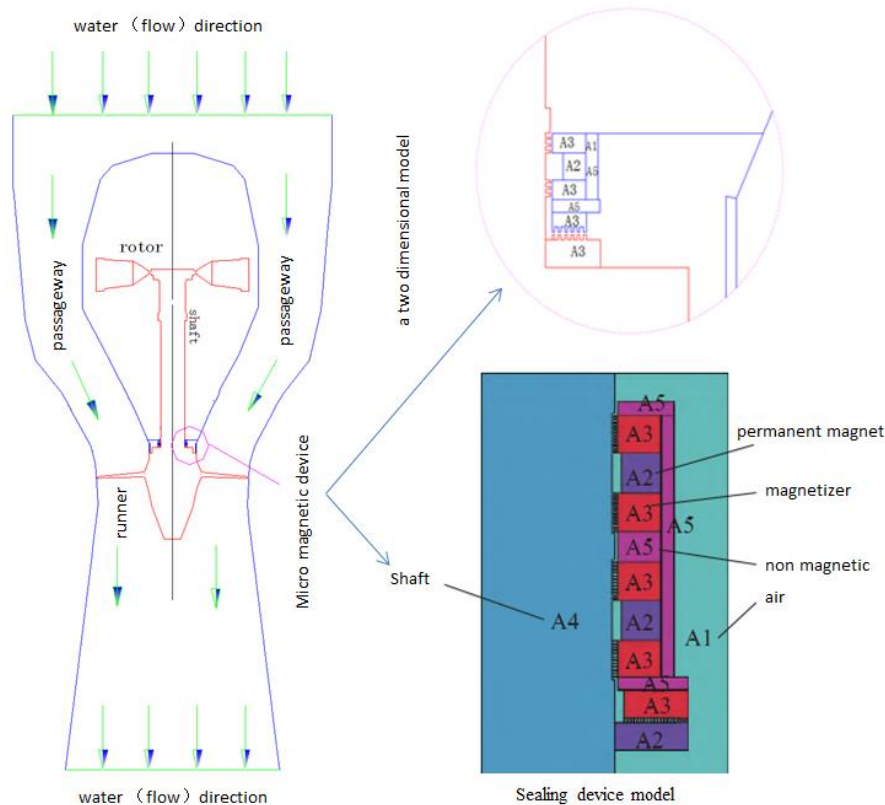


Figure 2. Sealing structure and device model for the main shaft in the water turbine.

3.2. Model and calculation analysis

Apply Ansys software to establish the model for the sealing device for the main shaft magnetofluid in a water turbine and divide the grids, load, and solve. During the simulation, the parameters and condition settings are as follow: the relative permeability is MURX 1.0; the coercive force MGXX and MGZZ are 0,975000, 0 A/m, respectively. Figure 3 is the sealing device model divided into grids. In order to analyze the friction power consumption caused by the imbalanced rotation of the micro-magnetofluid at different positions in the sealing clearance, 3 path curves have been defined in the sealing clearance of the device, as indicated in Figure 4. The positions are A, B, and C. Ten calculation points are set for each path curve.

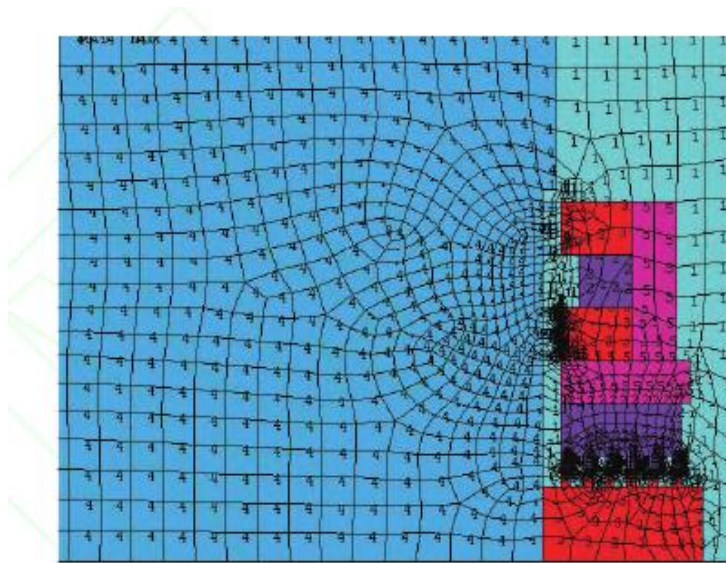


Figure 3. Sealing device divided into grids for the magnetofluid in the water turbine main shaft.

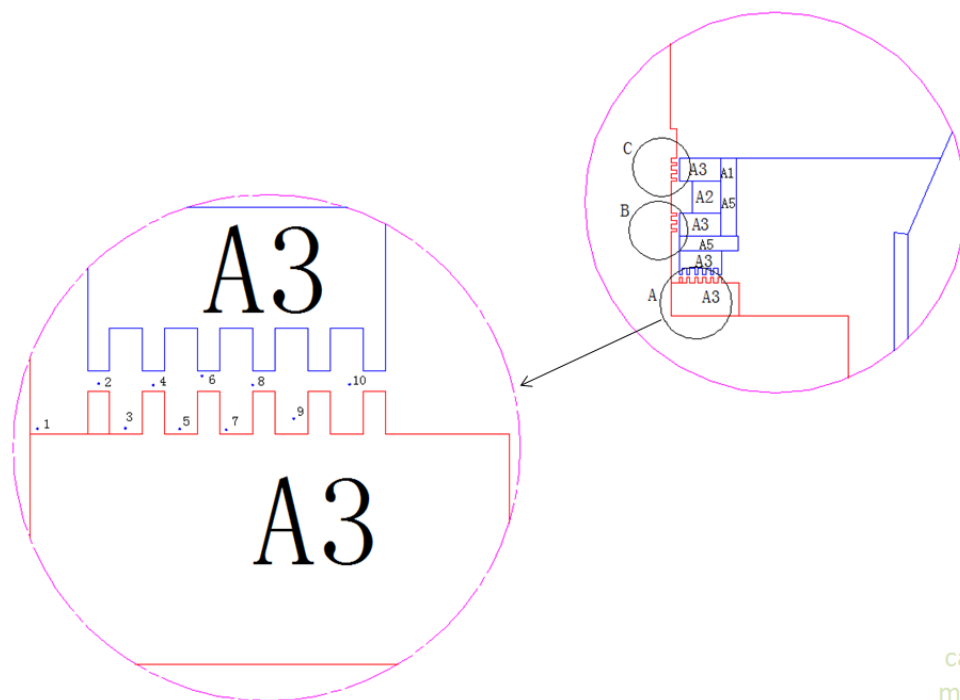


Figure 4. Calculation points Position A, B, and C from the simulation.

In the magnetofluid sealing device, magnetic induction is different at different positions. The imbalanced rotation of the magnetofluid will show different migrations caused by the dipole moment m . As indicated in figure 5, at the sealing clearance of the device, the magnetic lines of force are concentrated and there is little magnetic flux leakage around it, so under the influence of the magnetic field, the magnetofluid gathers at the pole tooth. If the magnetic field distribution is different at the sealing clearance, the sealing capacity will be different at various positions. As illustrated in Figure 6, the path curves in positions A, B, and C show similar trends in the structure. Different positions will see different friction power consumptions caused by different magnetic field distributions and imbalanced rotations of the micro magnetofluid. The wave crest is the magnetic strength on the

internal path curve of the sealing clearance indicating the location of the tooth pole; and the wave trough is the magnetic strength on the internal path curve of the sealing clearance indicating the location of the tooth space. The difference between the wave crest and wave trough is the magnetic field gradient ΔB_{sum} . The larger the ΔB_{sum} is, the more obvious the magnetofluid imbalanced rotation will be at that location; and the unbalanced torque will be larger, and so will the seal differential pressure; as a result, the sealing capacity will be better and the friction power consumption will be larger; and the reverse is also true. This variation trend fits the friction torque formula for the magnetic nanofluid increments.

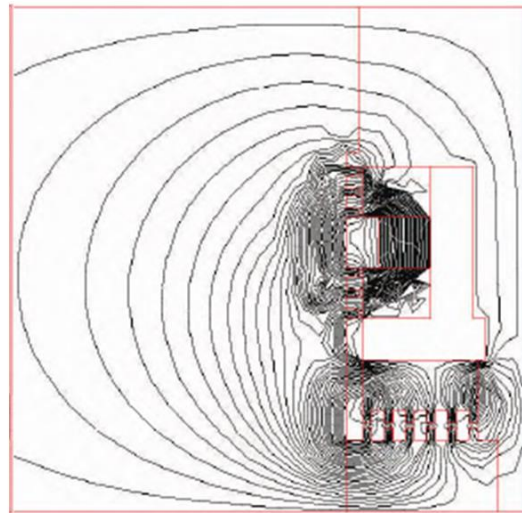
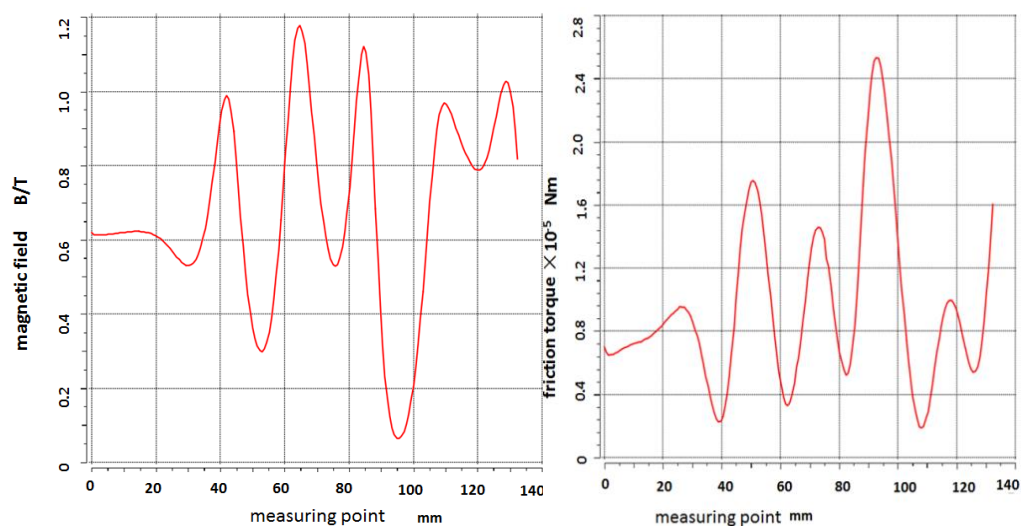


Figure 5. Magnetic lines of distribution of force for the sealing device.



(a) First measuring position A

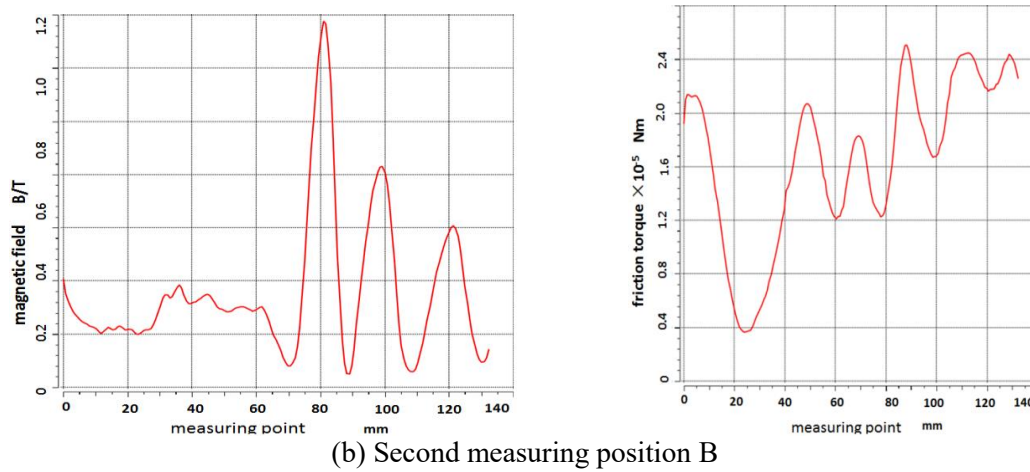


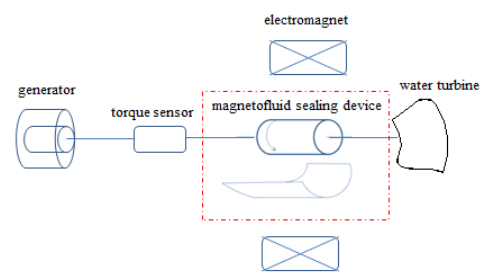
Figure 6. Magnetic field distribution and micro-magnetofluid particle torque changes at various positions in the sealing clearance.

4. Test model

The friction power consumption caused by the imbalanced rotation of the magnetic nanofluid sealing device can be broken down into two parts: the first part is the Newton internal friction power consumption of the carrier liquid; and the second part is the desynchrony between the ferromagnetic particles and the carrier liquid under the impact of the external magnetic field. In order to obtain the friction power consumption mentioned in the second part, the device shown in Figure 7 was applied in the test. The test was carried out in the T4 test stand for hydraulic machinery in the hydropower test room at the Sifang Company's research and test center (Fig. 7, Picture A); the model and actual size ratio was 21:1. Based on the schematic design in Picture B, there is ferrofluid inside the model water-turbine main shaft sealing device. The water turbine makes the main shaft rotate. Without an external magnetic field, the motion of the magnetofluid and the main shaft is synchronized. At that time, $\mathbf{v} = \boldsymbol{\Omega}_0 \times \mathbf{R}$ ($\boldsymbol{\Omega}_0$ is the rotational speed of the axle, and r is the distance from the axle centerline to the magnetofluid). When the rotational speed of the axle $\boldsymbol{\Omega}_0$ reaches a constant value, the motion system becomes comparatively balanced. If the electromagnet is electrified, a magnetic field will appear in the device. The magnetic field will cause the rotational friction of the magnetic particles inside the ferrofluid. The balance of the motion system will then be destroyed and the load torque increased. The torque increments caused by the rotational friction of the magnetic particles can be measured by the torque sensor.



(a) model test stand.



(b) Test schematic diagram.

Figure 7. Rotation friction system for the water turbine magnetofluid sealing device.

The positions chosen for the test are close to those in the simulation calculation. As shown in Figure 8, according to the 3 path curves defined in the simulation calculation, the friction power consumption caused by the micro-magnetofluid imbalanced rotation at three positions, A, B and C, was analyzed. The wave crest and trough appeared, respectively, at the tooth pole and the tooth space.

The curve variation trend was the same as the magnetic field distribution and the micro-magnetofluid particle torque. In the figure, the wave crest shows the magnetic strength at the position of the tooth pole of the sealing clearance, and the wave trough shows the magnetic strength at the position of the corresponding tooth space. The difference between the wave crest and wave trough is the magnetic field gradient. The larger the ΔB_{sum} is, the more obvious the magnetofluid imbalanced rotation will be at that location; and the unbalanced torque will be larger, and so will the seal differential pressure; as a result, the sealing capacity will be better and the friction power consumption will be larger; and the reverse is also true. From the perspective of the test, the calculation formula for the magnetic nanofluid friction torque is verified.

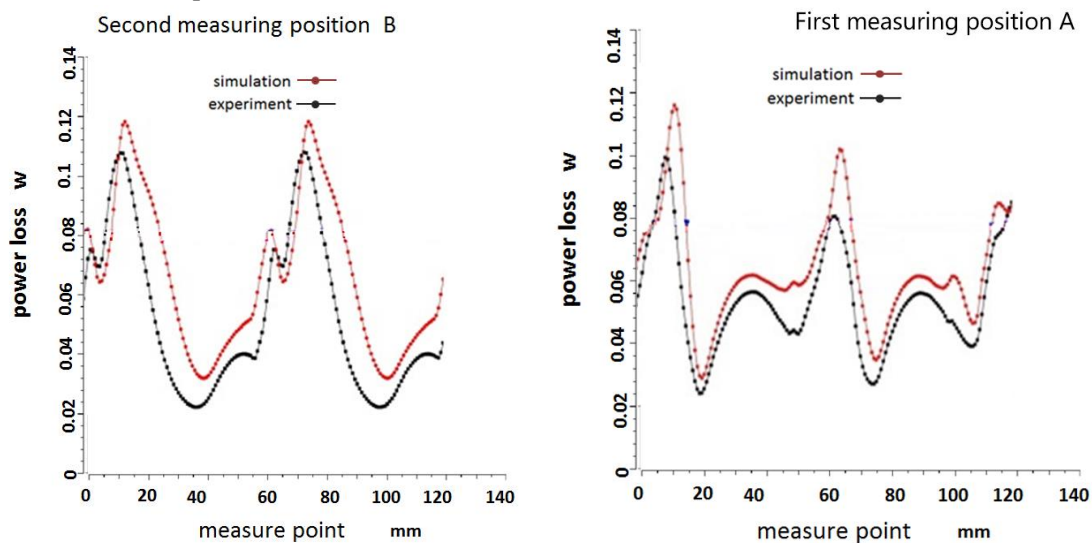


Figure 8. Friction power consumption fluctuation curves for the sealed micro-magnetofluid device.

5. Conclusion

In the magnetofluid seal in the water turbine, the imbalanced rotation $\Omega \neq 1/2$ (rot v) of the magnetic nanofluid will lead to rotational friction of the magnetic particles, with a magnetic dipole moment of m , suspended in the paramagnetic carrier liquid. The magnetic particles, originally oriented in the magnetic domain, change its direction due to the impact from the external magnetic field. The rotational friction torque formula of the magnetic particles derived from the Langevin and Navier–Stokes equations can be used to calculate the friction power consumption of the magnetic particles in the magnetofluid. The calculation result of the friction power consumption through the friction torque formula matches the simulation calculation and the test result. Further studies show that in a sealed magnetic nanofluid device in the water turbine, the magnetic field gradient ΔB_{sum} between the tooth space (wave trough) and the pole tooth (wave crest) has the most influence on imbalanced rotation. To be more specific, the larger the ΔB_{sum} is, the more obvious the magnetofluid imbalanced rotation will be at that location; and the unbalanced torque will be larger, and so will the seal differential pressure; as a result, the sealing capacity will be better and the friction power consumption will be larger; and the reverse is also true. The study results can be referred to when designing sealed magnetic nanofluidic devices. Specifically, under certain friction power consumptions, a cooling device may be needed for the device.

Acknowledgments

The authors would like to thank Sichuan Science and Technology Department (No 17CZ0034) for their financial support, Xihua University Natural fund (Z1510416), Ministry of education for fluid and power engineering (szjj2017-089).

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