

# Analytical Model of Steam Chamber Evolution from Vertical Well

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**Abstract.** This paper is aimed to check the possibility of applying the Steam Assisted Gravity Drainage in vertical wells. This challenge seems to be vital because most of the natural bitumen reservoirs are found to occur above the oil fields being developed so that a well system is already available at the stage of field management. The existing vertical wells are hard to be used for horizontal sidetracking in most of cases as the bitumen reservoir occurs at a shallow depth. The matter is to use the existing wells as vertical ones. At the same time, it is possible to drill an additional sidetrack as a producer or an injector.

## 1. Introduction

Steam assisted gravity drainage (SAGD) [1, 2] is one of the most successful thermal methods for heavy oil recovery [1, 2]. Well-known approximate SAGD methods are described by R.M. Butler [3, 4]. The vast majority of papers describe the development of a heavy oil field by a system of horizontal wells since this approach is the most technologically advanced [5, 6]. For such systems, simple integral estimates of the steam chamber parameters have been determined [7, 8].

This paper uses the methods based on integral correlations [9] to study how the steam chamber develops from a vertical well. The approach is based on the total balance correlations that result in an exact relationship for the integral characteristics, a weak dependence of the qualitative results on the local disturbances of the flow parameters and the shape of the steam chamber [8]. The approach makes it possible to get qualitative conclusions without performing sophisticated numerical calculations. For a pair of horizontal wells, it is shown in [8] how the results of a similar approach match well with the results of numerical simulation performed on a full-size hydrodynamic simulator.

## 2. Methodology

Test schematic is shown in Fig. 1. The assumption is made that the steam chamber is likely a circular cone with a constant height  $h$  and a variable radius  $R$ . The height  $h$  is determined primarily by the reservoir thickness, as well as by the distance to the production well  $h_{\min}$ , which is necessary to prevent steam breakthrough.



The production rate is assumed to be constant and even  $q$ . Such an assumption is entirely permissible within the integral model for the rates that do not vary greatly with time.

In all formulas, the index  $r$  stands for the reservoir, the index  $t$  stands for the top boundary, the index  $o$  stands for the oil,  $w$  stands for the water,  $s$  stands for the steam. The index  $T$  will stand for the heat flux.

It appeals to qualitatively estimate the volume  $V$  and typical dimensions  $R$ ,  $l$  of the steam chamber development from the vertical well, the measure of the produced oil  $q_o$  and the pattern of how the oil ratio changes in the production  $q_o/q$ , where  $q$  is the total production rate of the production well.

Let  $q$  be the volume flow rates of fluids per unit time.

The total flow rate is equal to the flow of water  $q_w$  and oil (bitumen)  $q_o$ :

$$q = q_w + q_o. \quad (1)$$

The volume of the produced water is:

$$q_w = q_s + q_{wr} - q_{sr} \quad (2)$$

where  $q_s$  is the volume of water injected as steam,  $q_{wr}$  is the volume of reservoir water swept,  $q_{sr}$  is the volume of water in the form of steam, which fills the steam chamber (all volumes are referred to a unit of time).

The formulas (1), (2) yield:

$$q_s = q - q_o - q_{wr} + q_{sr} \quad (3)$$

The last two terms are valid:

$$q_{wr} = m \cdot \Delta s_w \frac{dV}{dt} \quad (4)$$

$$q_{sr} = \frac{\rho_s}{\rho_w} m \cdot \Delta s_m \frac{dV}{dt} \quad (5)$$

where  $\rho$  is the density,  $m$  is the porosity,  $V$  is the volume of the steam chamber,  $t$  is the time,  $\Delta s_w = s_{w0} - s_{w\min}$ ,  $\Delta s_m = 1 - s_{w\min} - s_{o\min}$ ,  $s_{w0}$ ,  $s_{w\min}$ ,  $s_{o0}$ ,  $s_{o\min}$  are initial and residual water saturation and oil saturation.

The change in the volume of the steam chamber is proportional to the oil production rate:

$$q_o = m \Delta s_o \frac{dV}{dt}, \quad (6)$$

where  $\Delta s_o = s_{o0} - s_{o\min}$ .

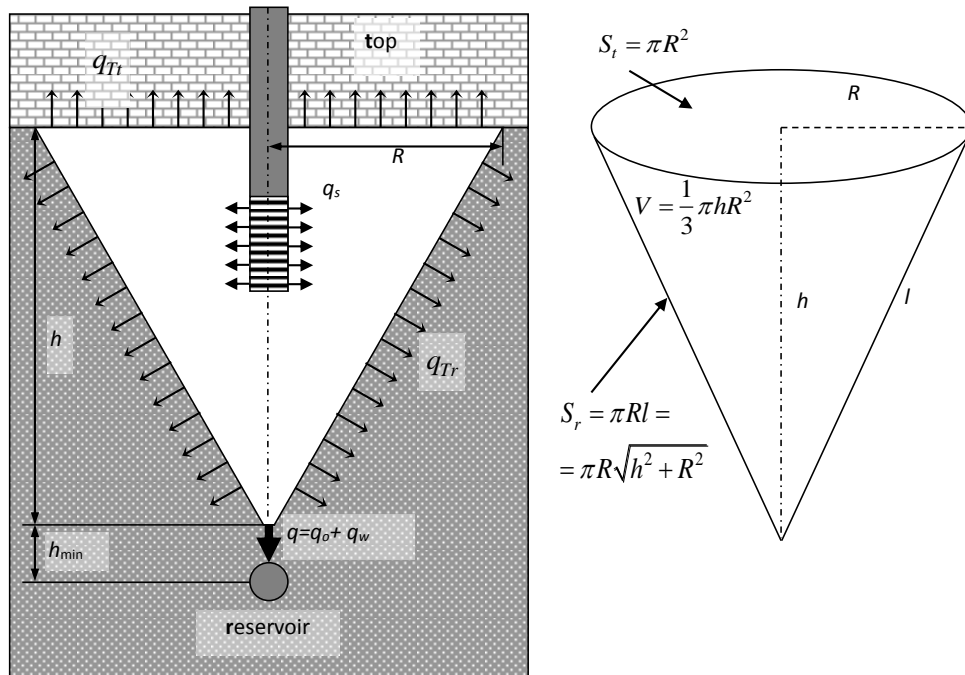


Fig. 1 Steam chamber in bitumen reservoir. Fluid and heat fluxes and the correlations between geometric characteristics

Then the total oil production is

$$Q_o = \int_0^t q_o dt = m \Delta s_o (V - V_0) = m \Delta s_o \left( \frac{1}{3} \pi h R^2 - V_0 \right), \quad (7)$$

where  $V_0$  is the volume of the steam chamber formed during preheating  $t_0$ , before the bitumen production starts.

The formulas (3)-(7) result in:

$$\frac{q_s}{q} = 1 - \frac{q_o}{q} \left( 1 + \frac{\Delta s_w}{\Delta s_o} - \frac{\rho_s}{\rho_w} \frac{\Delta s_m}{\Delta s_o} \right) \quad (8)$$

Let's write the energy mass balance equation of the energy entering the reservoir. The energy that releases during steam condensation heats up the steam chamber and the heat loss to the reservoir and to the top boundary:

$$L \left( q_s \rho_w \varphi - m \rho_s \Delta s_m \frac{dV}{dt} \right) = k_r C_{vr} dT \frac{dV}{dt} + S_t q_{Ti} + S_r q_{Tr}, \quad (9)$$

where  $L$  is the latent heat of vaporization of water,  $\varphi$  is the mass fraction of dry steam in the vapor,  $dT = T_s - T_0$  is the difference between the temperature of the condensing vapor and the initial reservoir temperature,  $C_{vr}$  is the volumetric heat capacity of rocks,  $S_t$ ,  $S_r$  are the areas of steam chamber contact with the top boundary and the reservoir, respectively,  $q_{Ti}$ ,  $q_{Tr}$  are the average heat flux through unit of the corresponding area.

The average flow through the surface of contact with the top boundary is well defined by the formula [10]:

$$q_{Ti} \approx \lambda_t \frac{dT}{b_t \sqrt{a_t (t-t_0)}} \quad (10)$$

where  $\lambda_t$  is the thermal conductivity of the top boundary,  $a_t$  is the thermal diffusivity of the top boundary ( $a_t = \lambda_t / C_{vt}$ ),  $b_t$  is the coefficient within  $0.7 \div 1.5$ .

Determination of the average flux through the contact surface with the reservoir required solution to a supplementary homogeneous problem of simulation resulting in an approximate dependence:

$$q_{Tr} \approx \lambda_r \frac{dT}{b_r (a_r (t-t_0))^{0.312}} \quad (11)$$

where  $\lambda_r$  is the thermal conductivity of the reservoir,  $a_r$  is the thermal diffusivity of the reservoir,  $b_r$  is the coefficient within  $0.05 \div 0.1$ .

The (6)-(11) and geometric relations for areas yield:

$$L \left( \rho_w \varphi \left( 1 - \frac{q_o}{q} \left( 1 + \frac{\Delta s_w}{\Delta s_o} - \frac{\rho_s}{\rho_w} \frac{\Delta s_m}{\Delta s_o} \right) \right) - \rho_s \frac{\Delta s_m}{\Delta s_o} \frac{q_o}{q} \right) = C_{vr} dT \frac{1}{m \Delta s_o} \frac{q_o}{q} + \frac{1}{q} \pi R^2 \lambda_t \frac{dT}{b_t \sqrt{a_t (t-t_0)}} + \pi R \sqrt{h^2 + R^2} \lambda_r \frac{dT}{b_r (a_r (t-t_0))^{0.312}} \quad (12)$$

The formula of the cone volume produces:

$$R = \sqrt{3V/\pi h} = \sqrt{3(V_0 + (Q_0/m\Delta s_o))}/\pi h \quad (13)$$

The formula (12) with the substitution (13) is a differential equation to determine the function  $Q_o(t)$ . The equation contains the first derivative of the desired function  $q_o(t) = Q'_o(t)$ , which can be expressed explicitly. The initial condition for this equation is written as:

$$Q_o(0) = 0. \quad (14)$$

Let's reduce the system (12)-(14) to the dimensionless form:

$$Ay' = 1 - B_t \frac{(y_0 + y)}{\sqrt{\tau - \tau_0}} - B_r \frac{\sqrt{(y_0 + y)(H + y_0 + y)}}{(\tau - \tau_0)^{0.312}}, \quad (15)$$

$$y(0) = 0, \quad (16)$$

where  $\tau = t/t_x$  is the dimensionless time;  $y(\tau) = Q_o/qt_x$  is the dimensionless value of accumulated oil recovery;  $y'(\tau) = q_o/q$  is the share of oil in the production

$$A = 1 + \frac{\Delta s_w}{\Delta s_o} + \frac{1-\varphi}{\varphi} \frac{\Delta s_m}{\Delta s_o} \frac{\rho_s}{\rho_w} + \frac{C_{vr} dT}{L \rho_w \phi m \Delta s_o} , \quad B_t = \frac{3b_t dT \sqrt{\lambda_t C_{vt} t_x}}{L \rho_w \phi h m \Delta s_o} , \quad B_r = \frac{3b_r dT \sqrt{\lambda_r C_{vr} t_x}}{L \rho_w \phi h m \Delta s_o} ,$$

$$H = \frac{\pi h^3}{3q t_x} m \Delta s_o , \quad y_0 = \frac{V_0 m \Delta s_o}{q t_x} ,$$

The typical time  $t_x$  is chosen from the proportionality of the dimensionless constants,  $A$ ,  $B_t$  and  $B_r$ . Lesser times produce the initial moment of growth of the chamber. Larger times yield a quasi-steady growth typical of an asymptotic equation:

$$Ay' = 1 - B_r \frac{y}{\tau^{0.312}} ,$$

the solution of which can be determined in quadratures [11]:

$$y = \frac{1}{A} \int_0^\tau \exp\left(\frac{B}{0.688A} (\xi^{0.688} - \tau^{0.688})\right) d\xi$$

Solution to the resulting equation (15) was done numerically on four orders of accuracy by the Runge-Kutta method [12].

The numerical experiments studied how the geometric dimensions of the pore chamber and the performance of the production well (the share of oil in the production, the total oil production, the steam oil ratio) had been changed depending on the reservoir characterization and the injection well regime. The alteration of the following parameters was observed in calculations: initial temperature, temperature drop  $dT$ , pore chamber height  $h$ , dry steam mass fraction in steam-water mixture  $\varphi$ , thermal conductivity of rocks  $\lambda_r$ , total well flow rate  $q$ .

The calculations were carried out on the following basic values of the parameters:  $dT = 100^\circ\text{C}$ ;  $\lambda_r = \lambda_t = 2\text{W/m}^\circ\text{C}$ ;  $C_{vr} = 2.5\text{MJ/m}^3\text{C}$ ,  $C_{vt} = 0.65C_{vr}$ ;  $L = 2.3\text{MJ/kg}$ ,  $m = 0.3$ ,  $\Delta s_o = 0.55$ ,  $\Delta s_w = 0.1$ ,  $\Delta s_m = 0.65$ ,  $h = 15\text{m}$ ,  $\varphi = 0.8$ ,  $q = 20\text{m}^3/\text{d}$ ,  $V_0 = 15\text{m}^3$ ,  $t_0 = 10\text{d}$ ,  $t_x = 500\text{d}$ .

### 3. Results and Discussions

The primary results of the numerical tests are shown in Figures 2-6. For all the tests, the figures (a) show the change in the share of oil in the production with time, the figures (b) show the change in the oil production rate, (c) show the growth of the steam chamber volume, and (d) show the change in the steam-oil ratio.

The figures demonstrate that the development is distinctly intensified, and the steam-oil ratio grows more slowly if  $dT$  drops, that is, at a higher initial temperature of the formation (Fig. 2.a-2.d). On the one hand, the result is obvious, but may be understood in a different way. The conclusion is that when the reservoir is heated up, it does not make sense to heat the steam to the temperatures higher than those of SAGD process; the excessive increase in temperature leads only to excess heat losses and premature growth of the steam-oil ratio.

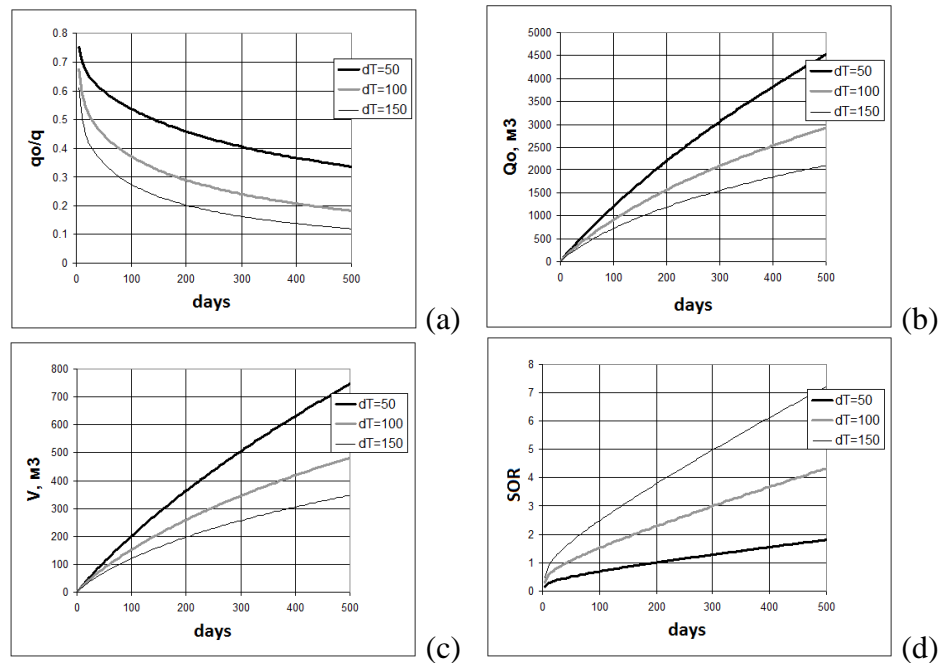


Fig. 2 Changes in measures vs. temperature drops

The increase of the steam chamber in the height results in a proportional increase in oil production and inversely proportional to the reduction in the steam-oil ratio for the same times (Figure 3.a-3.d). This result confirms how the SAGD process is effective for thick reservoirs.

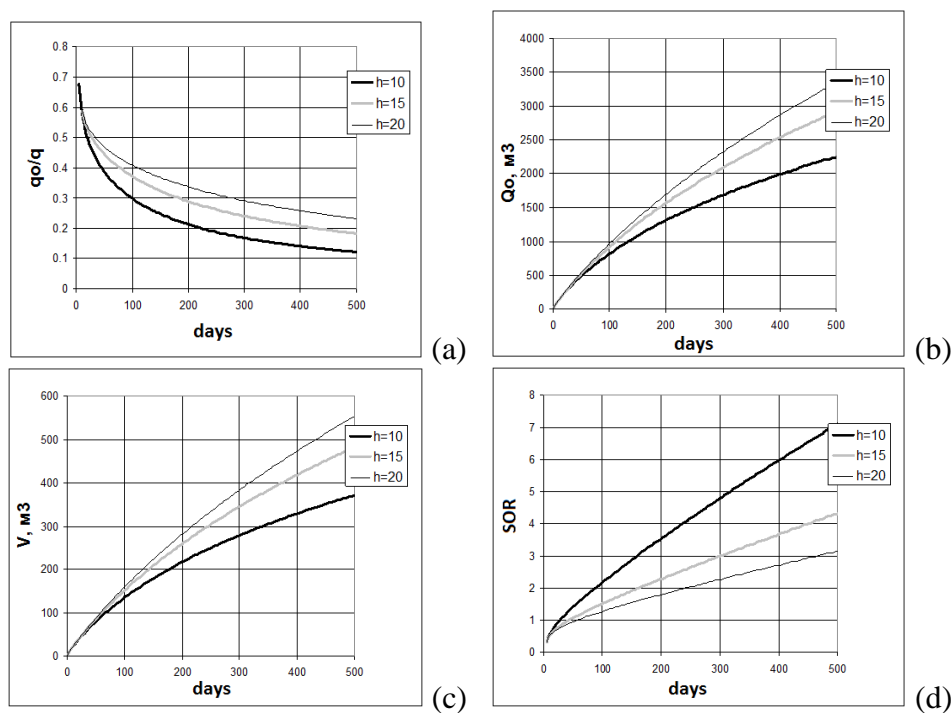


Fig. 3 Changes in measures vs. the maximum height of the steam chamber

Better results of all performance figures were expected with the improvement in quality (degree of dryness) of the injected steam (Fig. 4.a-4.d).

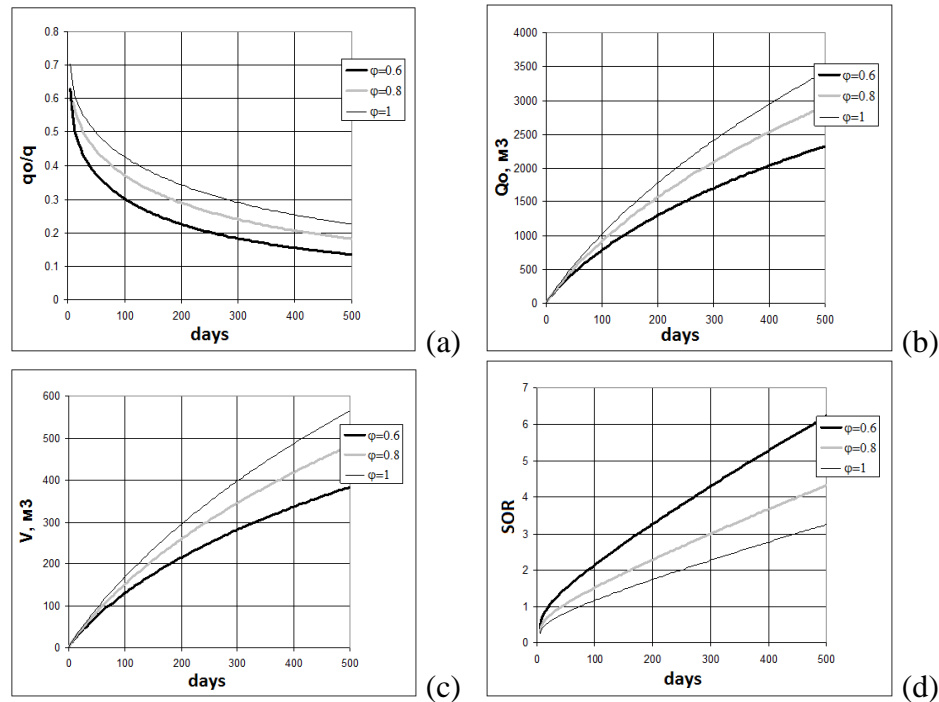


Fig. 4 Changes in measures vs. mass fraction of dry steam

Higher thermal conductivity of rocks increases heat loss from the steam chamber, which, in turn, provokes degradation of the figures. The dependence on  $\lambda$  is quite strong (Figure 5.a-5.d). Therefore, the exact definition of this parameter is particularly challenging.

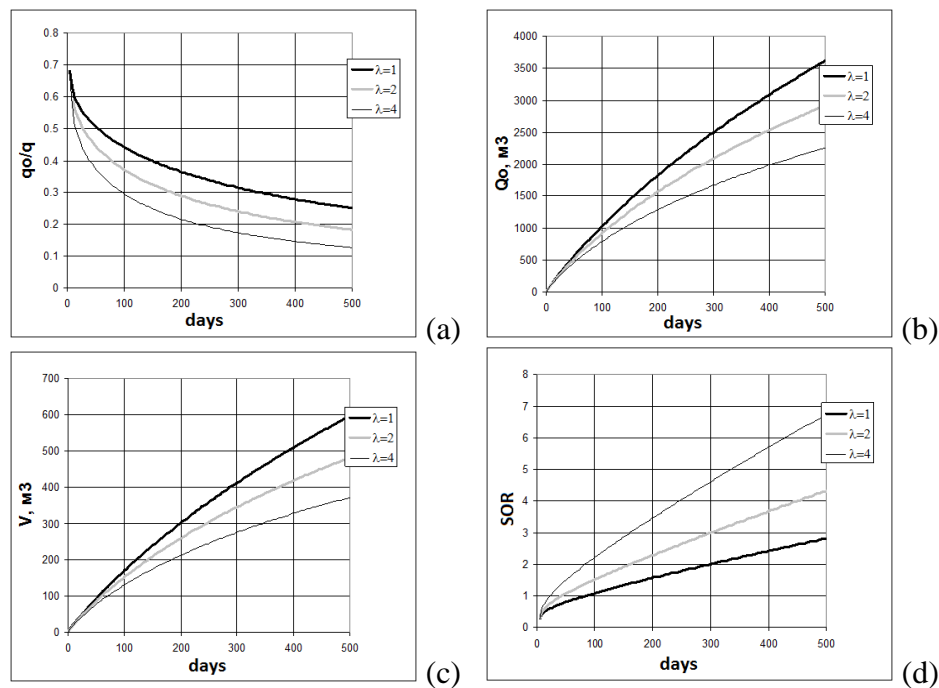


Fig. 5 Changes in measures vs. thermal conductivity of rocks

The change in the total production rate yields a proportional intensification of the production (growth  $Q_o$  and  $V$ ). However, relative figures (water cut, steam-oil ratio) vary slightly (Fig. 6.a-6.d).

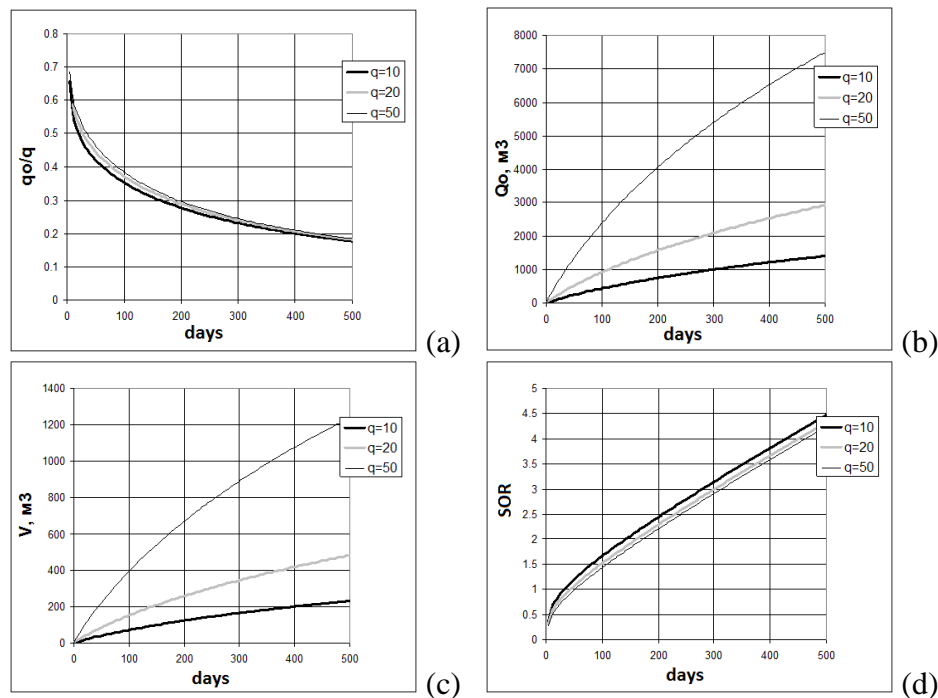


Fig. 6 Changes in measures vs. total production rate

#### 4. Conclusions

The result is an analytical model that is capable to estimate the parameters of the steam chamber near the vertical well. Unlike the numerical calculations in a full-scale formulation, the calculations by the resulting model take an essentially shorter time.

The model can be applied for rapid assessments of the relevance of the SAGD process using in vertical wells as injection ones.

The results of the parametric analysis allow making conclusions about the qualitative dependence of the key development parameters both on the reservoir characterization and on the technological conditions of development.

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