Heating of the porous medium in gas hydrates formation process

I Gimaltdinov[1], **M Khasanov**[2], **M Stolpovsky** [1]
[1]Ufa State Petroleum Technological University,
[2]Sterlitamak Branch of Bashkir State University

E-mail: iljas_g@mail.ru

Abstract. This paper examines the mathematical model of gas hydrates formation in the porous medium with an establishment of the extended region of the phase transition. It also studies the influence of source parameters and parameters on the boundary of the medium in the bed.

1. Introduction

Various technological processes in oil and gas industry accompanied by the formation and the deposition of gas hydrates both in the reservoir and the systems of surface and subsurface equipment of oil and gas fields. Currently, most of the time gas hydrate formation is having negative effect such as emergency stops caused by plants «sclerosis» and also following reconstruction costs. However, hydrate formation may have useful applications such as increasing capacity of underground tanks for carbon dioxide storage. This possibility is connected with sharp growth of gas by weight in the gas hydrate per unit of volume in the compassion with the free state of gas at the same temperature and pressure. [8].

Processes of hydrate formation are accompanied by latent heat release of hydrate formation that could lead to the significant raise of temperature in the porous medium [9]. This effect can be used for cleaning of the porous medium from plaque or for dilution of hydrocarbon systems, which are initially solid for subsequent extraction from subsurface.

The mathematical model of gas hydration and decomposition in the porous medium of an infinite length is examined in papers [1-3, 5-7, 10]. There were obtained automodel decisions and it were shown that depending on the initial and boundary conditions formation and decomposition of the gas hydrate may occur both on frontal surface and extended region. This work describes the mathematical model of formation of gas hydrate in the porous medium of finite-length with an establishment of extended region of the phase transition. There was studied the influence of initial parameters and also parameters on the board of the medium on the evolution of temperature fields.

2. Basic assumptions and equations

Let us consider a linearly-parallel problem of gas hydrate formation in the porous bed of length L ($0 \le x \le L$) in case injecting cold gas through its left boundary $x = 0$. The intensity of the gas hydrate formation process is limited by latent heat elimination of hydrate formation. To describe the processes of heat and mass transfer in the porous medium while injecting gas, in the case of gas injection, which are accompanied by gas hydrate formation, we made the following assumptions: the medium porosity is constant, the skeleton of the porous medium, water, and gas hydrate are incompressible and motionless. The assumption about immobility of the liquid is justified by the superiority of the gas filtration velocity over the water filtration velocity with the exception of cases when water saturation is close to one. So in most cases when water saturation of pores is no more than a half, the assumption about its immobility is justified. In addition, we adopt the mono-temperature model of the porous

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medium that is the model where the temperature of the medium and of the saturating substance coincides at all points. The hydrate is a two-species system with the mass fraction of the gas G., and the gas is calorically perfect.

If we orientate the X-axis in the direction of gas flow, system of equations describing the processes of filtration and heat transfer accompanied by gas hydrate formation in the porous medium, which consist of law of conservation of mass and energy; with those assumptions Darcy's law and Ideal gas law looks like [3]:

$$\frac{\partial}{\partial t}\left(mS_g\rho_g + mGS_h\rho_h\right) + \frac{\partial}{\partial x}\left(mS_g\upsilon_g\rho_g\right) = 0\,,\quad \frac{\partial}{\partial t}\left(mS_l\rho_l + m(1-G)S_h\rho_h\right) = 0,$$

$$\frac{\partial}{\partial t}(\rho c\cdot T) + c_g\rho_g mS_g\upsilon_g\frac{\partial T}{\partial x} = \frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial t}\left(m\rho_h L_h S_h\right),\ mS_g\upsilon_g = -\frac{k_g}{\mu_g}\frac{\partial p}{\partial x},\ p = \rho_g R_g T\,. \quad (1)$$

Here, m is the porosity; ρ_j and S_j ($j = sk, h, l$) are the true density and saturation of the jth phase pores; υ_g, k_g, c_g and μ_g are respectively velocity, permeability, specific heat and dynamic viscosity of the gas phase; p is the pressure; T is the temperature; L_h is the specific heat of hydrate formation; ρc and λ are the specific volume heat and the heat transfer coefficient of the system consisting of the porous medium and gas hydrate, which with those assumptions can be considered as constant values; subscripts h, l, g relates to parameters of hydrate, water and gas respectively

This system is complemented by dependence of the gas permeability coefficient on the gas saturation

$$k_g = k_0 S_g^3,$$

where k_0 – is the absolute permeability of the bed.

When the gas hydrate is formed, regions where the gas, hydrate, and water can be in different states can appear in the porous bed. Following mass and heat balance conditions are being realized on discontinuity surface between regions which have a shock of phase saturation, and also the flows of mass and heat.

$$\left[m\left(\rho_l S_l\left(\upsilon_l - \dot{x}_{(i)}\right) - \rho_h S_h(1-G)\dot{x}_{(i)}\right)\right] = 0,\qquad \left[m\left(\rho_g S_g\left(\upsilon_g - \dot{x}_{(i)}\right) - \rho_h S_h G\dot{x}_{(i)}\right)\right] = 0,$$

$$\left[\lambda\frac{\partial T}{\partial x}\right] = \left[m\rho_h L_h S_h \dot{x}_{(i)}\right]. \quad (2)$$

Here $[\psi] = \psi^+ - \psi^-$ – a shock of parameter ψ on interface $x = x_{(i)}$, $\dot{x}_{(i)}$ — velocity of this boundary. We also will consider the temperature and the pressure on these interfaces are assumed to be continuous.

The phase equilibrium condition is adopting in three-phase region where gas, gas hydrate and water present at the same tame.

$$T = T_0 + T_* \ln\left(\frac{p}{p_{s0}}\right). \quad (3)$$

Here T_0 – the initial temperature of the system, p_{s0} – the equilibrium pressure corresponding to initial temperature, T_* – is an empirical parameter depending on the gas hydrate type. It follows from (3) that

$$\frac{\partial T}{\partial x}=\frac{T_*}{p}\frac{\partial p}{\partial x},\quad \frac{\partial^2 T}{\partial x^2}=\frac{T_*}{p}\frac{\partial^2 p}{\partial x^2}-\frac{T_*}{p^2}\left(\frac{\partial p}{\partial x}\right)^2. \tag{4}$$

Let a porous bed of length L at the initial state be saturated by gas and water under the pressure p_0 and the temperature T_0 meeting conditions of the free existing and which are identical in the whole bed:

$$p=p_0,\ T=T_0,\ S_l=S_{l0}\quad (0\le x\le L,\ t=0). \tag{5}$$

The conditions on the left boundary (x = 0) which is being injected with the p_e and the temperature T_e meeting the conditions of the stable existing of gas and hydrate can be represented in the form:

$$p=p_e,\ T=T_e\quad (x=0,\ t>0). \tag{6}$$

Let us set the conditions on the right boundary ($x = L$), modeling absence of heat flow though it and the pressure equal to inital:

$$\frac{\partial p}{\partial x}=0,\ \frac{\partial T}{\partial x}=0\quad (x=L,\ t>0) \tag{7}$$

3. Reduction of the system of equations to a convenient form for numerical solution and analysis of the results

Let us consider the process of hydrate formation in volume region. There are three characteristic regions: the near region where pores are filled by gas and hydrate, intermediate region where gas, hydrate and water are in equilibrium and the distant one which is filled by gas and water. Process of hydrate formation occurs in the intermediate region process. Respectively, two movable surfaces form: between distant and intermediate regions where the process of water to hydrate transformation begins, and between distant and intermediate regions where the process of water to hydrate transformation ends.

Boundary where conditions (6) are valid will be called impermeable.

From (1) the equations of piezoconductivity and thermal conductivity have the form:

$$\frac{\partial}{\partial t}\left(\frac{p}{T}\right)=\frac{k_g}{m\mu_g S_g}\frac{\partial}{\partial x}\left(\frac{p}{T}\cdot\frac{\partial p}{\partial x}\right), \tag{8}$$

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\kappa^{(T)}\frac{\partial T}{\partial x}\right)+\frac{c_g k_g}{\mu_g \rho c}\frac{p}{R_g T}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}.$$

The analogical equations for the intermediate region:

$$\frac{\partial}{\partial t}\left(\frac{p}{T}\right)=\frac{k_g}{m\mu_g S_g}\frac{\partial}{\partial x}\left(\frac{p}{T}\frac{\partial p}{\partial x}\right)-\Re\frac{\partial S_h}{\partial t}, \tag{9}$$

$$\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\kappa^{(T)}\frac{\partial T}{\partial x}\right)+\frac{c_g k_g}{\mu_g \rho c}\frac{p}{R_g T}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}+\Delta T\frac{\partial S_h}{\partial t},$$

where $\Re=\dfrac{G\rho_h R_g}{1-S_{l0}}$, $\Delta T=\dfrac{m\rho_h L_h}{\rho c}$, $\kappa^{(T)}=\dfrac{\lambda}{\rho c}$ –thermal conductivity coefficient of the bed. If we exclude the time derivative of hydrate saturation from (9) with allowance of equations (5) we will obtain the system for finding the pressure distribution (and relatively the temperature distribution) and hydrate saturation in the extended region:

$$\frac{\partial}{\partial t}\left(\frac{p}{T}\right)=\frac{k_g}{m\mu_g S_g}\frac{\partial}{\partial x}\left(\frac{p}{T}\frac{\partial p}{\partial x}\right)-\frac{\Re}{\Delta T}\frac{\partial T}{\partial t}+\frac{\kappa^{(T)}\Re}{\Delta T}\frac{\partial^2 T}{\partial x^2}+\frac{c_g k_g \Re}{\mu_g \rho c \Delta T R_g}\frac{p}{T}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}, \qquad (10)$$
$$\frac{\partial S_h}{\partial t}=\frac{1}{\Delta T}\left[\frac{\partial T}{\partial t}-\frac{\partial}{\partial x}\left(\kappa^{(T)}\frac{\partial T}{\partial x}\right)-\frac{c_g k_g}{\mu_g \rho c}\frac{p}{R_g T}\frac{\partial p}{\partial x}\frac{\partial T}{\partial x}\right].$$

We will consider that gas injection to the bed on the surface $x = x_{(n)}$ dividing the distant and intermediate region leads to the shock of hydrate saturation from $S^-_{h(n)} = S_{he} > 0$ to $S^+_{h(n)} > 0$ reasoned by the transfer of part of water to hydrate on this boundary. Conditions of mass equilibrium (for gas and water) and energy equilibrium on this surface like it follows from (2) have the form:

$$mS^-_{g(n)}\rho^-_{g(n)}\left(\upsilon^-_{g(n)}-\dot{x}_{(n)}\right)=mS^+_{g(n)}\rho^+_{g(n)}\left(\upsilon^+_{g(n)}-\dot{x}_{(n)}\right)+m\left(S^-_{h(n)}-S^+_{h(n)}\right)\rho_h G\dot{x}_{(n)},$$
$$mS^+_{l(n)}\rho_l\dot{x}_{(n)}=m\left(S^-_{h(n)}-S^+_{h(n)}\right)\rho_h(1-G)\dot{x}_{(n)}, \qquad (11)$$
$$\lambda\left(\frac{\partial T}{\partial x}\right)^-_{(n)}-\lambda\left(\frac{\partial T}{\partial x}\right)^+_{(n)}=m\rho_h L_h\left(S^-_{h(n)}-S^+_{h(n)}\right)\dot{x}_{(n)}.$$

Here $\dot{x}_{(n)}$ – is the velocity of motion of the boundary $x = x_{(n)}$; the plus and minus superscripts refer to the values before the interface and behind it. It can be showed that on the surface $x = x_{(d)}$ dividing the intermediate and distant region, the hydrate saturation value is continuous and equal to zero: $S^-_{h(d)} = S^+_{h(d)} = 0$. Then from (2) equations for finding the pressure and the temperature on this boundary have the form:

$$\left(\frac{\partial p}{\partial x}\right)^-_{(d)}=\left(\frac{\partial p}{\partial x}\right)^+_{(d)}, \qquad (12)$$
$$\left(\frac{\partial T}{\partial x}\right)^-_{(d)}=\left(\frac{\partial T}{\partial x}\right)^+_{(d)}$$

System of equations (8) and (10) with initial and boundary (5) – (7) conditions on the phase transfer boundary (11) and (12) in the finite-difference form is being solved with using the method of front capturing in nodes of the spatial grid [4].

Fig. 2 shows calculation dependences in hydrate formation in the extended region for the case of impermeable right boundary while gas is being injected to the model bed of length L=2m. with the absolute permeability $k_0 = 10^{-14}$ m^2. There are assumed following values characterizing the system consisting of the porous medium, solid gas hydrate, and gas: $m = 0.1$, $G = 0.12$, $S_{l0} = 0.2$, $\mu_g = 10^{-5}$ Pa·s, $\lambda = 2\ \text{W/(m}\cdot\text{K)}$, $L_h = 5\cdot10^5$ J/kg, $\rho_h = 900\ \text{kg/m}^3$, $\rho_l = 1000\ \text{kg/m}^3$, $T_0 = 280$ K, $T_* = 10$ K, $p_0 = 4$ MPa, $p_{s0} = 5.5$ MPa, $c_g = 1560\ \text{J/(kg}\cdot\text{K)}$. Figure shows that injection of the cold gas, which is accompanied by hydrate formation, leads to bed heating. Hydrate formation occurs in two. On the first quick stage partial formation in the volume region occurs. On the second stage full hydration of the porous medium occurs. The distant movable region reaches the right boundary of the porous bed as the result. There are two regions exist, the near one contains gas and gas hydrate, the second one contains gas, gas hydrate and water at the thermodynamic equilibrium state. The distribution of hydrate saturation and temperature in three-phase region becomes homogeneous over the time.

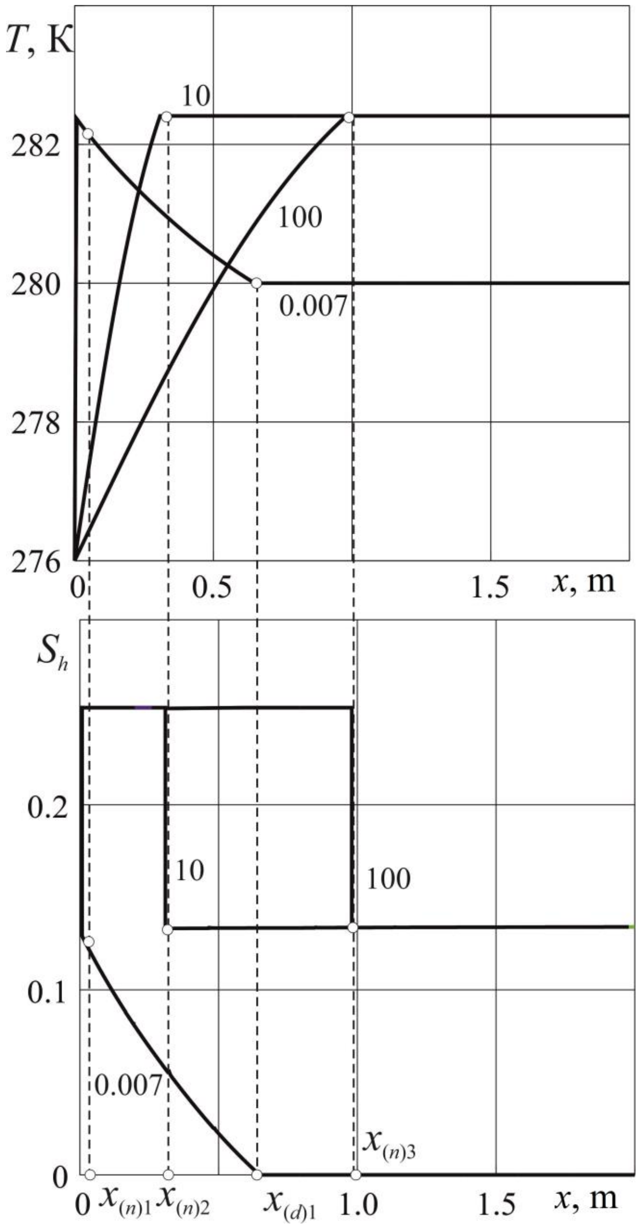

Fig.1. Distribution of temperature and hydrate saturation by X-coordinate in the case of hydrate formation. Values on the curves are the time in hours.

Fig. 1 shows dependence of temperature $T_{(n)}$ on the near boundary from the side of the extended region on the pressure of injecting gas to the bed. Lines 1 and 2 corresponding to values of permeability of the bed $k_0 = 10^{-13}$, $k_0 = 10^{-15}$ m^2. Fig. 2 shows that increasing of the injecting gas pressure and permeability the temperature on the near boundary of hydrate formation becomes higher than the initial temperature of the bed. The reason is that the pressure in the bed is significantly exceeds the equilibrium pressure corresponding to initial temperature T_0. Equilibrium temperature of hydrate formation increases with the growing of pressure. Increasing of pressure of the injecting gas and decreasing of permeability of the bed leads to growing of pressure in the. Therefore the feed of cold gas to the bed leads to its heating.

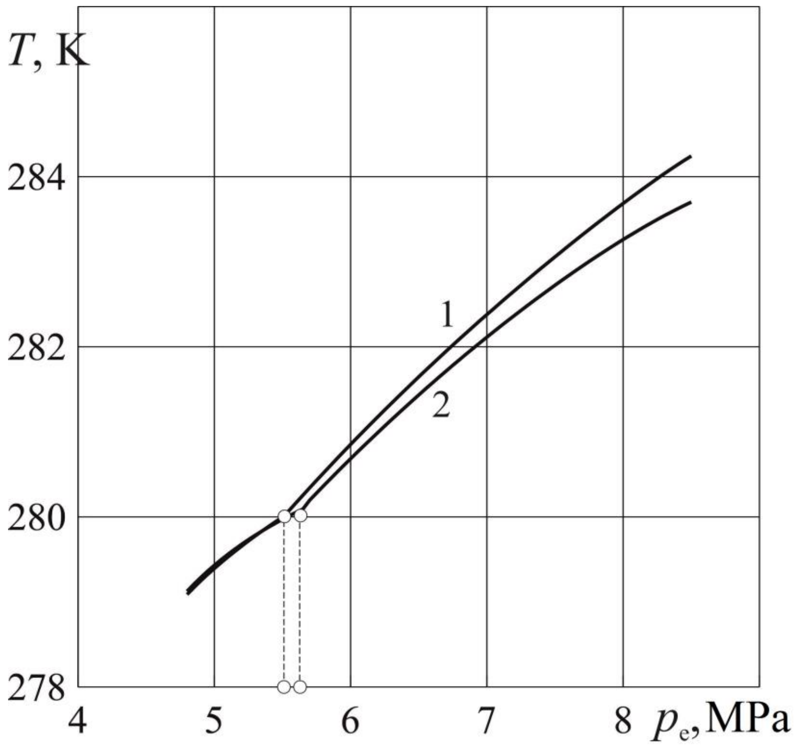

Fig. 2. Distribution of temperature on the near boundary (from the side of extended region) on the pressure of injecting gas: 1 – $k_0 = 10^{-13}$ m^2, 2 – $k_0 = 10^{-15}$ m^2

4. Conclusion

Numerical simulations of the hydrate formation process in the porous medium with the establishment of the extended region of hydrate have been carried out. It have been shown that injecting of cold gas leads to heating of the bed, and the higher the injecting gas pressure is, the higher heating of the bed.

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