

# Research on astronomical tide and tsunami coupled model

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**Abstract.** An astronomical tide and tsunami coupled numerical model of East China Sea was established by adding tidal boundary conditions into COMCOT tsunami model. Two different tsunami wave, positively leading wave and negatively leading wave, were designed according to the dislocation direction of earthquake plates during an undersea earthquake with magnitude of 7.6 which occurs in Ryukyu Trench. The tsunami wave crest and astronomic high tide level occurs at the same time through the modification of phase of tsunami wave. Comparison about the summation of astronomic tide level and tsunami wave level was made between the result of coupling model and linear sum. It demonstrated that the tsunami wave was in advance in both cases abovementioned whether the leading wave is positive or negative. And it's safer for the result of linear sum with respect to tsunami wave height.

## 1. Introduction

Tsunamis are one of the most severe marine disasters. Once a tsunami occurs, it causes substantial destruction. Accurate tsunami forecasting is a prerequisite for maximizing tsunami mitigation, and tsunami numerical simulation is an important tool for tsunami forecasting. The existing tsunami forecasting model generally does not contain tidal modules, and it generally does not consider the nonlinear effects of tides and tsunamis. In this study, the tsunami wave height and the tidal value were linearly summed to obtain the water level at the concerned site. However, the nonlinear effects of tides and tsunamis will increase, which will significantly affect shallow coastal waters. Therefore, it is observed that there are obviously several errors in traditional tsunami forecasting methods.

With regard to coupling tsunami waves and astronomical waves, there has been several successful achievements. Kowalik et al. [1] investigated the nonlinear effects of tides and tsunamis using a simple one-dimensional model and set up continental shelves with different widths for the study. The results showed that the interaction between tsunamis and tides is transient when propagating over a narrower shelf, while the amplitude and flow of tsunamis decrease over a wider shelf due to the effect of bottom friction when the tidal water level and flow size remain almost constant. Dao et al. [2] simulated tidal water level changes during an Indian Ocean tsunami in 2004 and compared the magnitude and arrival time of the tsunami against those for tsunamis in Thailand and Malaysia during high and low tide levels. The results were significantly different. Yao et al. [3] used the HAMSOM model, which is based on the tidal background, to simulate a tsunami event in the Taiwan Strait in 1994. In this paper, a coupled mathematical model between astronomical tides and tsunamis in the East China Sea is established. The differences in tsunami arrival times and near-shore tsunami waves between the coupled and linear models are analyzed.

## 2. methods



### 2.1. Introduction of COMCOT

The COMCOT [4] (Cornell Multi-Grid Coupled Tsunami) model is a numerical calculation model for tsunamis based on the shallow water long wave equation developed by Cornell University (Liu PL-F), which simulates the entire process of a tsunami effectively, including the propagation and inundation. The basic equation is as follows:

$$\frac{\partial \eta}{\partial t} + \frac{1}{R \cos \phi} \left[ \frac{\partial P}{\partial \psi} + \frac{\partial}{\partial \phi} (\cos \phi Q) \right] = 0 \quad (1)$$

$$\frac{\partial P}{\partial t} + \frac{gH}{R \cos \phi} \frac{\partial \eta}{\partial \psi} - fQ = 0 \quad (2)$$

Where  $\eta$  represents the free surface displacement relative to mean sea level,  $P$  represents the flux along a unit of latitude,  $Q$  represents the flux along a unit of longitude,  $\phi$  and  $\psi$  represents longitude and latitude, respectively,  $f$  is the Coriolis coefficient,  $g$  represents acceleration due to gravity,  $R$  represents the radius of the earth,  $H$  represents the water depth, and  $t$  represents time.

When tsunamis spread over nearshore regions, cartesian coordinates are used in the nonlinear shallow wave equation, and the effect of bottom friction is considered. The continuity equation and the momentum equation are as follows:

$$\frac{\partial \eta}{\partial t} + \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 0 \quad (3)$$

$$\frac{\partial P}{\partial t} + \frac{\partial}{\partial x} \left( \frac{P^2}{H} \right) + \frac{\partial}{\partial y} \left( \frac{PQ}{H} \right) + gH \frac{\partial \eta}{\partial x} + \frac{gn^2}{H^{7/3}} P(P^2 + Q^2)^{1/2} = 0 \quad (4)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial y} \left( \frac{Q^2}{H} \right) + \frac{\partial}{\partial x} \left( \frac{PQ}{H} \right) + gH \frac{\partial \eta}{\partial y} + \frac{gn^2}{H^{7/3}} Q(P^2 + Q^2)^{1/2} = 0 \quad (5)$$

Where  $P$  and  $Q$  represent the volumetric fluxes in the  $x$  and  $y$  directions, respectively;  $n$  is the Manning roughness coefficient; and  $H$  represents the water depth. The COMCOT model has been used many times to model historical tsunami events, such as the Flores Islands (Indonesia) Tsunami in 1992 [5] and the Indian Ocean tsunami in 2004 [6]. Li et al. [7] applied COMCOT to study tsunami propagation in the South China Sea and obtained good results. Ying et al. [8] used this model to verify the Japan 3/11 tsunami and proved its applicability in the East China Sea.

### 2.2. Astronomical tide model establishment and verification

The COMCOT model uses a longwave shallow water equation, which can also be used to describe the movement of tidal waves. In this paper, tidal boundary conditions are added to the COMCOT model to simulate the East China Sea tide. The tidal boundary is obtained from TPXO7. The tide is simulated using the amplitudes and phase shifts of ten major tidal constituents ( $M_2$ ,  $S_2$ ,  $K_1$ ,  $O_1$ ,  $N_2$ ,  $P_1$ ,  $K_2$ ,  $Q_1$ ,  $M_f$ , and  $M_m$ ).

The astronomical tide calculation encompasses approximately 15°38'–41°47'N and 111°56'–135°32'E. The calculation grid is divided into four layers. From the first layer to the fourth layer, the grid scales are, 2°, 1800 m, 600 m, and 200 m, respectively (Figure 1). ETOPO1 data are used for the topographic data in the top-level grid. For the remaining grids, topographic data are obtained via interpolation between sea charts and measured data.

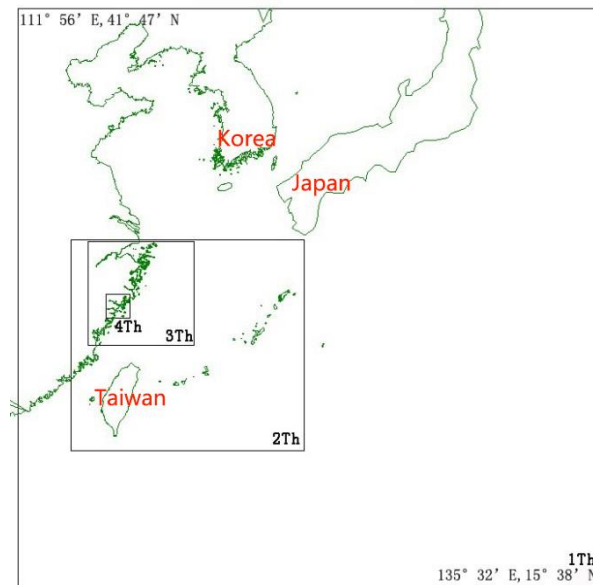


Figure 1. The layout of computational grids for the analysis of astronomical tides

The astronomical tidal model was verified with measured tide-level data for Dongtou and Nanji (Wenzhou Bay) in April 2013. For later analyses, a monitoring point, A ( $27^{\circ}45'58.38''\text{N}$ ,  $120^{\circ}57'49.73''\text{E}$ ), is utilized (Figure 2).



Figure 2. The locations of tide stations and monitoring points

Based on the tidal verification chart (Figure 3), it can be seen that the calculated tidal level coincides well with the measured tidal level in both phase and magnitude, which proves that the COMCOT model coupled with a tidal boundary can be applied to the simulation of astronomical tidal waves.

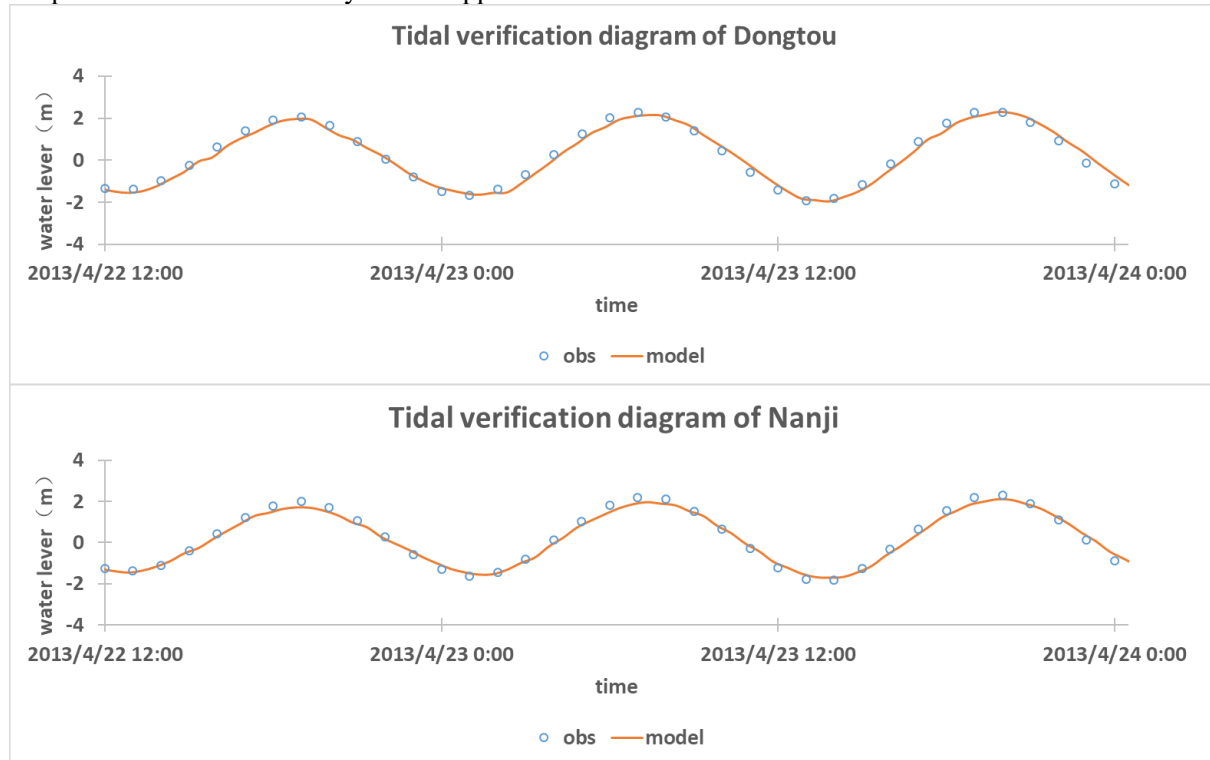


Figure 3. Verification of astronomical tides

### 3. Results and Discussions

#### 3.1. Tsunami source design

To enable tsunami waves to spread over the Wenzhou sea area and to not affect the astronomical boundary, the tsunami source is designed inside the Ryukyu trench region. Two different tsunami waves, positive leading waves and negative leading waves, were designed according to the dislocated direction of earthquake plates during an undersea earthquake with magnitude of 7.6. The designed source parameters are shown in Table 1. The tsunami wave is calculated by using the grid shown in Figure 1. The initial tsunami fluctuations and the maximum water surface that result from the positive leading wave condition are shown in Figure 4, while the initial tsunami fluctuations and maximum water surface that result from the negative leading wave condition are shown in Figure 5.

The water surface fluctuation process that extracts monitoring point A (Figure 2) during the tsunami is shown in Figure 6. As seen from the figure, under the negative leading wave condition, the water level at point A slightly decreases at the beginning of the tsunami wave event, then increase rapidly. The maximum water increase at point A is 0.23m, which occurs 3h and 50 min after the earthquake. Under the positive leading wave condition, the water level at point A increase slightly at the beginning of the tsunami wave event, then increases rapidly after the shock wave. The maximum water increase at point A is 0.22m, which occurs 4h and 15 min after the earthquake.

Table 1. The parameters of the designed tsunami source

Parameters	Positive leading wave	Negative leading wave
Epicentre (LON) (°)	124.544	124.544
Epicentre (LAT)(°)	24.8397	24.8397
Focal depth (km)	19.55	19.55
Length of Fault Plane (km)	100	100
Width of Fault Plane (km)	10	10
Dip angle (°)	17.16	17.16
Strike direction (°)	228.73	228.73
Rake (slip) angle (°)	90	90
Dislocation (m)	-10	10

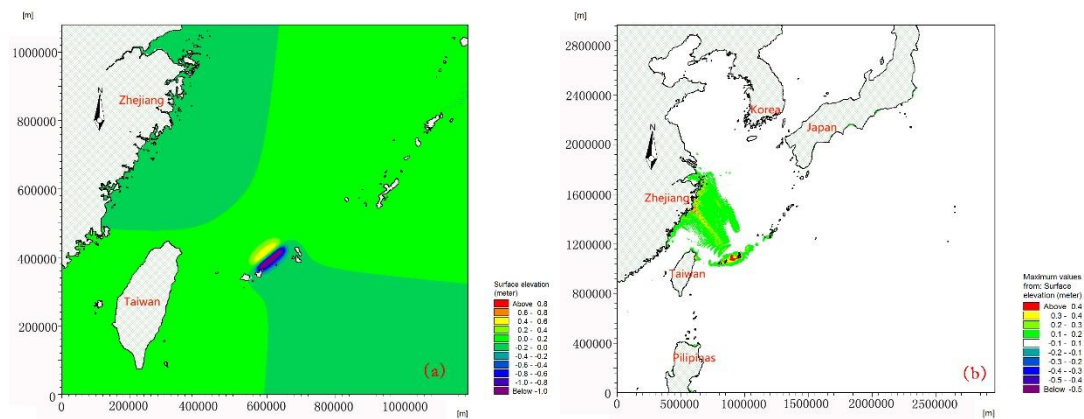


Figure 4. The initial water level (a) and maximum spin-up (b) of the designed tsunami with a positive leading wave

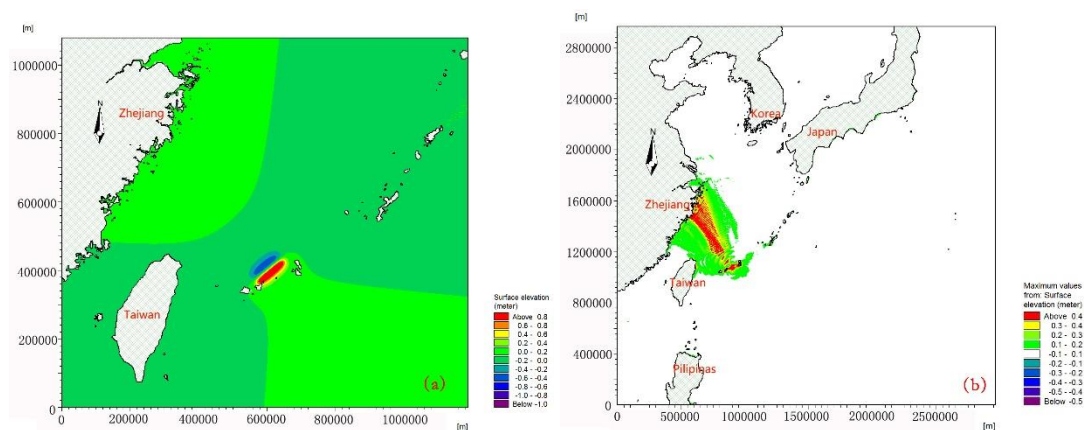


Figure 5. The initial water level (a) and maximum spin-up (b) of the designed tsunami with a negative leading wave

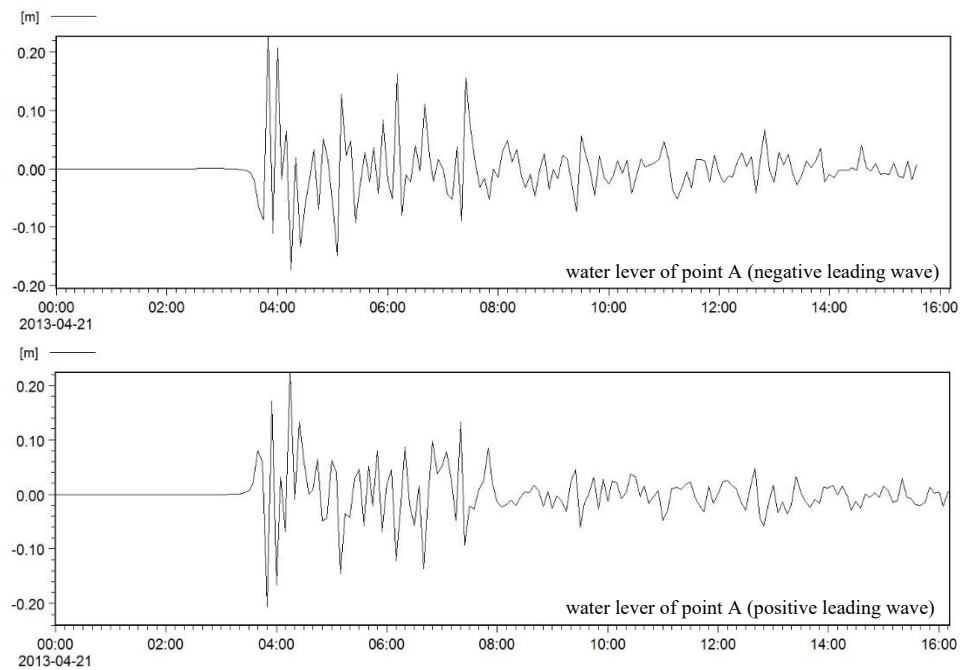


Figure 6. Tide-level time series at monitoring point A under tsunami condition

### 3.2. Coupling astronomical tides and tsunamis

After a spin-up time of 15 days, tsunami waves were added to the model. By adjusting the tsunami wave occurrence time, the tsunami wave peak was synchronized with the astronomical tide peak in Wenzhou Bay. A water level contour plot of the positive leading wave condition is shown in figure 7, while a water level contour plot of the negative leading wave condition is shown in figure 8. It can be seen from the figure that tsunami waves disrupt the original astronomical tidal system and form a long and narrow shock wave, which is propagated over the nearshore region.



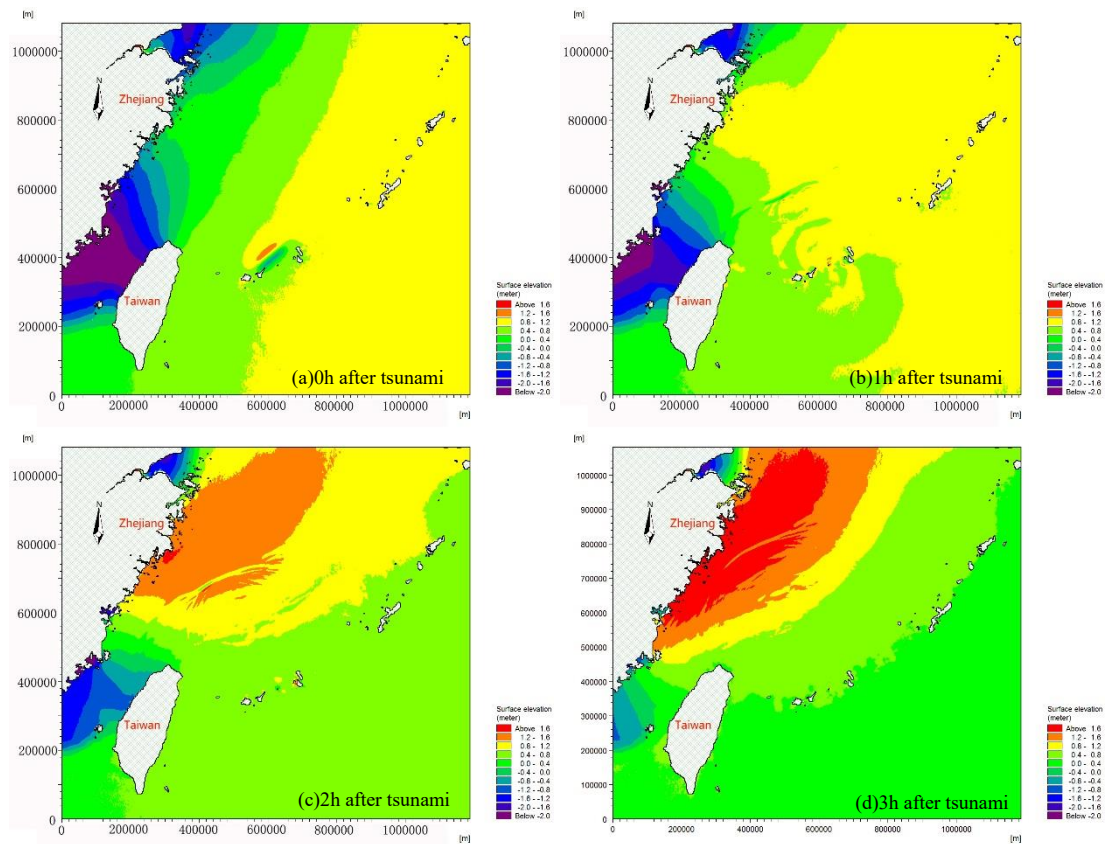


Figure 7. The Water level contour plot of the coupled astronomical tide and tsunami model (positive leading wave)

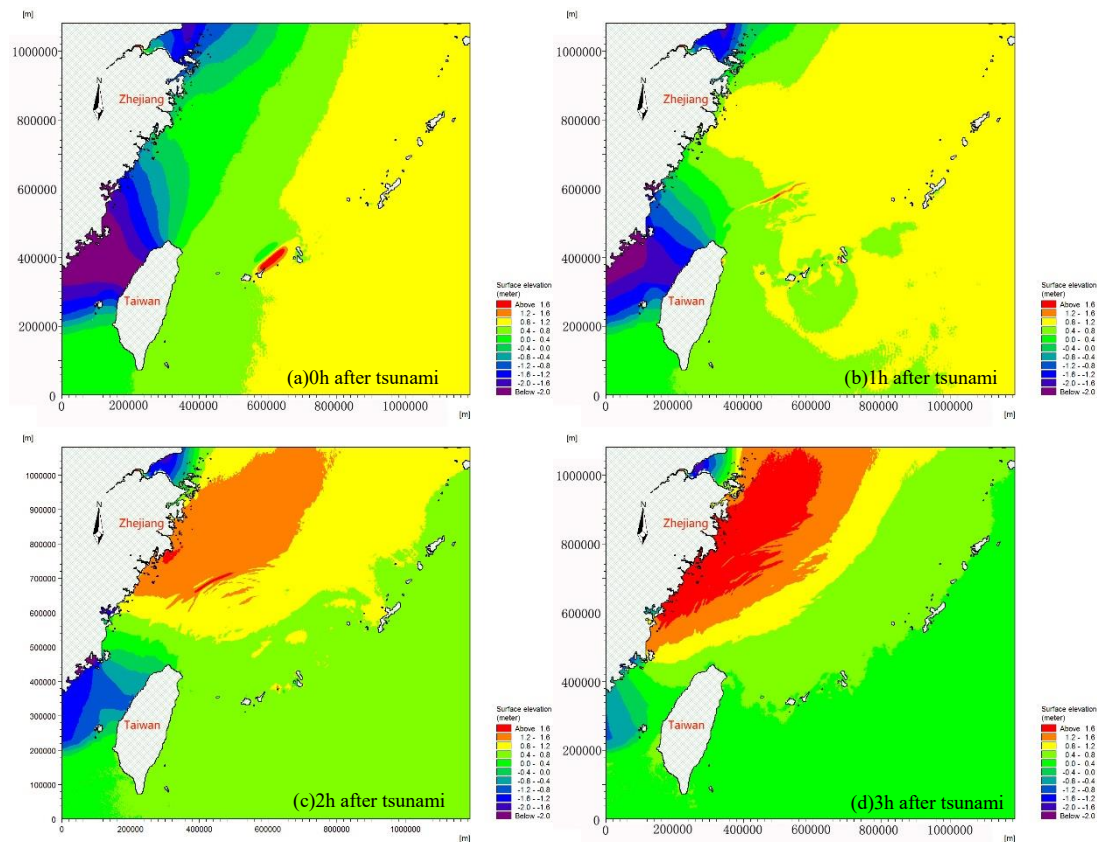
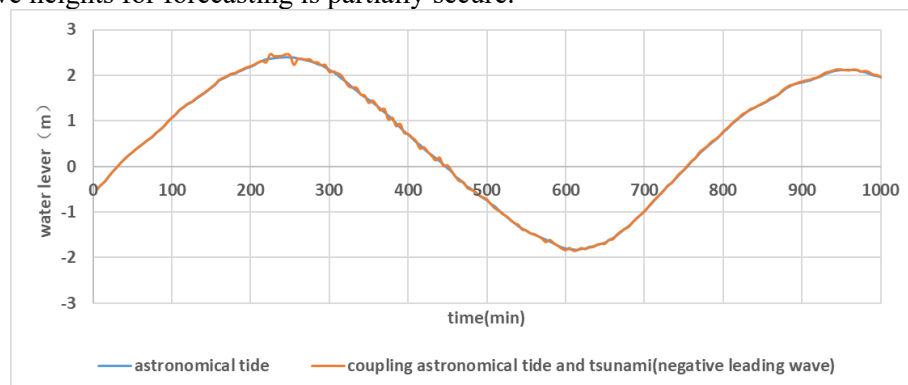


Figure 8. The Water level contour plot of the coupled astronomical tide and tsunami model (negative leading wave)

The tide-level time series at monitoring point A is shown in figure 9. As seen from the figure, the astronomical tide curve produces high-frequency, small fluctuations due to the impact of tsunami waves. The arrival time of the tsunami wave is approximately 10 min in advance when the coupled tsunami wave and astronomical tide results are compared to those from the linear superposition. The results are consistent with the theoretical analysis. The speed of a tsunami wave is approximately. In the coupling calculation, the total water depth,  $h$ , is greater than that in the linear superposition condition. Therefore, the tsunami propagation speed increases, and the propagation time becomes shorter. Judging by the tsunami wave height, the result of the linear superposition is larger than that of the coupled calculation result, with a maximum deviation of approximately 12 cm. This shows that the linear superposition of tsunami wave heights for forecasting is partially secure.





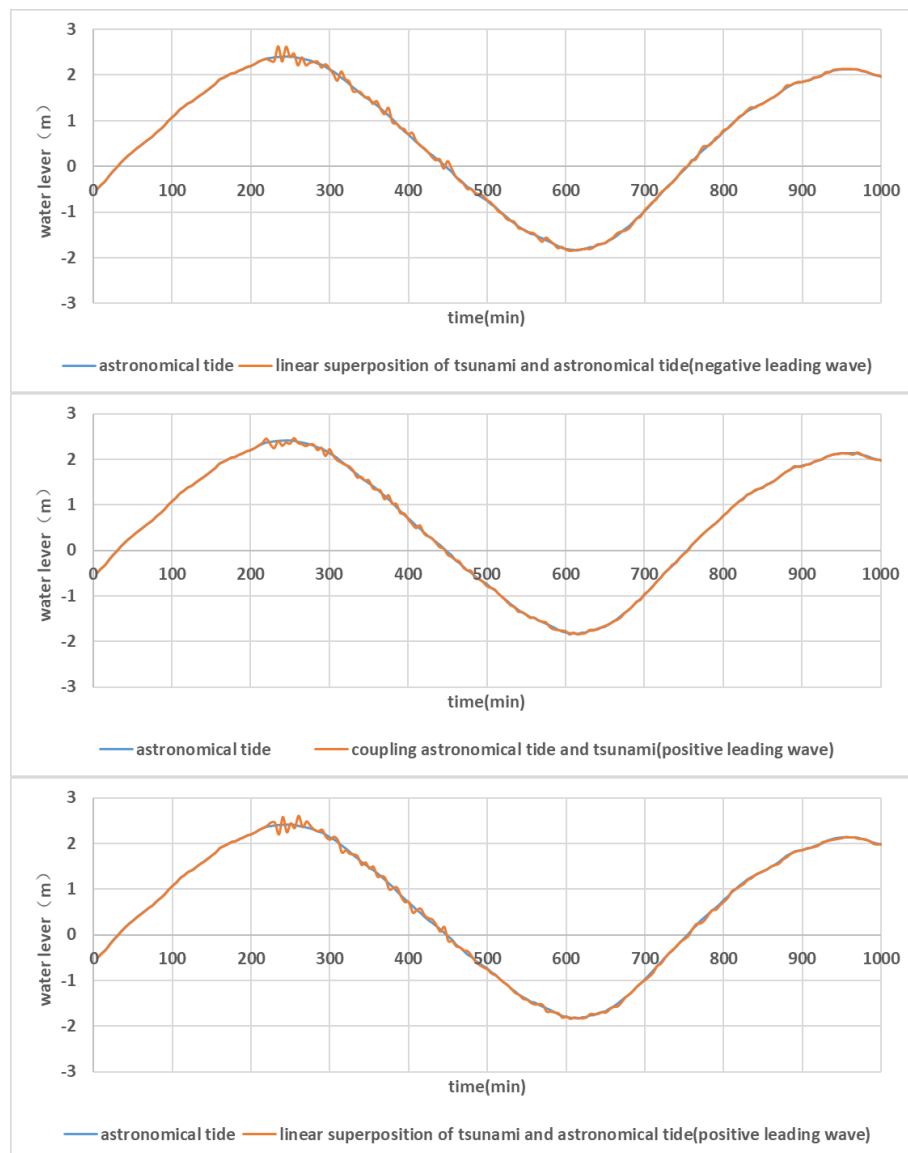


Figure 9. Tide-level time series at monitoring point A

#### 4. Conclusion

In this paper, tidal boundary conditions are added into a mathematical tsunami model (COMCOT), and a coupled mathematical model using astronomical tides and tsunamis in the East China Sea is established. Two different tsunami waves (positive leading waves and negative leading waves) are designed according to the dislocated direction of earthquake plates during an undersea earthquake with magnitude of 7.6, which occurs in the Ryukyu Trench. A comparison regarding the summation of astronomical tide levels and tsunami wave levels was made between the result of the coupling model and that of the linear superposition. The results demonstrated that tsunami waves occur in advance of astronomical tides in both of the mentioned cases, regardless of whether the leading wave is positive or negative. In addition, the result from the linear superposition is more secure with respect to the tsunami wave height.

#### Acknowledgments

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