

A New Method for Calculating the Critical Force of Irregular Frames --The Axial Force Area Ratio Method

Shu Wei LAN, Dong Hua ZHOU

School of Civil Engineering, Kunming University of Science and Technology,
Kunming, Yunnan650000, China

Email: lanshuwei2000@163.com

Abstract: Based on the activation degree of the frame column stiffness, this paper studies the influence of the axial force area on the critical bearing capacity of the irregular frame and deduce some rules. Based on these rules, this paper corrects the critical bearing capacity of the regular frame and establishes the critical bearing capacity calculation method of the irregular frame. The axial force area ratio method can make up for the current codes can only calculate the critical force of the regular frame, which has great value in engineering application. Compared with the results of ANSYS, the axial force area ratio method has good precision. This method can consider the interaction between columns of the same story and the inter-story interaction.

1. Introduction

The difference between the irregular and the regular framework is that the axial force of the same layer's column is different, and the axial force of the upper and lower columns is different, which leads to the difference between the critical force of the irregular frame and the regular frame. The traditional computational length coefficient method is obtained under the idealized assumption, and it is inevitable for the irregular frame to calculate the critical force. Therefore, the method of improving the precision of the length coefficient is continuously studied. There is a mutual support between the columns of the same column, and there is already a formula to consider the mutual support of the same layer to correct the calculation length coefficient [1]. This topic of the different column axial force causing the inter-story interaction has been studied by many researchers[2 3 4]. Professor Liang Qizhi [6] proposed the cumulative algorithm to consider the interaction between layers and layers in order to obtain the calculated length coefficient of the weak layer. Professor Tong Genshu [7 9] proposed the consideration of the interaction between layers and layers using the concept of axial stiffness obtaining the calculation length coefficient. Although the above study is more accurate to solve the establishment of the critical force of the frame and the stability equation has been improved, but still need to be iterative calculation, or multiple equations to solve, and therefore cannot be widely used. In this paper, the axial force distribution of the frame is different. The influence of the axial force area on the critical force is studied, and some rules are found from the activation degree of the column stiffness, and the corresponding calculation method is established. The method to calculate the critical bearing capacity of the irregular frame is obtained by correcting the critical bearing capacity of the regular frame.

2. Upper Top Load Frame System

There is little difference between the critical forces of the frame of the variable load. The frame system



can be regarded as the activation of the stiffness, and the load is applied only to the upper top of the frame, that is, the upper and lower columns of the frame have the same axial force, as shown in Figure1.

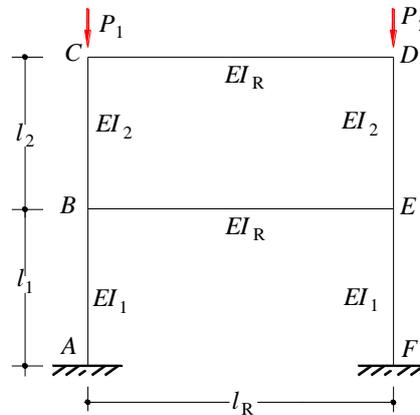


Figure1 Frame system diagram.

The basic assumption: the stiffness of the regular frame = the stiffness of the irregular frame, that is, the axial force area of the regular frame = the axial force area of the irregular frame:

$$2N(l_1 + l_2) = (N_1 + N_2)(l_1 + l_2) \tag{2.1}$$

$$2P_{cr}(l_1 + l_2) = (P_{cr1} + P_{cr2})(l_1 + l_2) \tag{2.2}$$

$$P_{cr1} = \frac{2P_{cr}}{1 + \frac{P_2}{P_1}}, P_{cr2} = \frac{2P_{cr}}{1 + \frac{P_1}{P_2}} \tag{2.3}$$

In the above formula, P_1 : the point load of the left column, P_2 : the point load of the right column, P_{cr1} : the critical load of the left column, P_{cr2} : the critical load of the right column, P_{cr} : the critical load of the regular frame[9].

The following example can verify the correctness of the formula 2.3. Assuming a single span two-layer frame, the lower part is fixed, as shown in figure1. Steel elastic modulus $E_s = 2.1 \times 10^4 \text{KN/cm}^2$, Poisson's ratio $\mu = 0.3$. Columns and beams are international hot rolled H-section steel. Column section is HM400x300x10x16, $I = 38900 \text{cm}^4$, $A = 136.7 \text{cm}^2$, $l_1 = 5 \text{m}$, $l_2 = 3 \text{m}$; Beam section is HM450x300x11x18, $I = 56100 \text{cm}^4$, $A = 157.4 \text{cm}^2$, $l_R = 6 \text{m}$. Changing the load of the frame, as the following table 1, calculate the critical load on the left and right columns.

Table 1 Critical load calculation correctness check

Frame type	P_2 / P_1	① The Axial Force		② ANSYS		error%	
		Area Ratio method		P_{cr1}^*	P_{cr2}^*	①-② / ②	
		P_{cr1}	P_{cr2}	P_{cr1}^*	P_{cr2}^*		
Regular	2.0/2.0	31711	31711	31902	31902	0.599	0.599
Irregular 1	1.8/0.2	57080	6342	56918	6324	0.285	0.285

Irregular 2	1.6/0.4	50738	12684	50790	12698	0.102	0.110
Irregular 3	1.5/0.5	47566	15856	47690	15897	0.260	0.258
Irregular 4	1.4/0.6	44395	19027	44565	19099	0.381	0.377
Irregular 5	1.2/0.8	38053	25369	38262	25508	0.546	0.545
Irregular 6	1.0/1.0	31711	31711	31903	31903	0.602	0.602
Irregular 7	0.8/1.2	25369	38053	25508	38262	0.545	0.546
Irregular 8	0.6/1.4	19027	44395	19099	44565	0.377	0.381
Irregular 9	0.5/1.5	15856	47566	15897	47690	0.258	0.260
Irregular 10	0.4/1.6	12684	50738	12698	50790	0.110	0.102
Irregular 11	0.2/1.8	6342	57080	6324	56918	0.285	0.285

According to the above table 1, changing frame load ratio from irregular frame 1 to irregular frame 11, the critical load of the frame calculated by the axial force ratio method is less than 1% in the ANSYS software results. It is proved that the calculation of the axial force area ratio has high precision, and the axial force area ratio method is the accuracy of the mutual support of the same story. The derivation process of the bottom hinged double frame is the same as that of the bottom fixed frame. The axial force area ratio of the multi-layer irregular frame is similar to that.

3. Upper and Lower Top Load Frame System

The sum of the critical force of the frame columns by changing upper and lower column load ratio is very different [10]. Such a system can be regarded as that the stiffness is not fully activated. The load acts on the upper and lower top of the frame, that is, the upper and lower columns have different axial force. In this case, a large part of the lateral displacement of the upper column is from the rigid body rotation of the lower column. The stiffness of the upper column is not fully activated, as shown in Figure2.

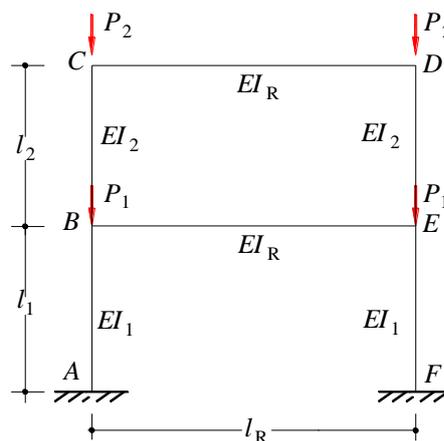


Figure2 Frame system diagram.

According to this rule, the regular frame axial force area is equal to the axial force of the irregular frame area, calculating the critical load of the irregular frame is quite different from the regular frame calculation result. In this case, the calculation assumptions need to be adjusted, the critical force area of the regular frame multiplied by the stiffness reduction factor φ .

The basic assumption: the stiffness of the regular frame* the stiffness reduction factor φ = the stiffness of the irregular frame, that is, the axial force area of the regular frame * the reduction factor

φ = the axial force area of the irregular frame:

$$2N(l_1 + l_2) \cdot \varphi = 2(N_1l_1 + N_2l_2) \tag{3.1}$$

$$P_{cr}(l_1 + l_2) \cdot \varphi = P_{cr1}l_1 + P_{cr2}(l_1 + l_2) \tag{3.2}$$

$$P_{cr1} = \frac{P_{cr}(l_1 + l_2) \cdot \varphi}{\frac{P_2}{P_1}(l_1 + l_2) + l_1}, P_{cr2} = \frac{P_{cr}(l_1 + l_2) \cdot \varphi}{(1 + \frac{P_2}{P_1})l_1 + l_2} \tag{3.3}$$

In the above formula, P_1 : the point load of the lower column, P_2 : the point load of the upper column, P_{cr1} : the critical load of the lower column, P_{cr2} : the critical load of the upper column, P_{cr} : the critical load of the regular frame[9].

The axial force distribution of the frame is different. The uneven distribution of the axial force of the frame is mainly reflected in the difference of the axial force area. In this case, the upper frame column stiffness is not fully activated, According to Figure 3, the proportion of the column axial force area is reflected as the proportion of column stiffness activated. This paper can obtain the reduction factor φ :

$$\varphi = \frac{1}{l_1 + l_2} \left(\frac{P_2}{2} l_2 + l_1 \right) \tag{3.4}$$

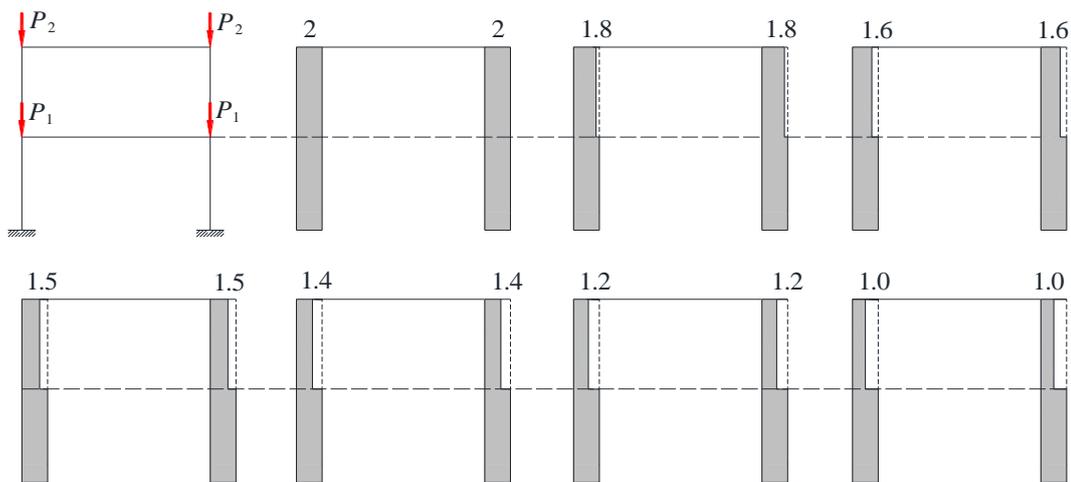


Figure3 Stiffness activation diagram.

The following example can verify the correctness of the formula 3.3 and the formula 3.4. Assuming a single span two-layer frame, the lower part is pinned, as shown in figure 1(b). Steel elastic modulus $E_s = 2.1 \times 10^4 \text{KN} / \text{cm}^2$, Poisson's ratio $\mu = 0.3$. Columns and beams are international hot rolled H-section steel. Column section is HM400x300x10x16, $I = 38900 \text{cm}^4$, $A = 136.7 \text{cm}^2$, $I_1 = 6\text{m}$, $I_2 = 3\text{m}$; Beam section is HM450x300x11x18, $I = 56100 \text{cm}^4$, $A = 157.4 \text{cm}^2$, $I_r = 6\text{m}$. Changing the load of the frame, as the following table 1, calculate the critical load on the left and right columns.

Table 2 Critical load calculation correctness check

Frame type	P_2 / P_1	① The Axial Force Area Ratio method	② ANSYS	error%
------------	-------------	-------------------------------------	---------	--------

		P_{cr1}	P_{cr2}	φ	P_{cr1}^*	P_{cr2}^*	①-② / ②
Regular	2.0/0.0	4582	0	1.000	4590	0	0.196 0.000
Irregular 1	1.8/0.2	4131	459	0.957	4135	459	0.174 0.000
Irregular 2	1.6/0.4	3672	918	0.914	3679	920	0.097 0.217
Irregular 3	1.5/0.5	3443	1147	0.893	3450	1150	0.190 0.261
Irregular 4	1.4/0.6	3213	1377	0.871	3222	1381	0.203 0.290
Irregular 5	1.2/0.8	2754	1836	0.829	2764	1842	0.279 0.326
Irregular 6	1.0/1.0	2295	2295	0.786	2305	2305	0.362 0.434
Irregular 7	0.8/1.2	1836	2754	0.743	1846	2768	0.434 0.506
Irregular 8	0.6/1.4	1377	3213	0.700	1385	3232	0.542 0.588
Irregular 9	0.5/1.5	1147	3443	0.679	1155	3465	0.578 0.635
Irregular10	0.4/1.6	918	3672	0.657	924	3697	0.693 0.676
Irregular 11	0.2/1.8	459	4131	0.614	463	4163	0.649 0.769
Irregular12	0.0/2.0	0	4590	0.571	0	4629	0.000 0.842

According to the above table 2, changing frame load ratio from irregular frame 1 to irregular frame 11, the critical load of the frame calculated by the axial force ratio method is less than 1% in the ANSYS software results. It is proved that the calculation of the axial force area ratio has high precision, and the axial force area ratio method is the accuracy of the mutual support of the inter-story. The derivation process of the bottom fixed double frame is the same as that of the bottom hinged frame. The axial force area ratio of the multi-layer irregular frame is similar to that.

4. Conclusion

(1) The different load modes of the frame load have a great influence on the bearing capacity of the lateral displacement frame column, which is manifested as the degree of activation of the column stiffness, and the reduction of the activation stiffness can be reflected in the decrease in the axial force area. The axial area ratio method by analyzing the activation degree of stiffness of the frame column, only need to obtain the critical load of the regular frame, this paper can use the axial force ratio method to push the simplified formula and calculate the critical load of the irregular frame. Compared with the traditional calculation length coefficient method, the axial area ratio method is more simple and quick, which can effectively solve the mutual support of the same layer and the mutual support of the inter-story.

(2) The axial force area ratio method can calculate the irregular frame with different axial forces in the left and right columns and calculate the irregular frame with different axial forces in the upper and lower columns. The critical load of the frame calculated by the axial force ratio method is less than 1% in the ANSYS software results, which is proved that the axial force area ratio method has high precision and great value in engineering application. The axial area ratio method can also be applied to multi-layer frame, the derivation process is similar.

Acknowledgments

This work was financially supported by the National Natural Science Foundation (51668027 and 51468026)

References

- [1] Salem, A. H. Discussion of “Buckling Analysis of One-Story Frames” by A. Zweig and H. Kahn, Journal of the Structural Division. ASCE, Vol. 95, No. ST5, May, 1969
- [2] Bridge, R. Q. and Fraser, D. J. (1986). “Improved G-factor method for evaluating effective

- lengths of columns.” J. Struct. Engrg. ASCE,113(6),1361-1356.
- [3] Hellesland, J. and Bjorhovde, R. (1996). "Restraint demand factors and effective lengths of braced columns.” J. Struct.Engrg. ASCE,122(10),1216-1224.
 - [4] Hellesland, J. and Bjorhovde, R. (1996). “Improved Frame Stability Analysis with effective lengths.”J.Struct.Engrg.ASCE,122(1),1275-1283.
 - [5] Duan, L. and Chen, W. F. (1989). “Effective length factor for columns in unbraced columns.” J. Struct. Engrg. ASCE,115(1),149-165.
 - [6] Liang Q. Tall building structure analysis and design [M], South China University of Technology Press, 1992
 - [7] Tong G. Stability of single span and two story frame[J]. Steel Structures, 2001 (9): 5-16
 - [8] Tong G, Shi Z. The physical meaning of calculating the length coefficient and the evaluation of various steel frame stability design methods [J]. Construction Steel Structures, 2004 (4)
 - [9] Wang J, Dong G. Consider the stability analysis of the inter-story interaction[D]. Zhejiang University Master 's Thesis, 2003:12-20
 - [10] Yang X, Zhou D. The practical calculation of the stability of the two story frame[D]. Kunming University of Science and Technology Master 's degree thesis, 2017:63-67
 - [11] GB. 50010-2003, Code for design of steel structures[S]. Beijing: China Architecture & Building Press, 2003.