

Study and Application of Structure Failure Simulation of Fracture Tracing by Manifold Method

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Abstract. By improving the simulation method of contact and contact forces, the function of tracing the propagation of Multi-cracks, the Numerical Manifold Method proposed by Genhua-Shi is developed which can simulate the discontinuity of block system, the tensile or shear failure, and the crack propagation. The least-square method to calculate SIF is presented in this paper for broken cracks. Also the calculation method of contact stiffness matrix is modified. For edge-to-edge contact, correct distribution of contact stress is guaranteed, namely the contact stiffness and contact length make direct proportion. The structure failure problems inducing by crack propagation in different engineering domains are simulated. It shows that the method put forward and the program developed in this paper can effectively simulate the structure failure and can give scientific suggestion for engineering supporting.

1. Introduction

The structure failures such as slope instability, ship lock and dam failure have brought large loss to the life and properties. To simulate the failure, the numerical method not only can simulate discontinuities such as joints and cracks, the large deformation after the failure of rock body but also can simulate the failure of intact rock mass, which cannot be solved well by methods based on discontinuous mechanics such as DEM and DDA or methods based on continuous mechanics such as FEM. Numerical Manifold Method (NMM) is a new numerical method proposed by Genhua Shi ^[1] in the early 1990s. This method takes the advantages of continuous method and discontinuous method. By using the concept of ‘Cover’ and two sets of grid (math grid and physical grid), not only can it calculate the deformation and stress inside blocks as Finite Element Method do, but also can it simulate block systems as the DDA ^[1] does. Guoxin Zhang ^[2-7] greatly extended the NMM of Genhua Shi, adopted the second-order Manifold Method, and the fracture mechanics algorithm is added to trace the propagation of cracks. The extended NMM not only can simulate multi-cracks propagation but can simulate the process of tension and shearing failure, as well as coupling analysis of seepage and deformation.

In this paper, based on the former study, the calculation method of stress intensity factor for broken cracks is studied further. The Singular Boundary Element Method can calculate the stress intensity factor is presented, but the error is brought when the SIF calculation of broken cracks. The least-square method to calculate SIF is presented in this paper for above problem, the crack integration method is also studied, and the calculation methods and processes of SIF of broken cracks are presented.

Also the crack tracing technic of NMM is illustrated, which include that the calculation of singular field at the crack tip, the criterion of crack propagation and propagation direction, the remeshing of physical meshes and mathematical meshes because of the crack propagation, the energy conservation after the element failure, the tracing and simulation methods of multi-cracks and the law of crack



propagation, coalescence and join each other. Last, by the extended Manifold method, some failure problems are simulated in this paper.

2. The crack tracing technique of Numerical Manifold Method

Multi-cracks often appear at the experiment of mechanical property of concrete, so the simulation of multi-cracks is very importance. This paper introduces a simulation method using singular boundary element method to calculate the stress intensity factor of the crack.

The manifold Method in this paper recurs to the fracture mechanics method combining with the singular boundary element method, and simulates the process of failure. This method can solve that the propagation of crack along arbitrary directions and the tracing of multi-cracks. Specific crack tracing technology can be divided as follows: (1) Trace all crack tips simultaneously, judge whether some crack tips meet crack criterion or not, and if it is, calculate the crack safety coefficient. (2) The smallest safety coefficient of crack is allowed to propagate. (3) If there is no crack propagate, search all the nodes, judge whether some nodes satisfy the new crack appearing criterion or not. (4) The nodes that satisfy the criterion and have a minimum safety factor is allowed to generate new crack. Repeat the above steps, and carry on the next circulation.

3. Calculation SIF by Least-Square Method

The least-square method to calculate SIF is presented. That is, simulate the stresses at a series of points of Crack Tip can use the least squares to find the appropriate simulation function, and by substitution of the function into the theoretical formula can solve the stresses of the crack tip, then find its extreme value to solve the SIF of Crack Tip. The specific method is as follows:

First, Change the coordinates to the local coordinate system of the crack, the stress of a series of points on the extension line of crack tip can be obtained by the above method, Secondly, the normal and tangential forces of a series of points on the crack extension line under the local coordinates of the crack are obtained by transforming the coordinates to the local coordinate system of the crack.

According to the Fracture Mechanics, the point coordinates of the crack extension is $\theta = 0^\circ$, and the crack tip stress field σ in I - II mixed mode is:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}, \quad \tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \quad (1)$$

The stress intensity factor of crack tip can be obtained separately,

$$K_I = \lim_{r \rightarrow a} \sqrt{2\pi r} \sigma_y, \quad K_{II} = \lim_{r \rightarrow a} \sqrt{2\pi r} \tau_{xy} \quad (2)$$

There is singularity of crack tip stress, as shown in Figure 1.

According to the form of stress in Figure 1, the simulation function is taken as follows:

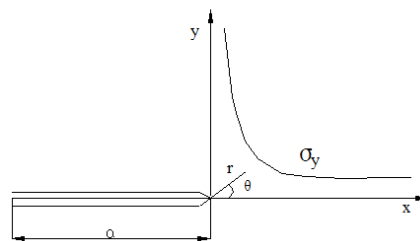


Figure 1 Stress distribution at crack tip

$$\sigma_y = \frac{m}{\sqrt{r}} \quad (3)$$

By the least-square method, we can get:

$$I = \left[\sum_{i=1}^n \sigma_{y_i} - \sum_{i=1}^n \left(\frac{m}{\sqrt{r_i}} \right) \right]^2 = 0 \quad (4)$$

Where n is the number of known points on the crack extension line.

For the minimum value:

$$\frac{\partial I}{\partial m} = 0$$

We can get:

$$\begin{aligned} 2* \left[\sum_{i=1}^n \sigma_{y_i} - \left(\sum_{i=1}^n \frac{m}{\sqrt{r_i}} \right) \right] * \left(-\frac{1}{\sqrt{r_i}} \right) &= 0, \quad -\sum_{i=1}^n \frac{\sigma_{y_i}}{\sqrt{r_i}} + \sum_{i=1}^n \frac{m}{r_i} = 0 \\ m &= \frac{\sum_{i=1}^n \frac{\sigma_{y_i}}{\sqrt{r_i}}}{\sum_{i=1}^n \frac{1}{r_i}} \end{aligned} \quad (5)$$

$$K_1 = \lim_{r \rightarrow a} \sqrt{2\pi r} \sigma_y = \lim_{r \rightarrow a} \sqrt{2\pi r} (m/\sqrt{r}) = \sqrt{2\pi} m \quad (6)$$

So the stress intensity factor K_I of type I crack can be obtained. For stress intensity factor of type II.

$$K_{\Pi} = \lim_{r \rightarrow a} \sqrt{2\pi r \tau_{xy}} \quad (7)$$

$$\tau_{xy} = \frac{m}{\sqrt{r}}$$

The same order,

Using the least squares simulation, we can get:

$$K_{\Pi} = \lim_{r \rightarrow a} \sqrt{2\pi r \tau_{xy}} = \lim_{r \rightarrow a} \sqrt{2\pi r} (m' / \sqrt{r}) = \sqrt{2\pi m'} \quad (8)$$

This method can solve the stress intensity factor of type I and type II, and mixed stress intensity factor of type I and II. And it only needs to calculate the stresses of the series points on the extension line, which is simple and practical. It can be widely used for the stress intensity factor calculation of the crack in multiple areas.

4. Simulation of bend-tension failure of rock slope

The bend-tension failure often happen on the front of slope, and the rock masses occur the cantilever bend accompanying with the tension cracks. The cracks propagate and parallel to the aspect at last. Taking the model in figure 2, simulate one simple example of bend-tension failure.

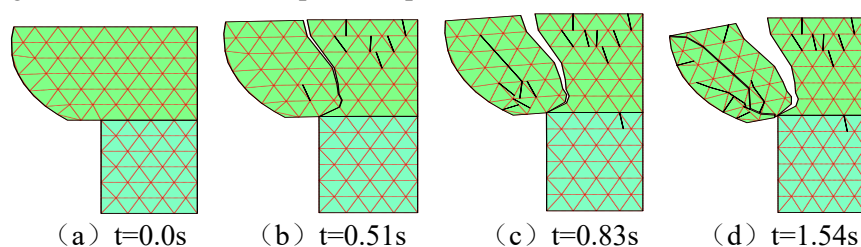
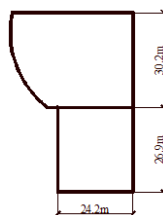


Figure2 Failure model

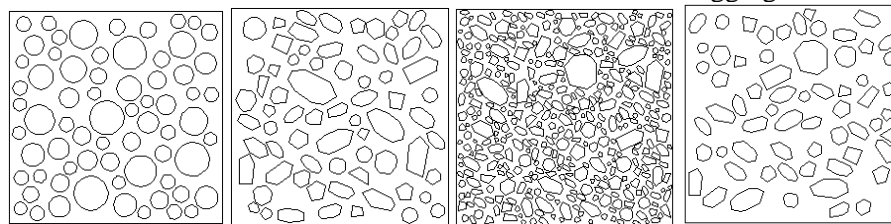
Figure3 Bend-tension failure of rock slope

The calculation parameters are, specific weight $P=24.5\text{kN/m}^3$, elastic modulus $E=1000\text{MPa}$, Poisson ratio $\nu=0.2$, Penalty= $98\,00000\text{kN/m}$, the maximal time step $\Delta t=0.01\text{s}$. The slope only bears weight in the first time, and along with the increasing of load at a fixed rate, we can see that the top block occur the bend-tension failure. The figure3 shows the failure state in the four different times.

5. Simulation of numerical test of concrete based on Numerical Manifold Method

5.1. Random Aggregate Model

A random aggregate model is got in Figure 4 based on Monte-Carlo method, and the profile is square with the size of $100\times 100\text{mm}$. The diameters of minimum and maximum aggregate are 5mm and 20mm .



(a) $R_{agg}=0.45, \gamma=0.25$ (b) $R_{agg}=0.45, \gamma=0.25$ (c) $R_{agg}=0.55, \gamma=0.35$ (d) $R_{agg}=0.35, \gamma=0.4$

Figure 4. Random Aggregate Model

5.2. Simulate of the failure process of concrete under uniaxial compression by NMM

In the simulation sample, the aggregate is reduced stone, the maximal diameter is 20mm , and compressive strength is 35MPa (uniaxial compressive strength).

On the top use the displacement control, the loading speed is 0.00005 m/s , and lower end is fixed in horizontal direction. The load model of concrete compression failure is shown in Figure 5. Figure 6 gives fracture models of concrete samples at different loading time.

Figure 6 shows the fracture process of concrete specimen at different loading time in the compression. The thread represents the interface is intact, and the thick line represents the interface has been fractured. At the beginning of load, the stress is small, aggregate interface is fractured for its low strength firstly, and then a few cracks appear in concrete. With the increasing of loading, interfaces are fractured and new cracks form persistently, in the end cracks begin to cut-through. When the peak load is got, as the result of the failure in the specimen, bearing capacity have already reached the terminal, once increasing external displacement, crack band come into being, as shown in figure 6 (c). Macroscopic crack along the loading direction basically, bypass aggregate and expand in the mortar matrix. Since then, cracks propagate steadily, new cracks also appear in the surround, until the last bearing capacity is lost and the specimen is fractured fully.

From figure 7 we can see that the stress-strain curve appears nonlinear obviously. When the stress reaches to the peak, the specimen is instability, and lost bearing capacity gradually, until the specimen is fractured completely. The concrete specimen's failure shape under uniaxial compression load can see Figure8. The failure mode and crack form that are got by NMM agree well with test results.

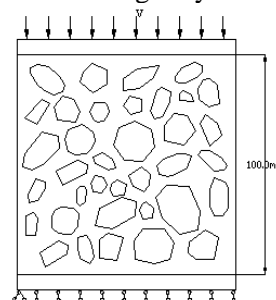


Figure 5 The loading model of concrete compression failure

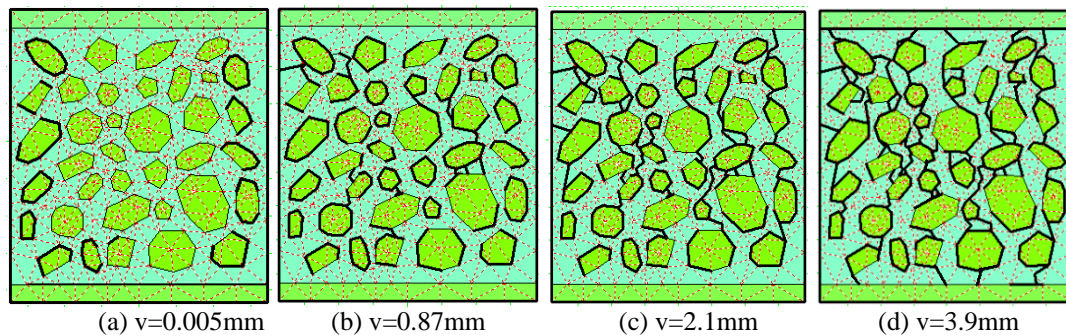


Figure 6 The fracture process of concrete samples in compression at different loading time

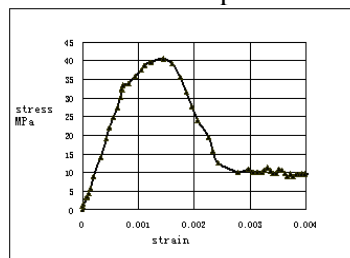


Figure 7 Stress-strain curve of concrete compression failure

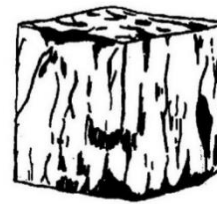


Figure 8 Specimen's failure shape by test

The simulation of failure process of concrete under uniaxial tension by NMM

The model in figure5 is used in this section, the tensile displacement $v=0.00005\text{m/s}$ applied to the top, and the interface tensile strength between aggregate and mortar is $R_e = 3.0\text{MPa}$. Second-Order Numerical Manifold Method is used to simulate the fracture process, and the simulation results of concrete can see Figure 9.

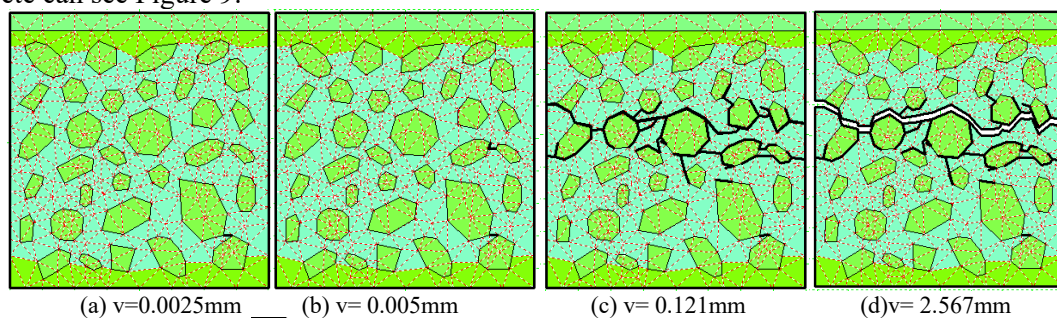


Figure 9 Fracture process of concrete uniaxial tensile failure

Figure9 show that the mainly are transverse cracks. Load-on initially, the interface between aggregate and mortar failure first, and then the cracks continue to propagate in the mortar matrix. The deformation localization is shown gradually, and at last, form one or more macro cracks that normal to the tensile load. Stress - strain curve shows significant nonlinear characteristics in Figure10. \boxtimes

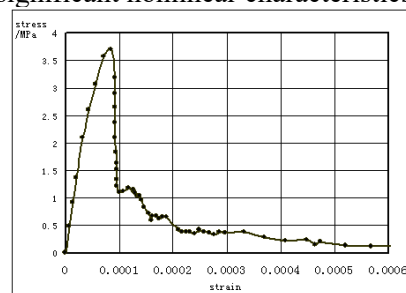


Figure 10 Stress - strain curve of concrete uniaxial tensile failure

At the mesoscopic level, the concrete is firstly assumed to be a multi-phase composite material composed of matrix and aggregates, and the interface between them is evaluated based on the test. The Monte Carlo method and Fuller Graded Formula are used to found the random aggregate model. The uniaxial compression test has been simulated of concrete in the mesoscopic level, based on the developed Second-Order Numerical Manifold Method. The failure mode and stress-strain curve were got. Compared with the test results, the failure mode and stress-strain curve are both correct. Thus, the difficulties existing in the numerical methods are solved. Analysis shows that the NMM has advantages and broad application prospects in simulating the mechanical property of concrete, especially for the failure process of concrete with various forces.

6. Conclusions

The extended second-order Manifold Method not only can simulate the failure course of structures from continuous to discontinuities, can simulate the contact and contact stresses, but can trace crack propagation successfully. The cracked section can automatically search and pre-setting of cracked section is not necessary, therefore it can accurately simulate those structure failure problems with highly discontinuous. From several case studies in this paper, it can be seen that NMM is objective and reasonable in simulating the failure mode of structure. It can solve problems that existing numerical methods cannot solve. NMM has advantages and broad application prospects in the simulating of the structure failure.

Acknowledgments

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